



$$\sum_{i=1}^n x_i z_i \leq Z$$

$x_i \in [0, 1]$



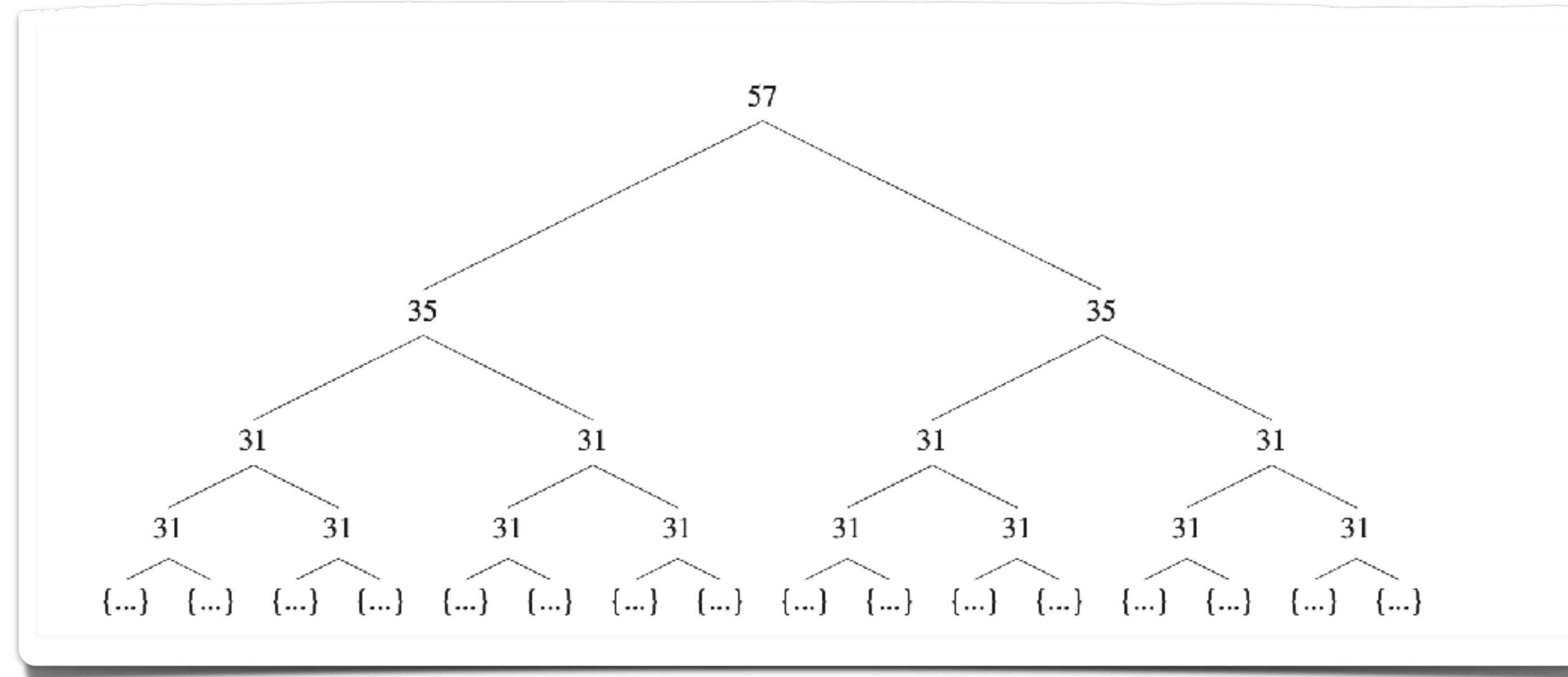
3 Branch-and-Bound

*Algorithmen und Datenstrukturen 2
Sommer 2022*

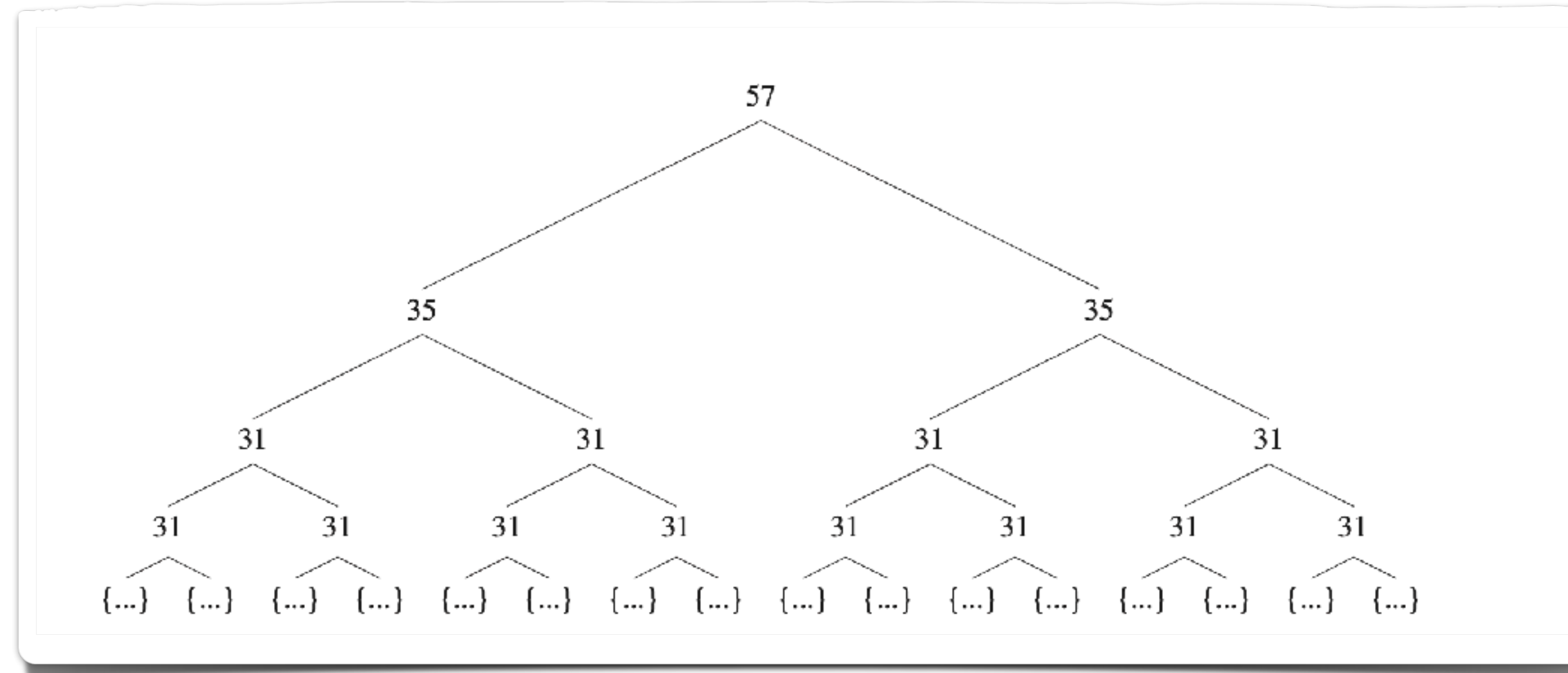
Prof. Dr. Sándor Fekete

3.1 Motivation

Enumerationsprinzip

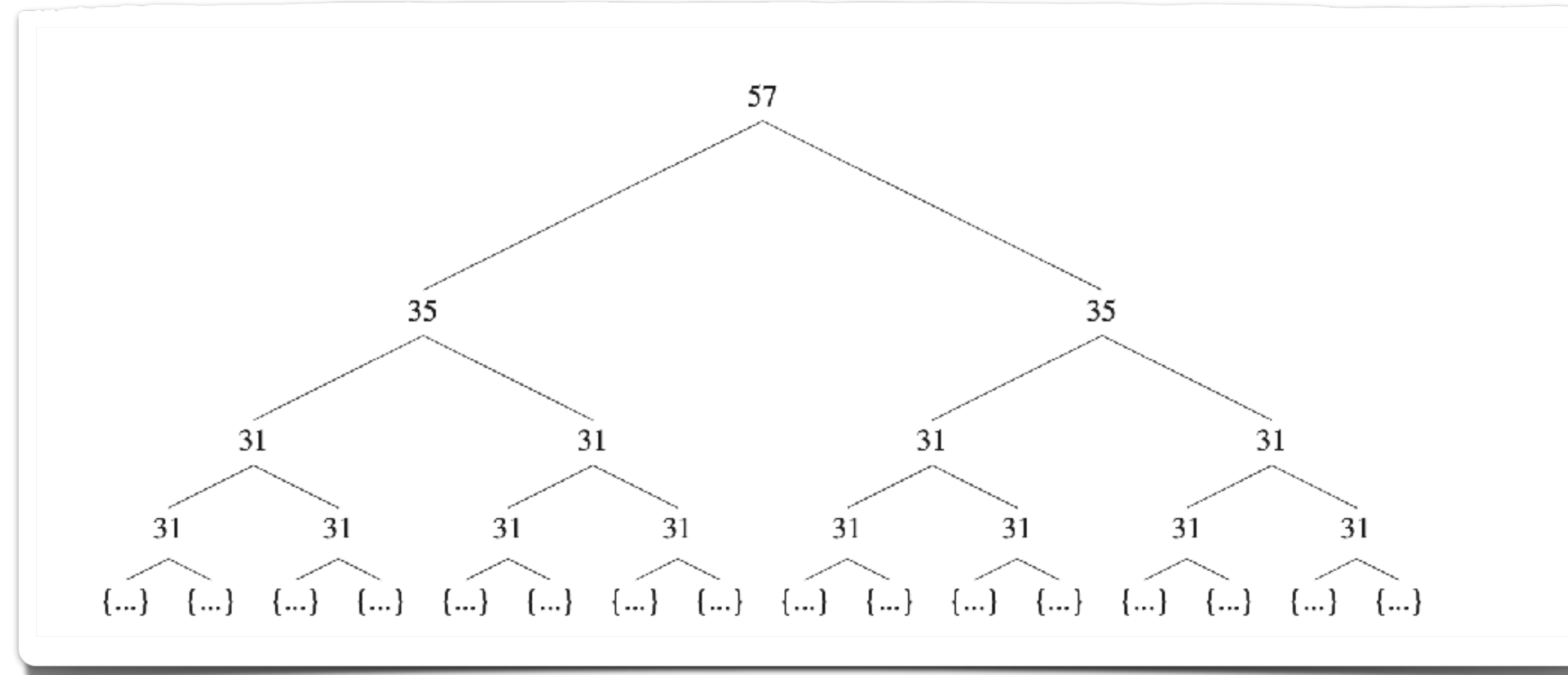


Enumerationsprinzip



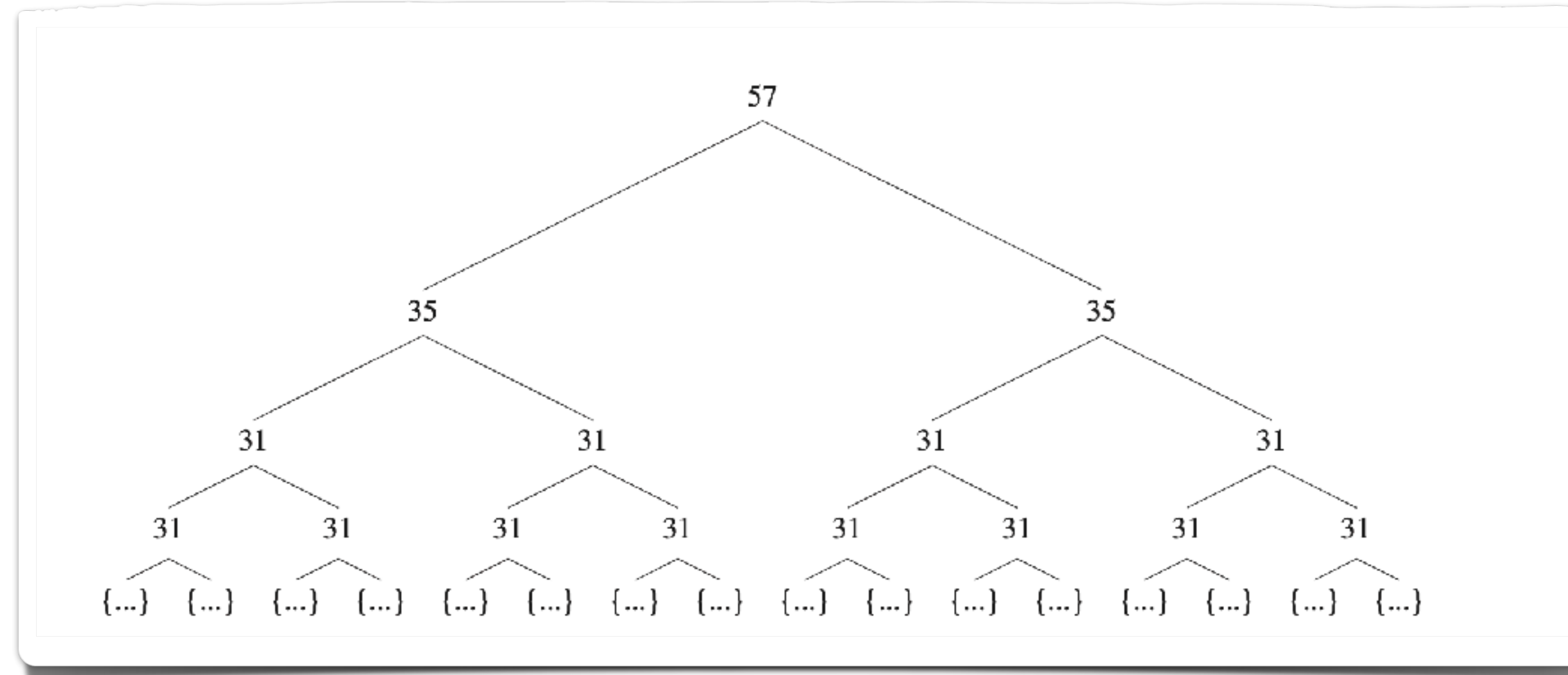
• Exponentiell viele Fälle!

Enumerationsprinzip



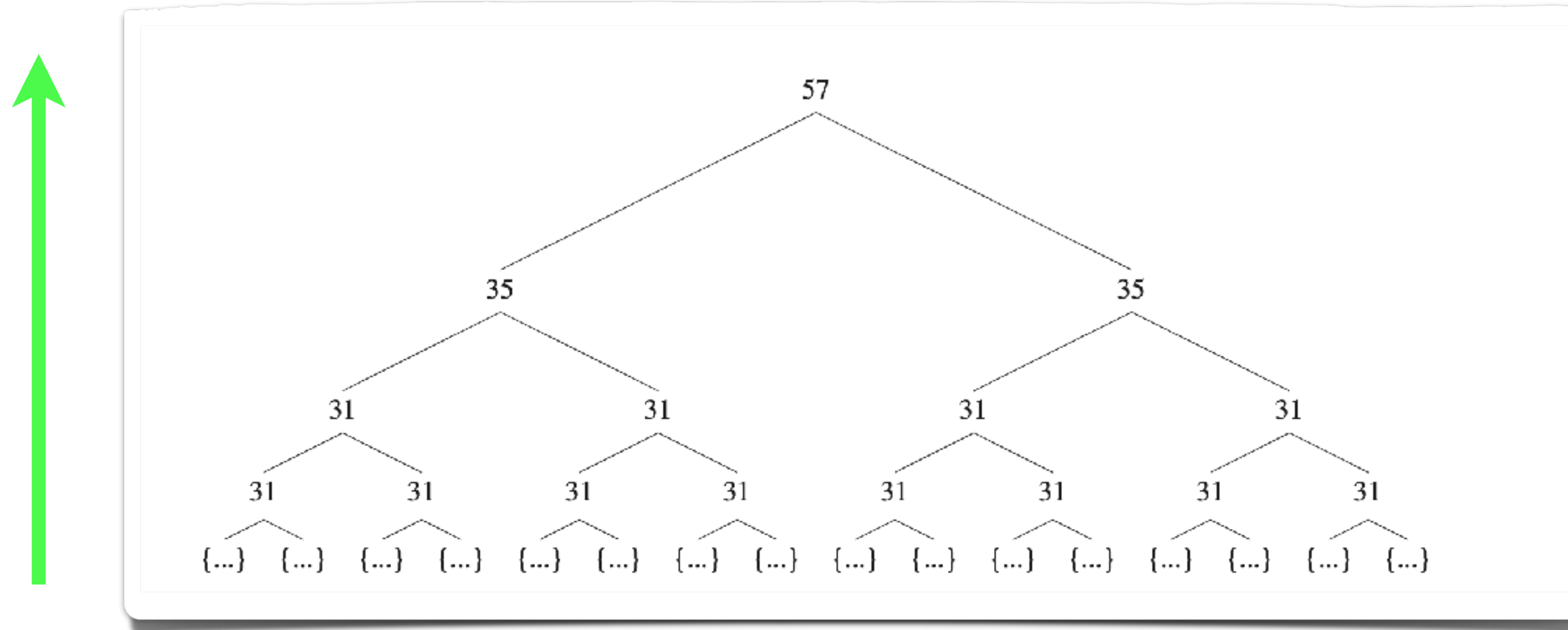
- Exponentiell viele Fälle!
- Wie geht das systematisch?

Enumerationsprinzip



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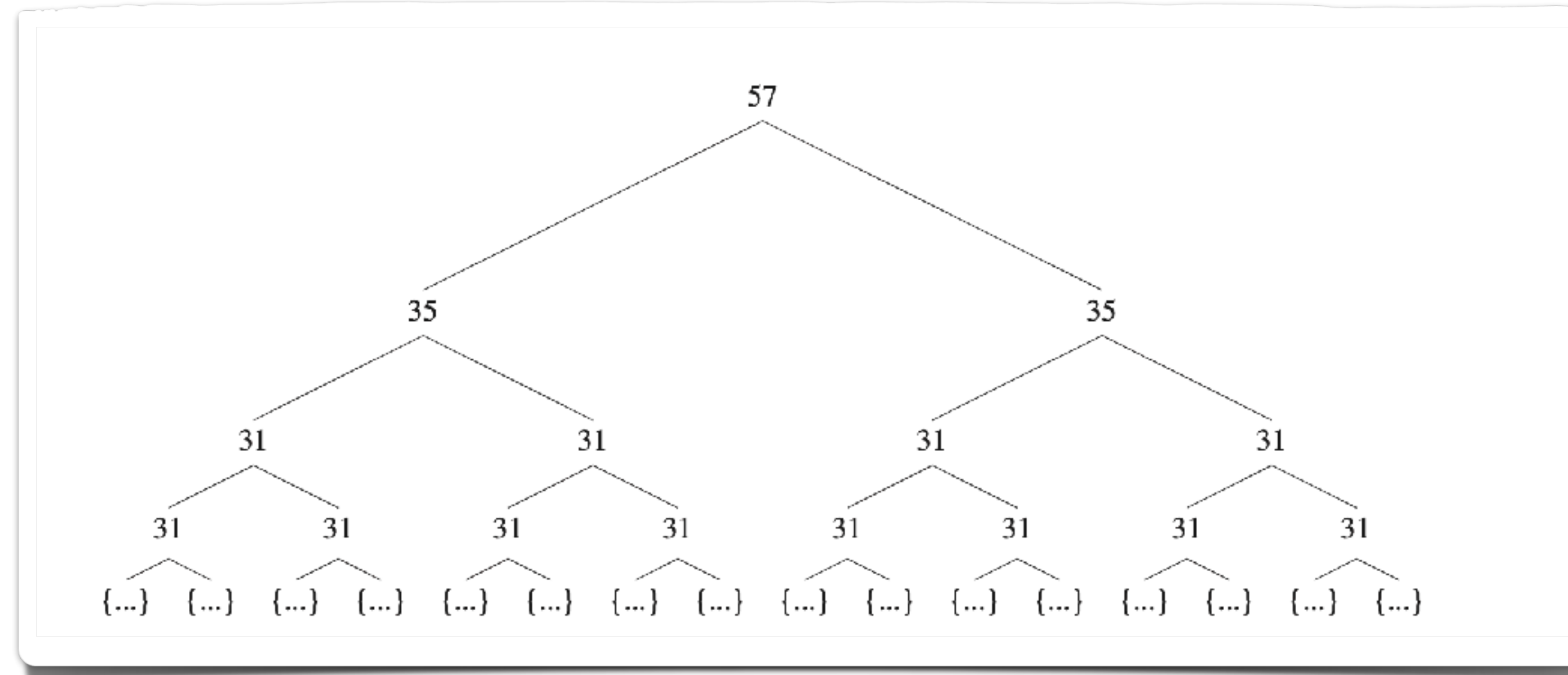
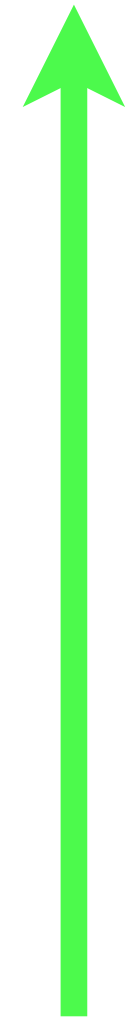
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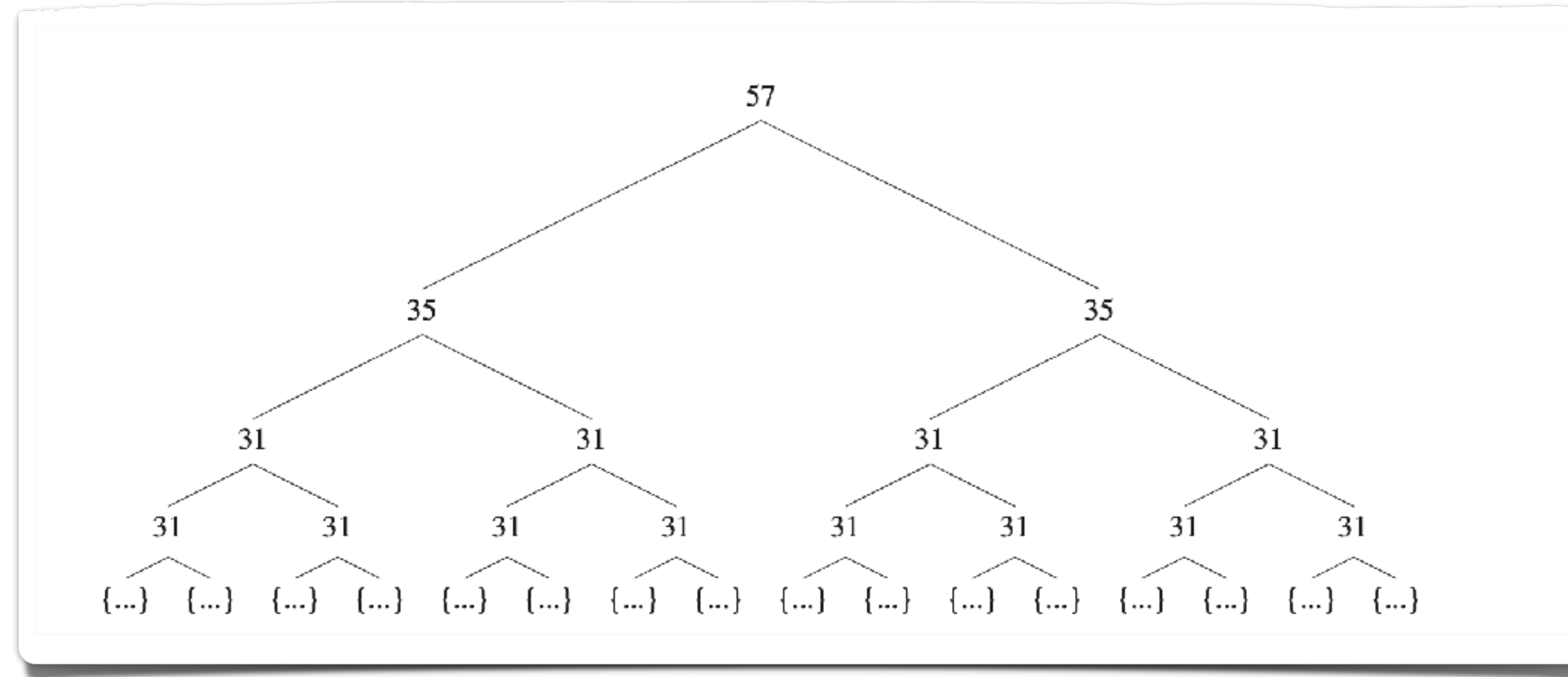
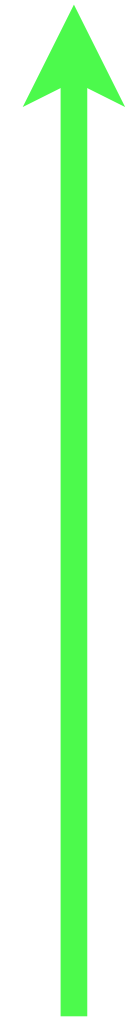
Dynamic
Programming



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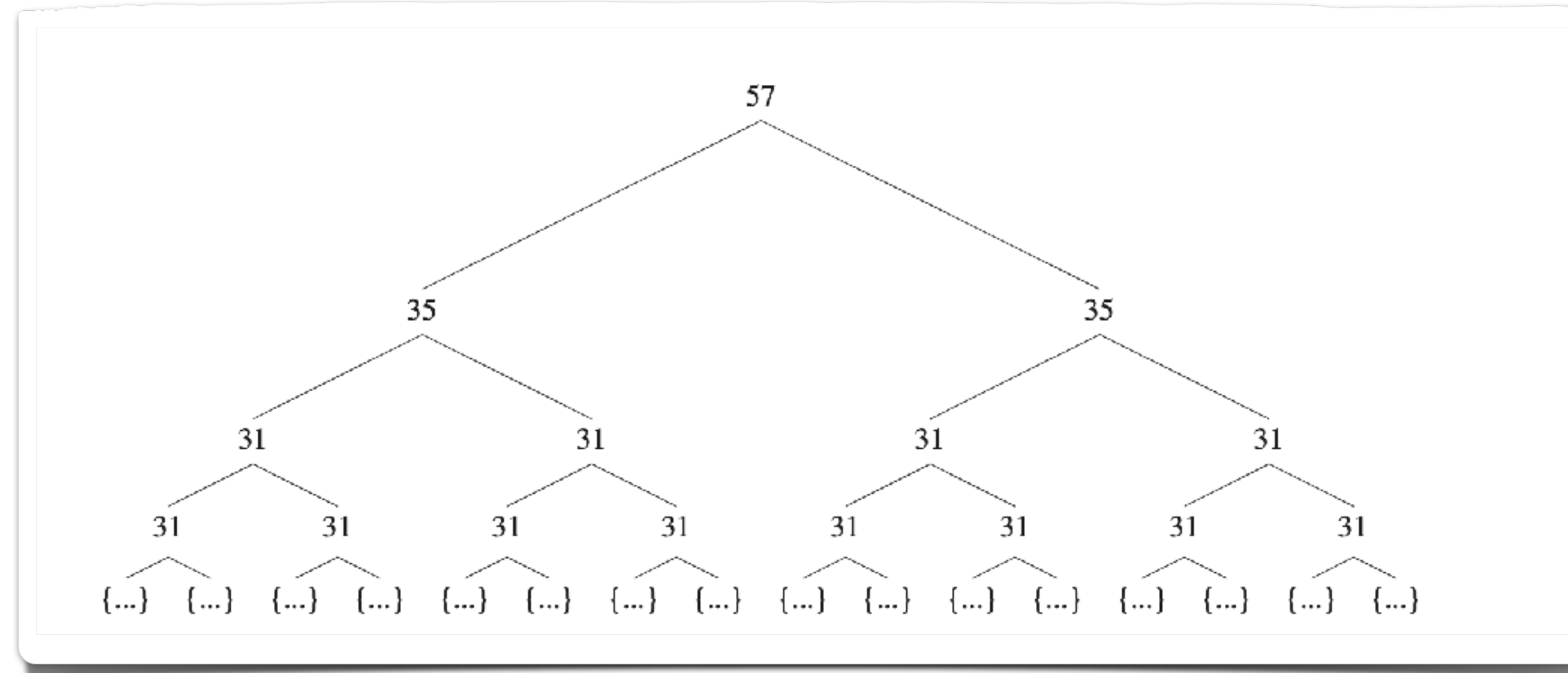
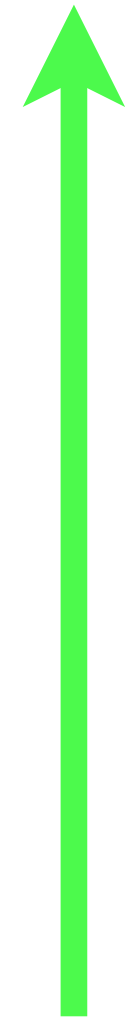
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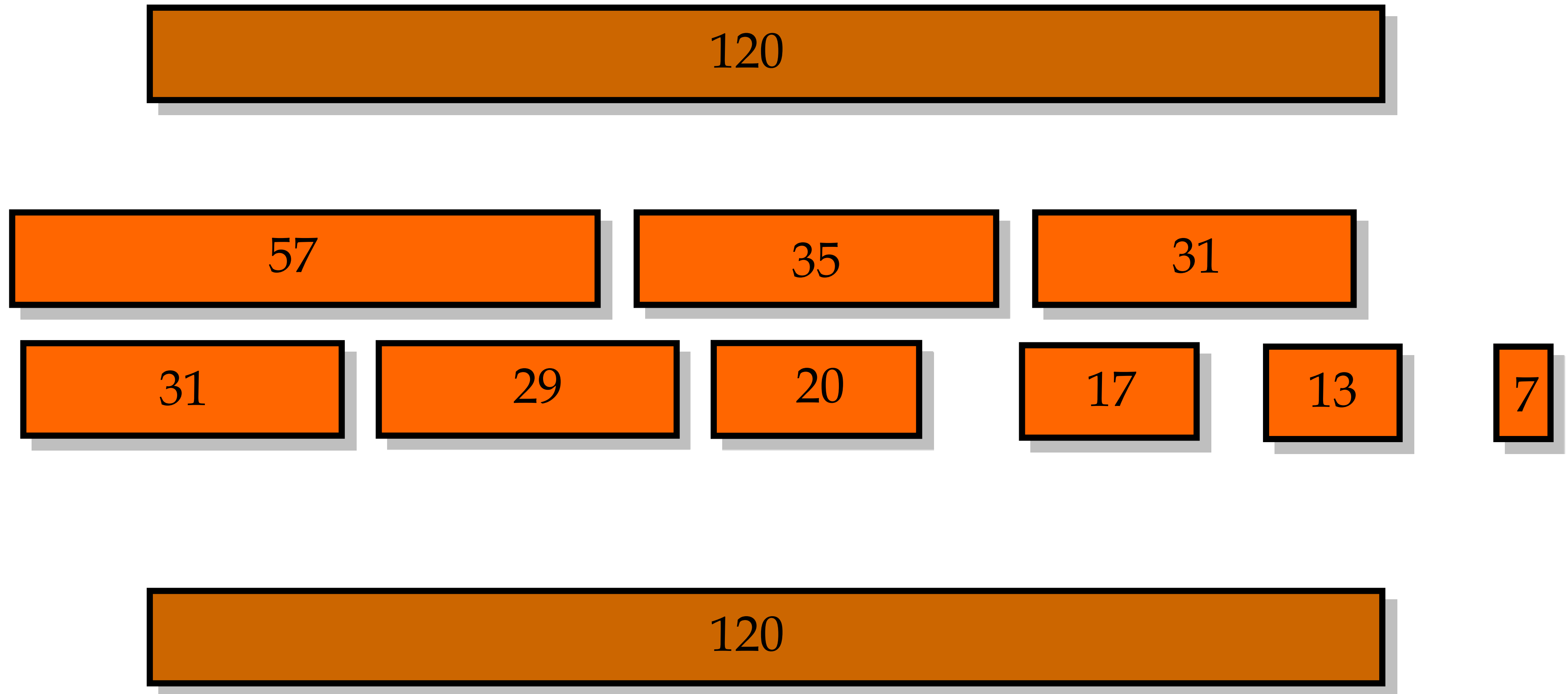
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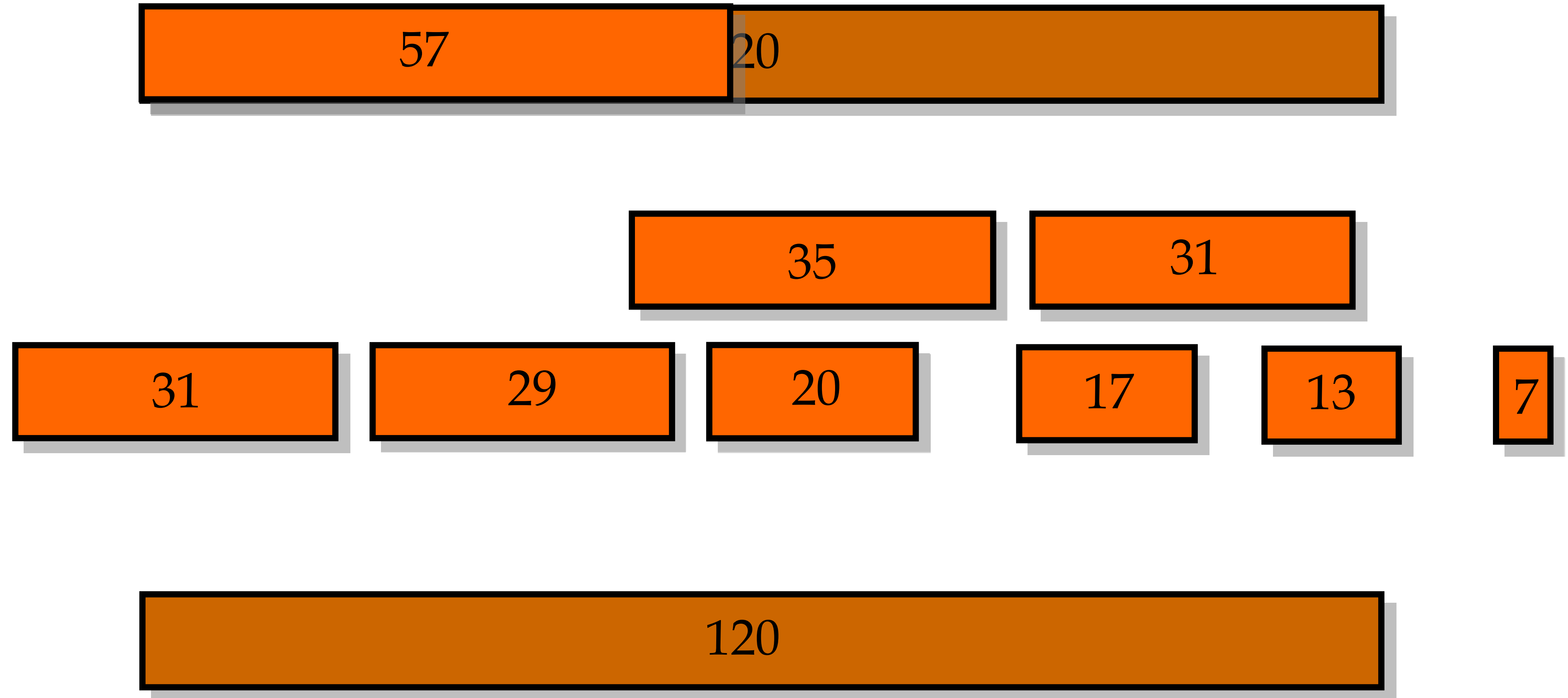
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Zur Erinnerung: Subset Sum

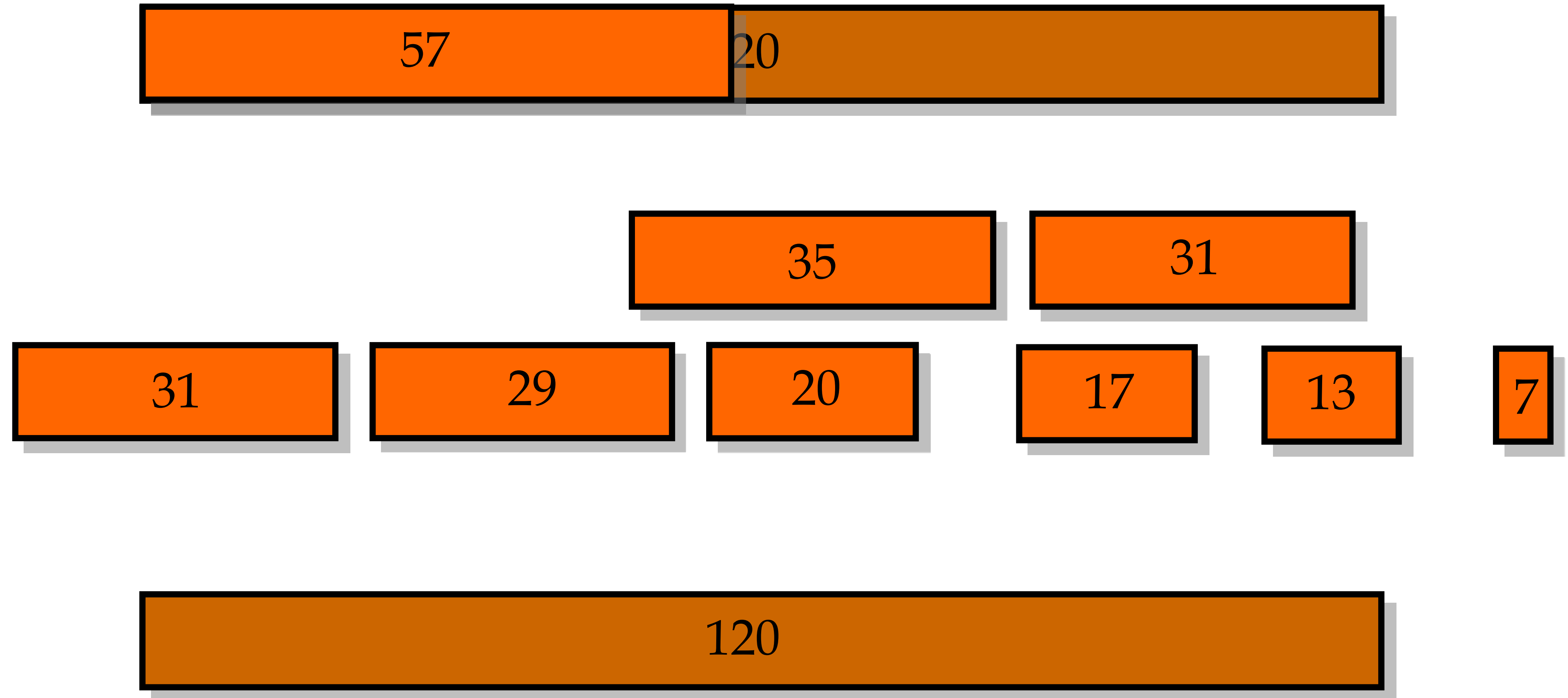
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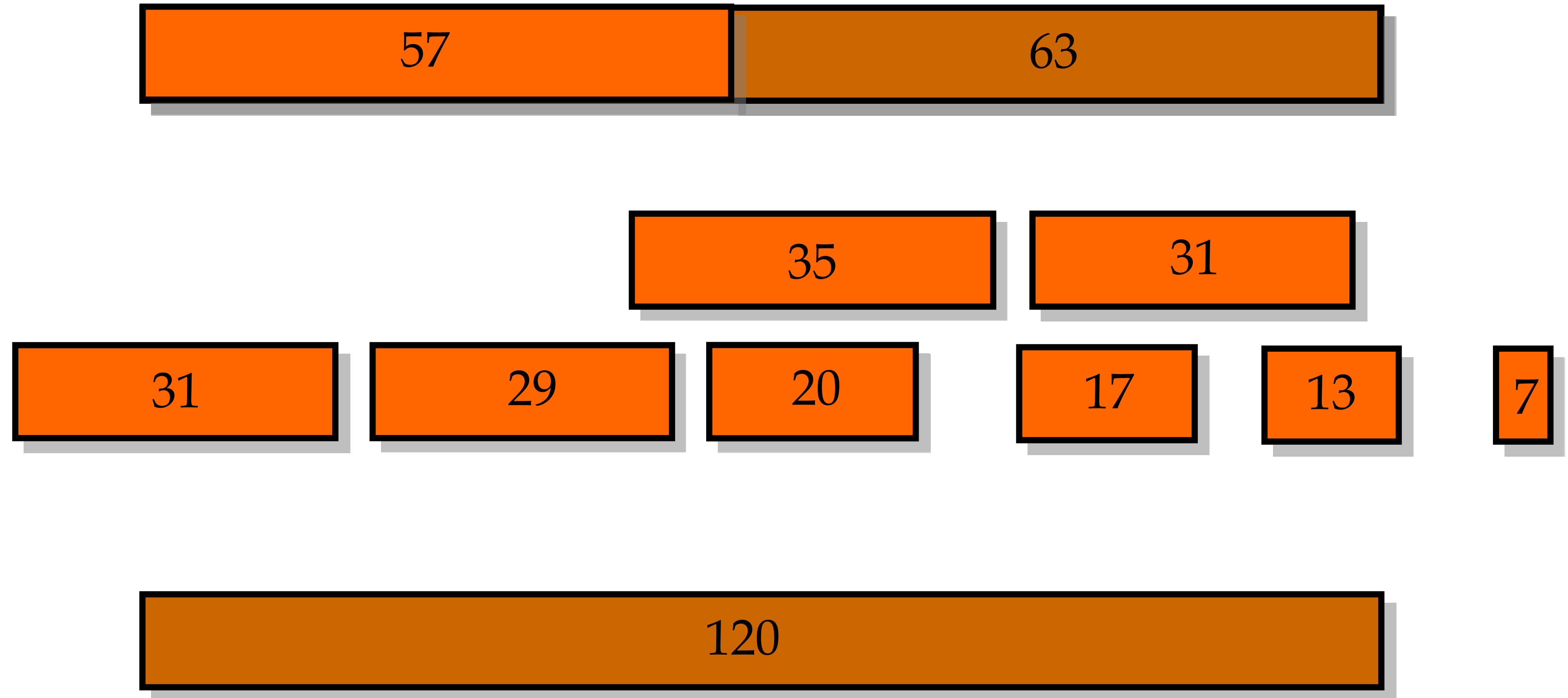
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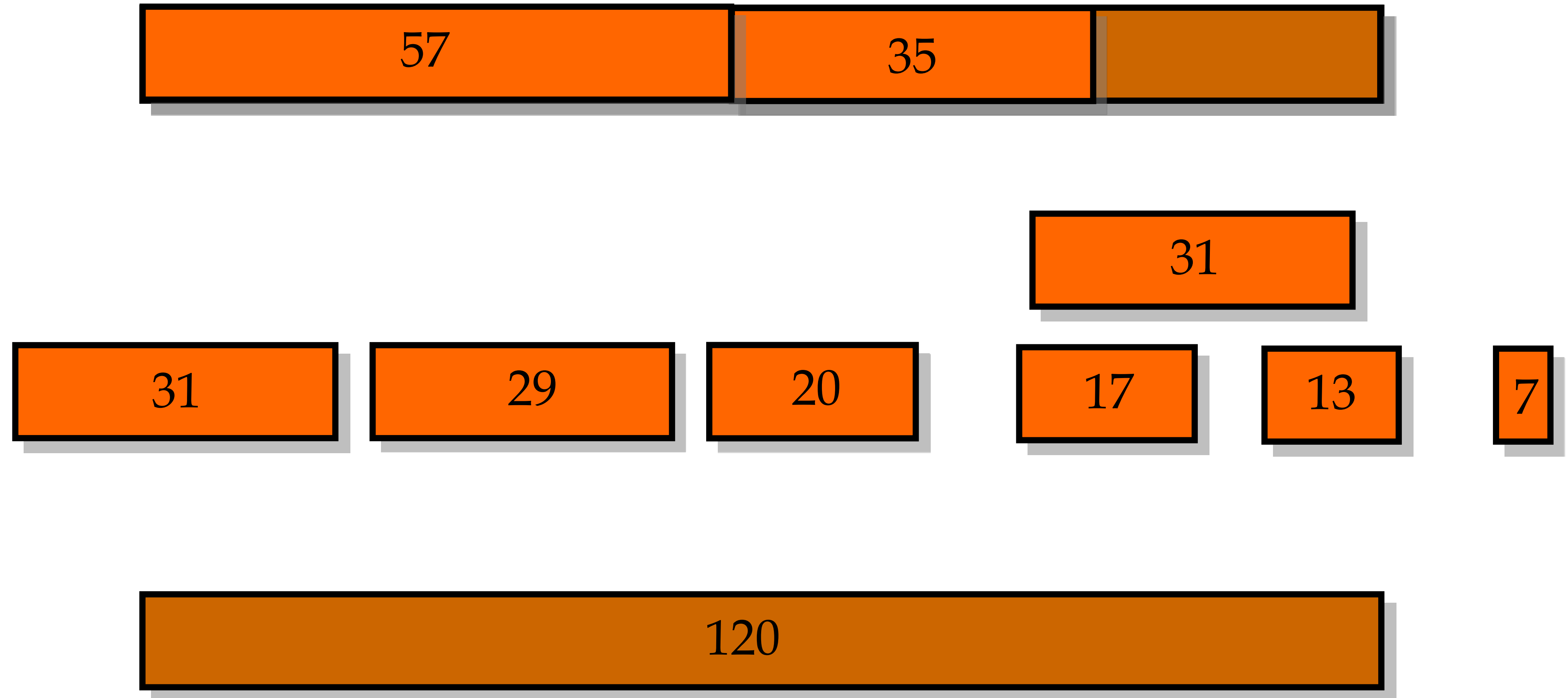
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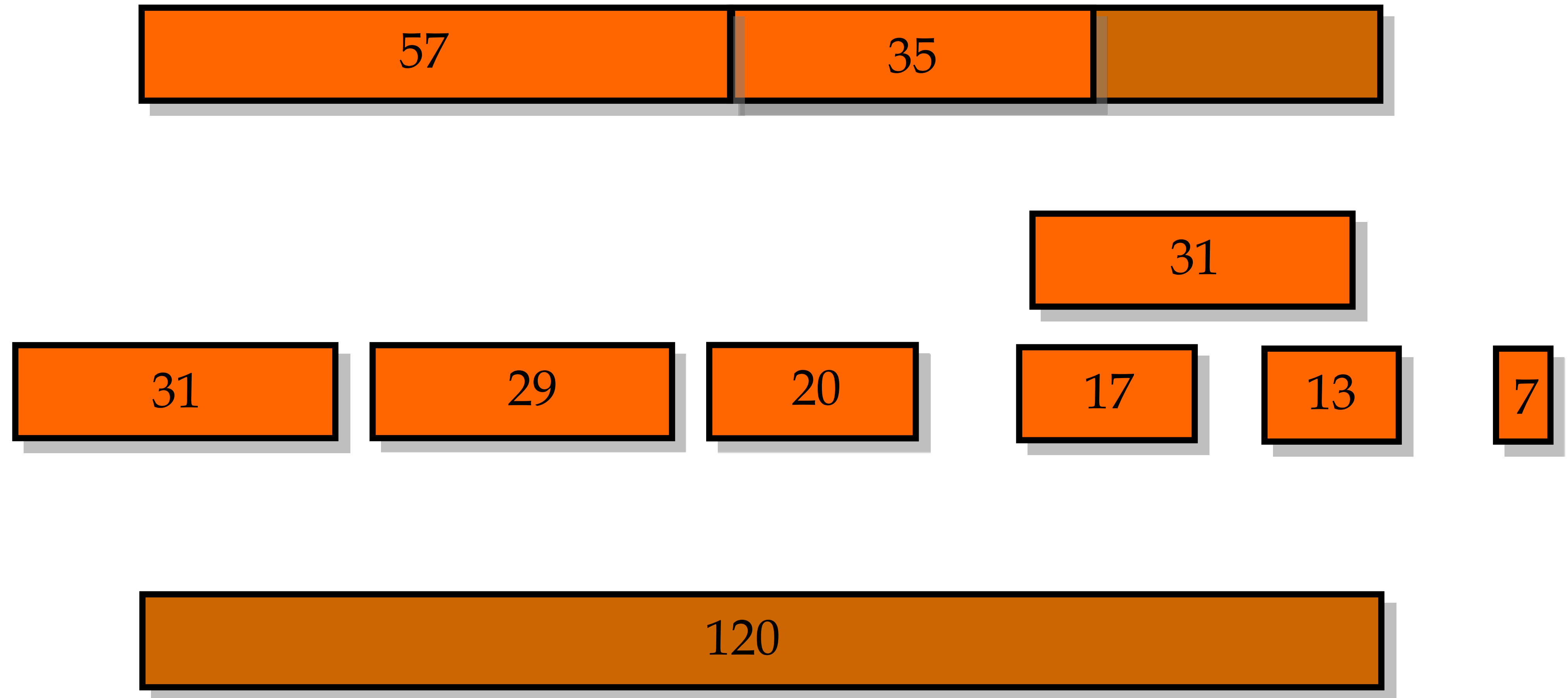
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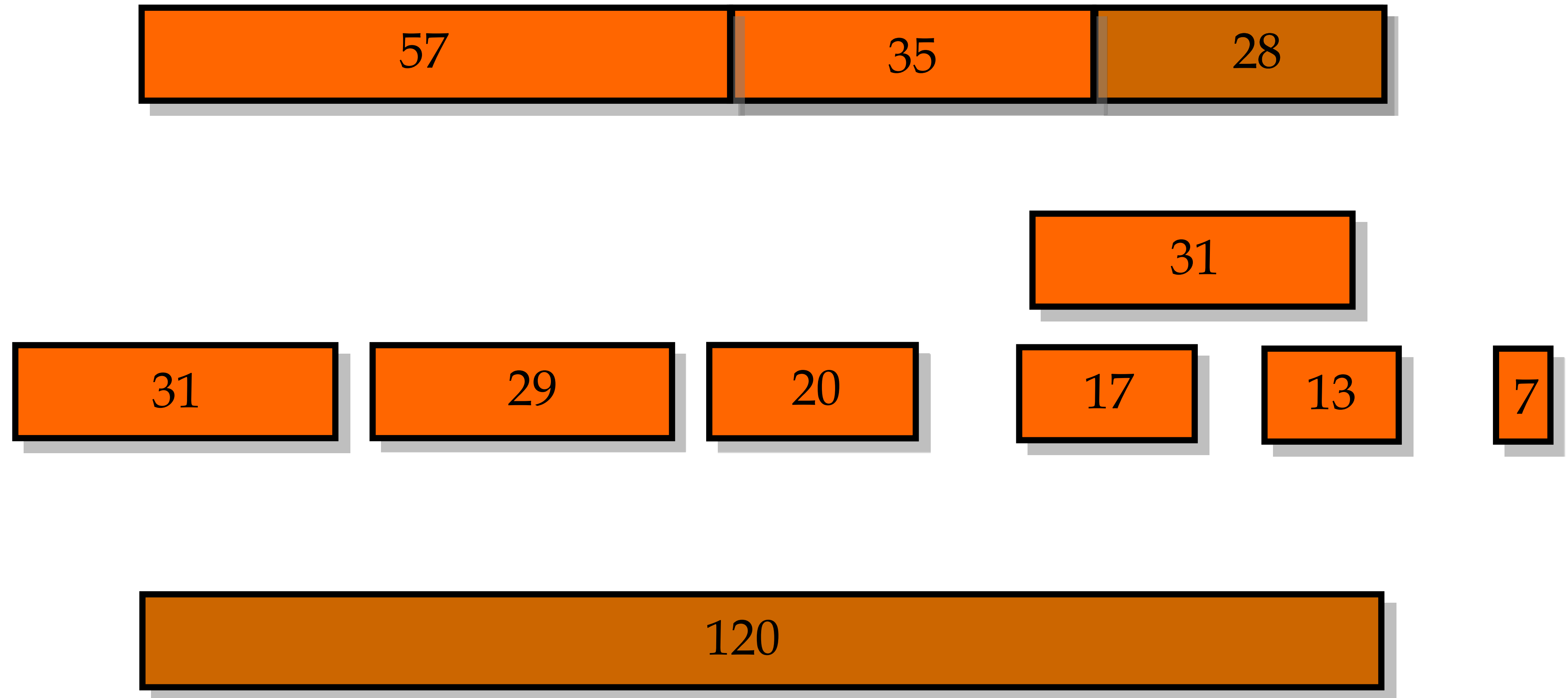
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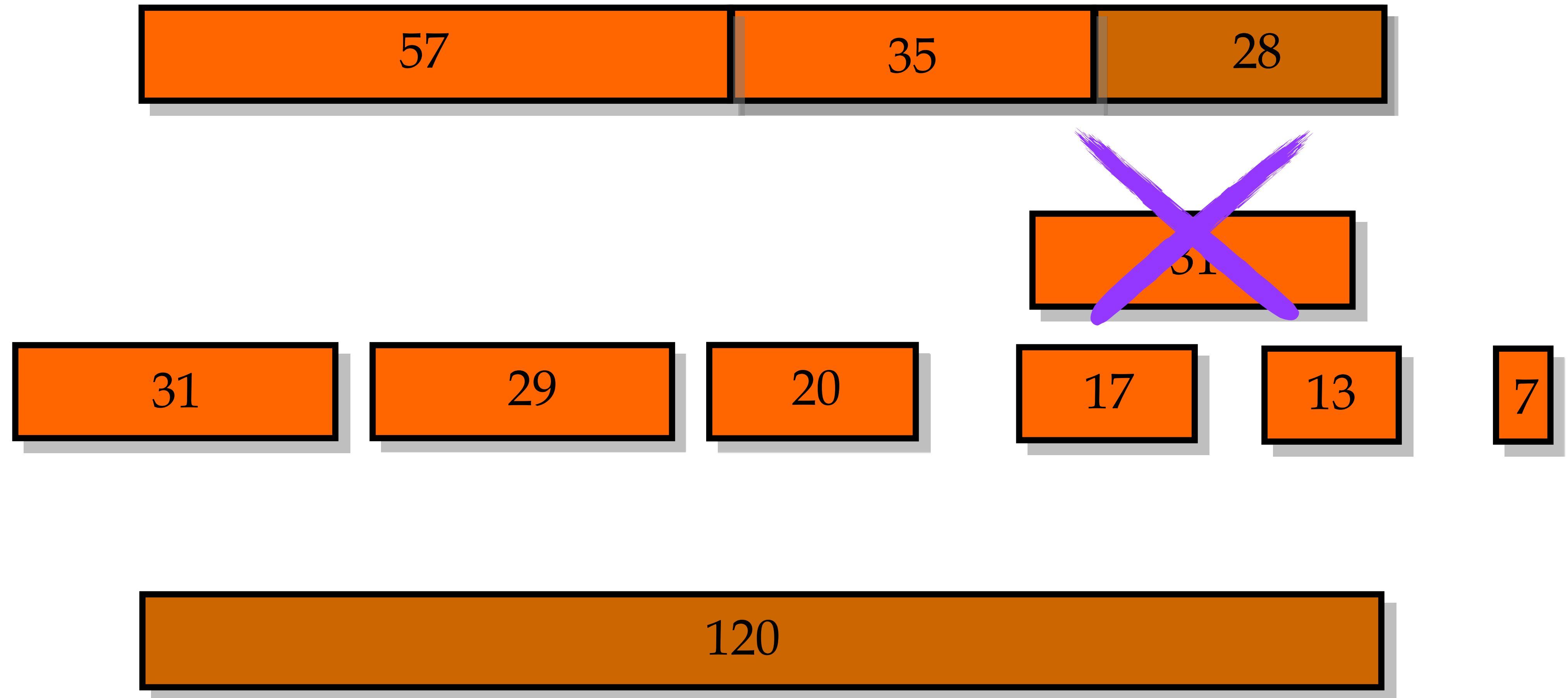
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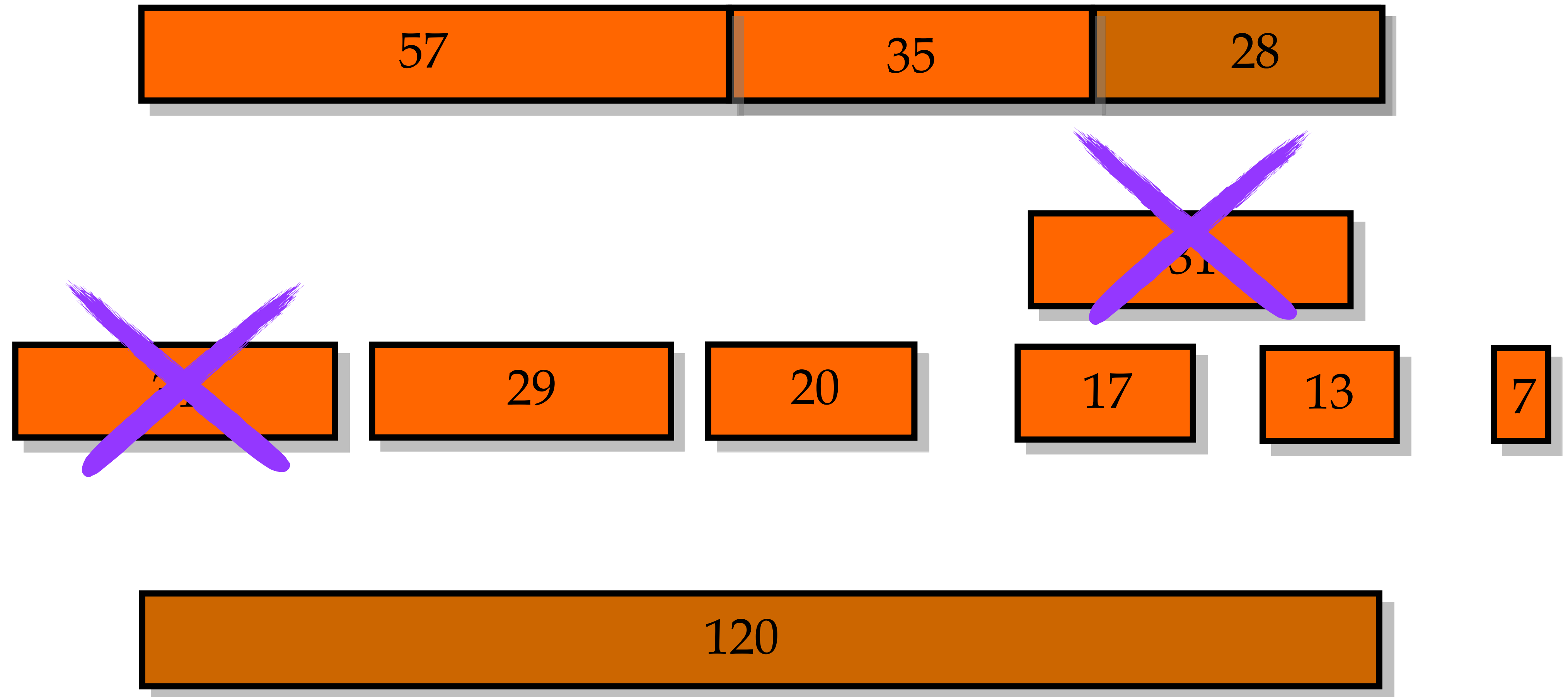
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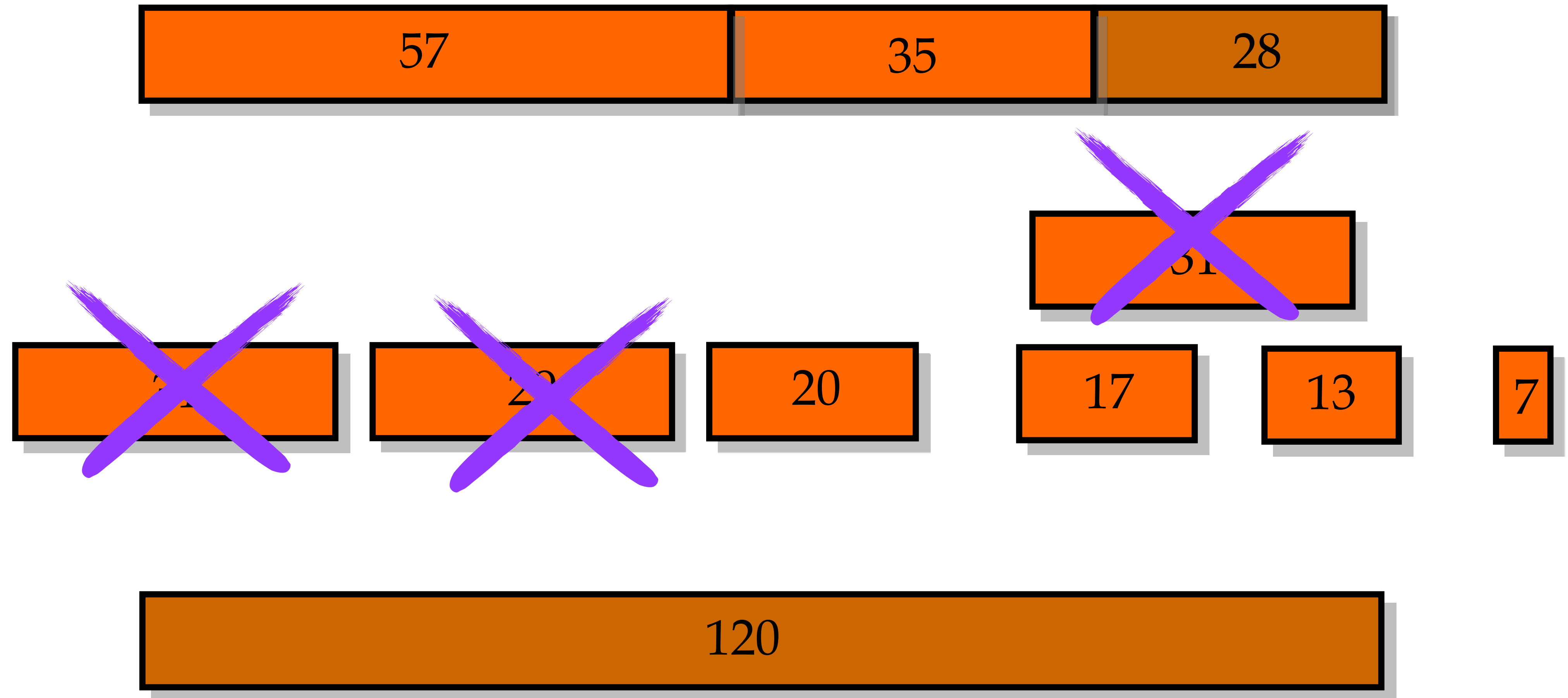
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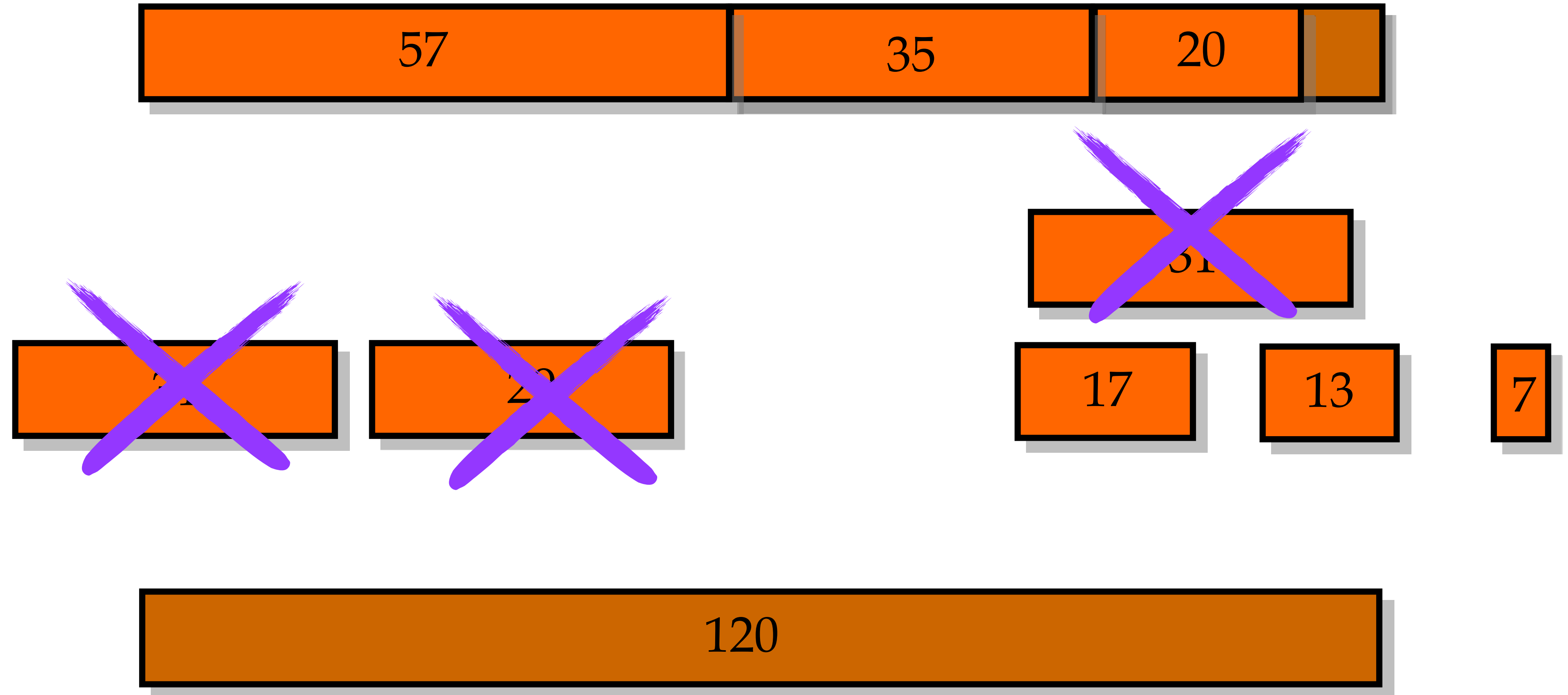
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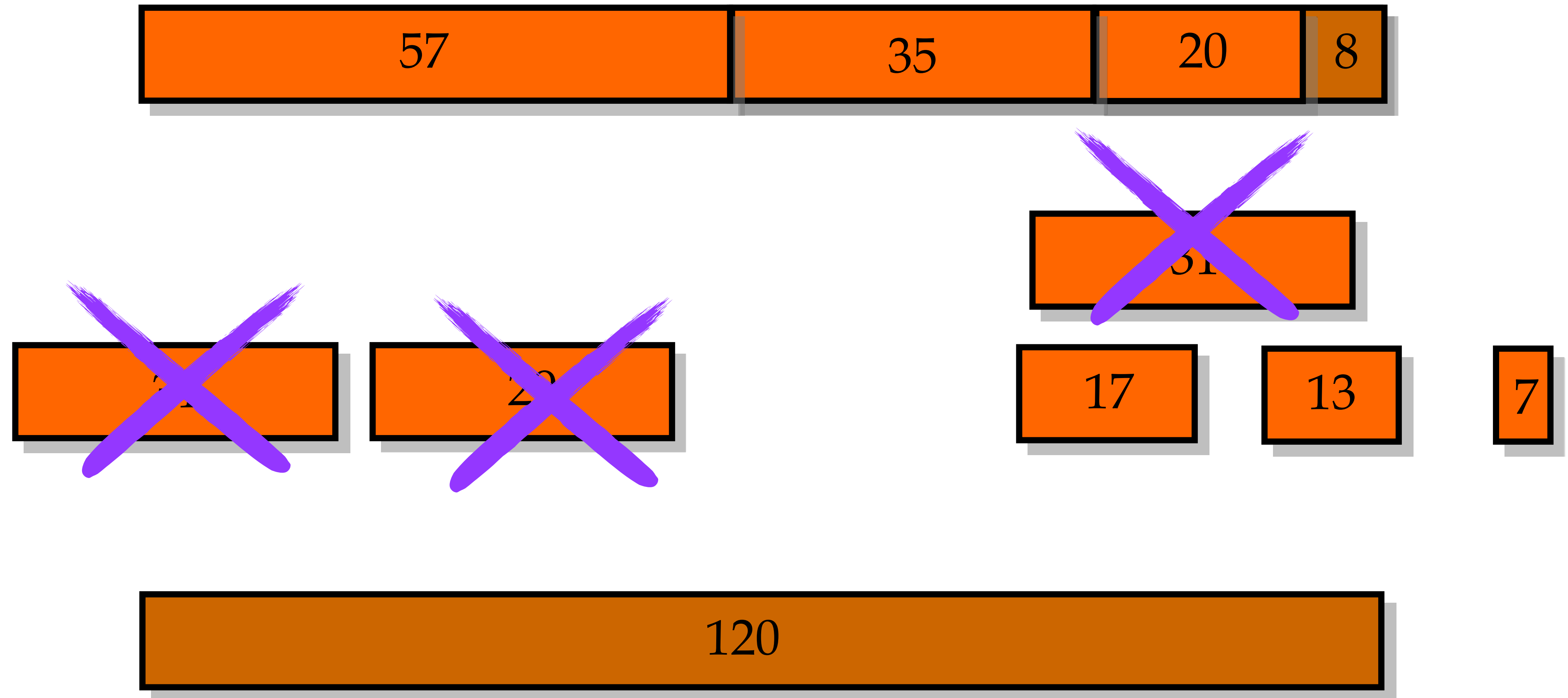
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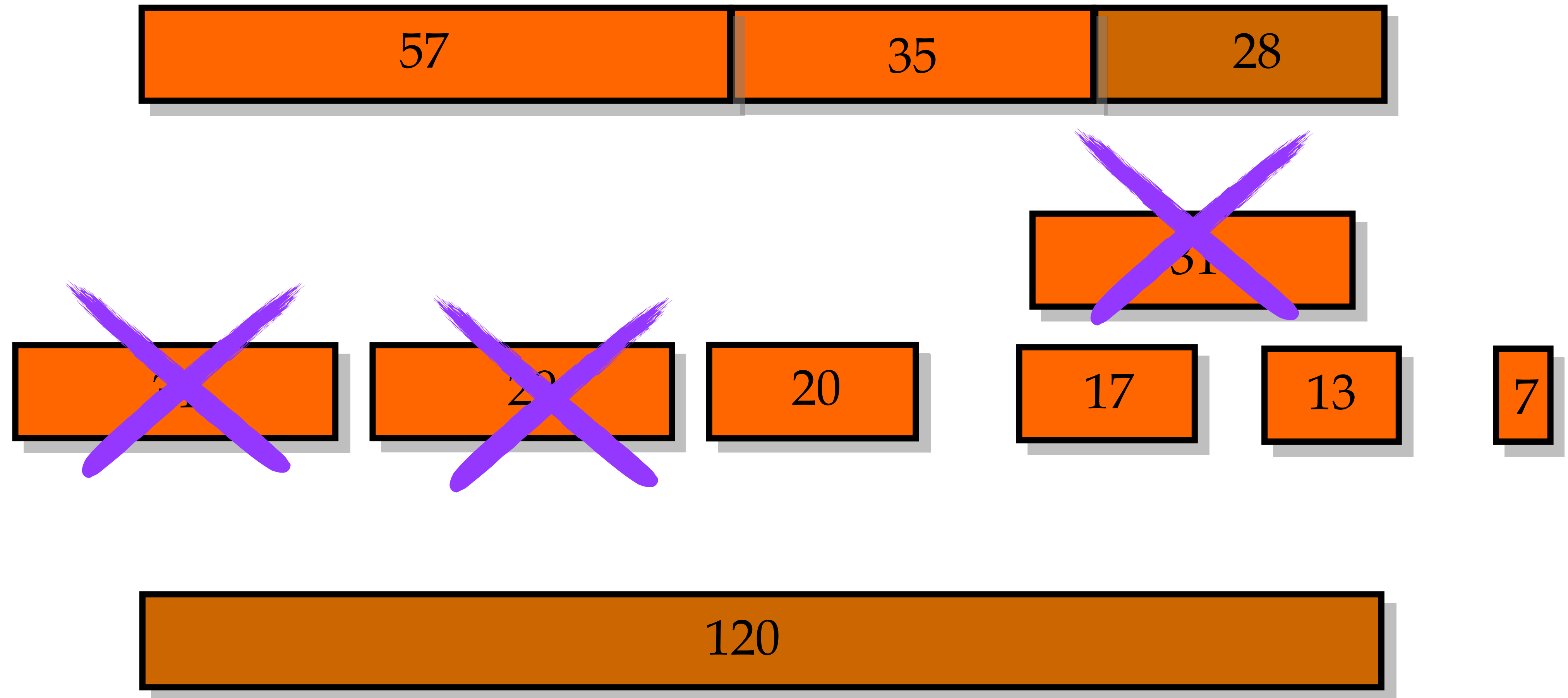
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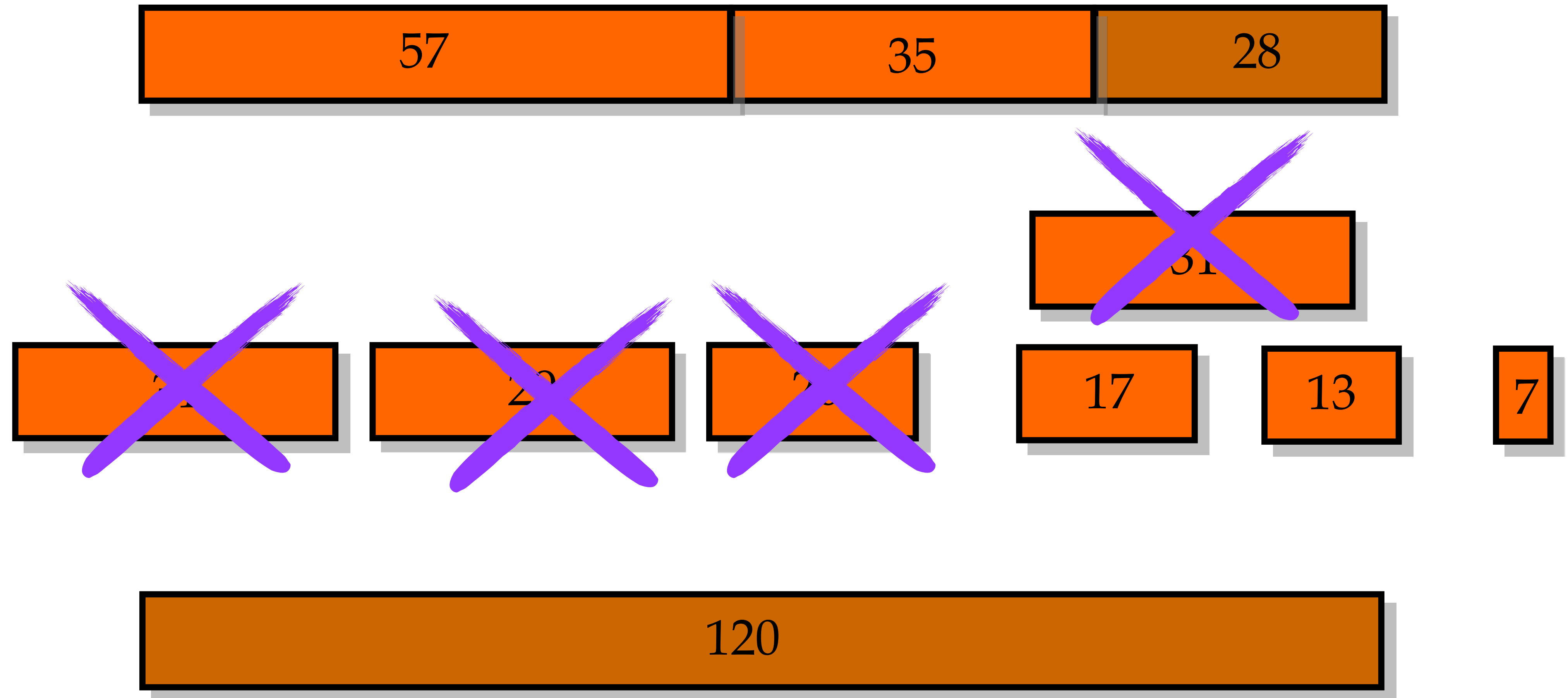
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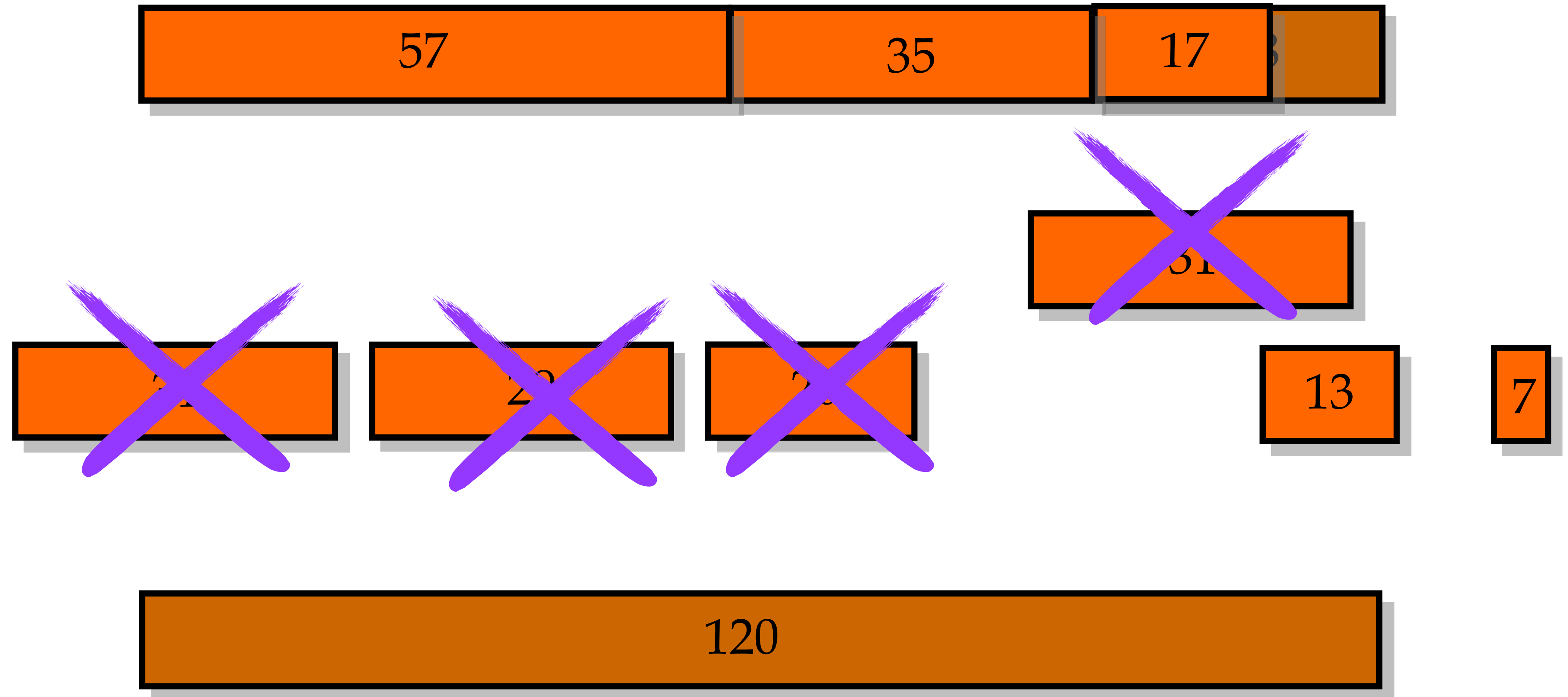
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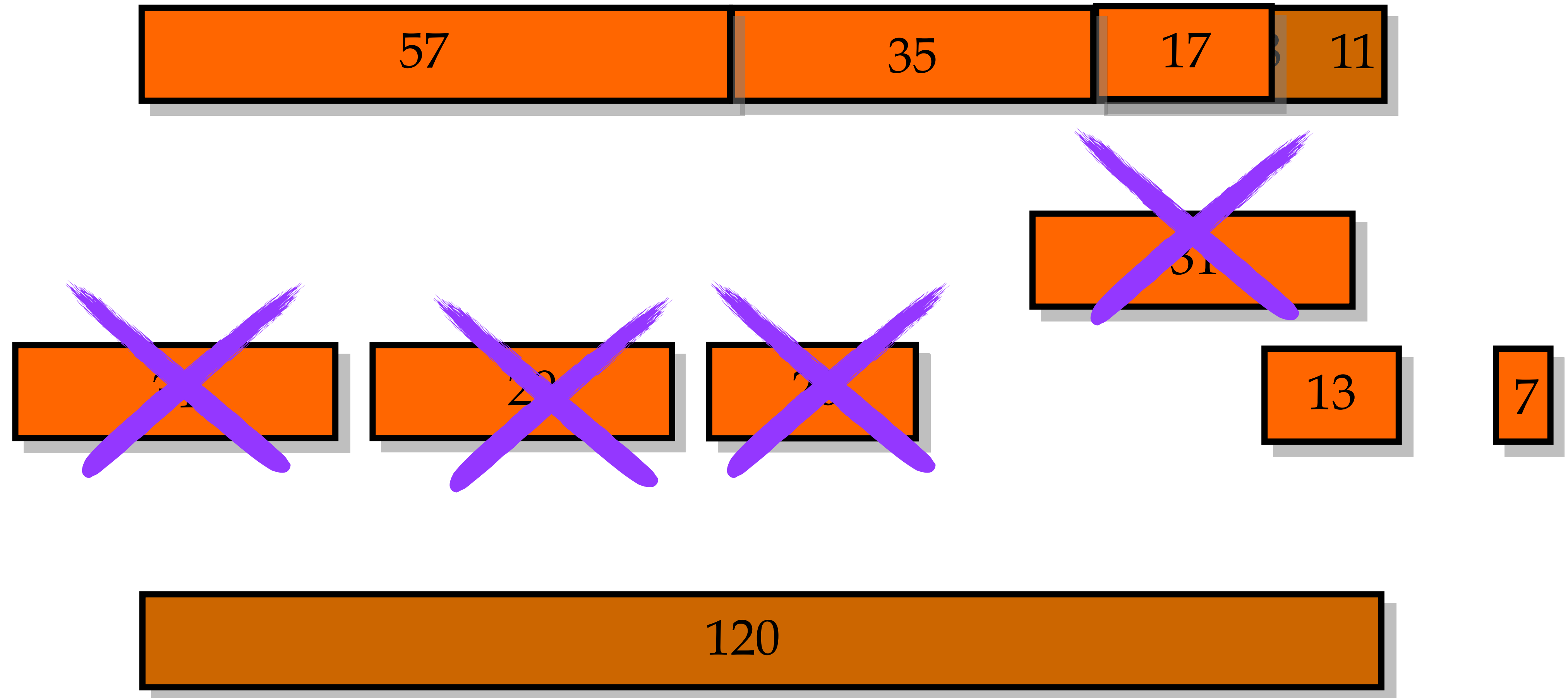
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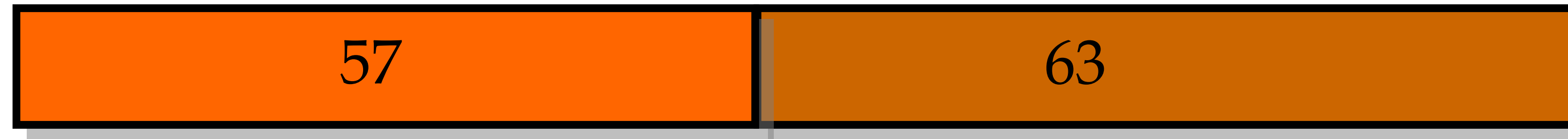
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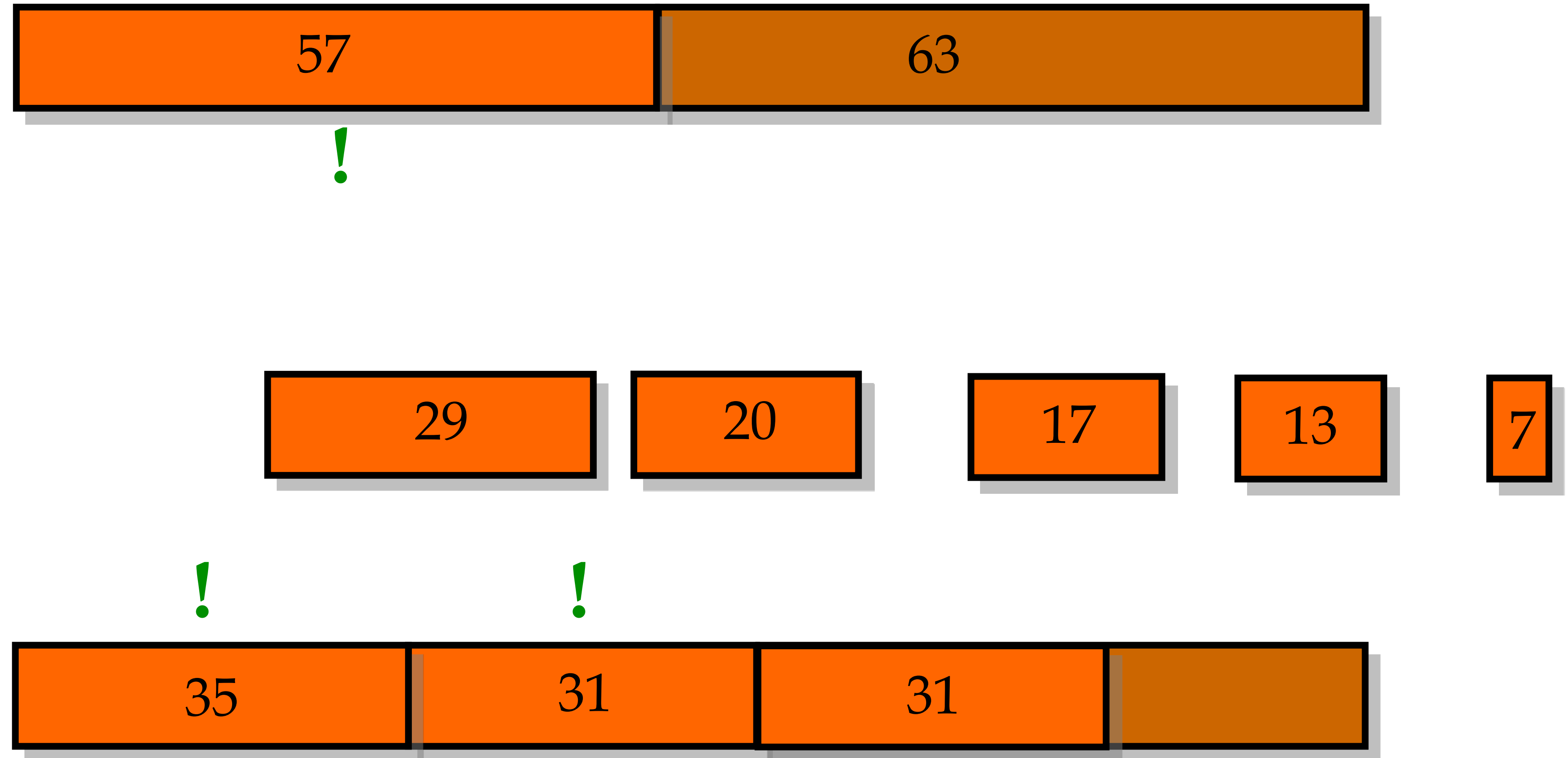
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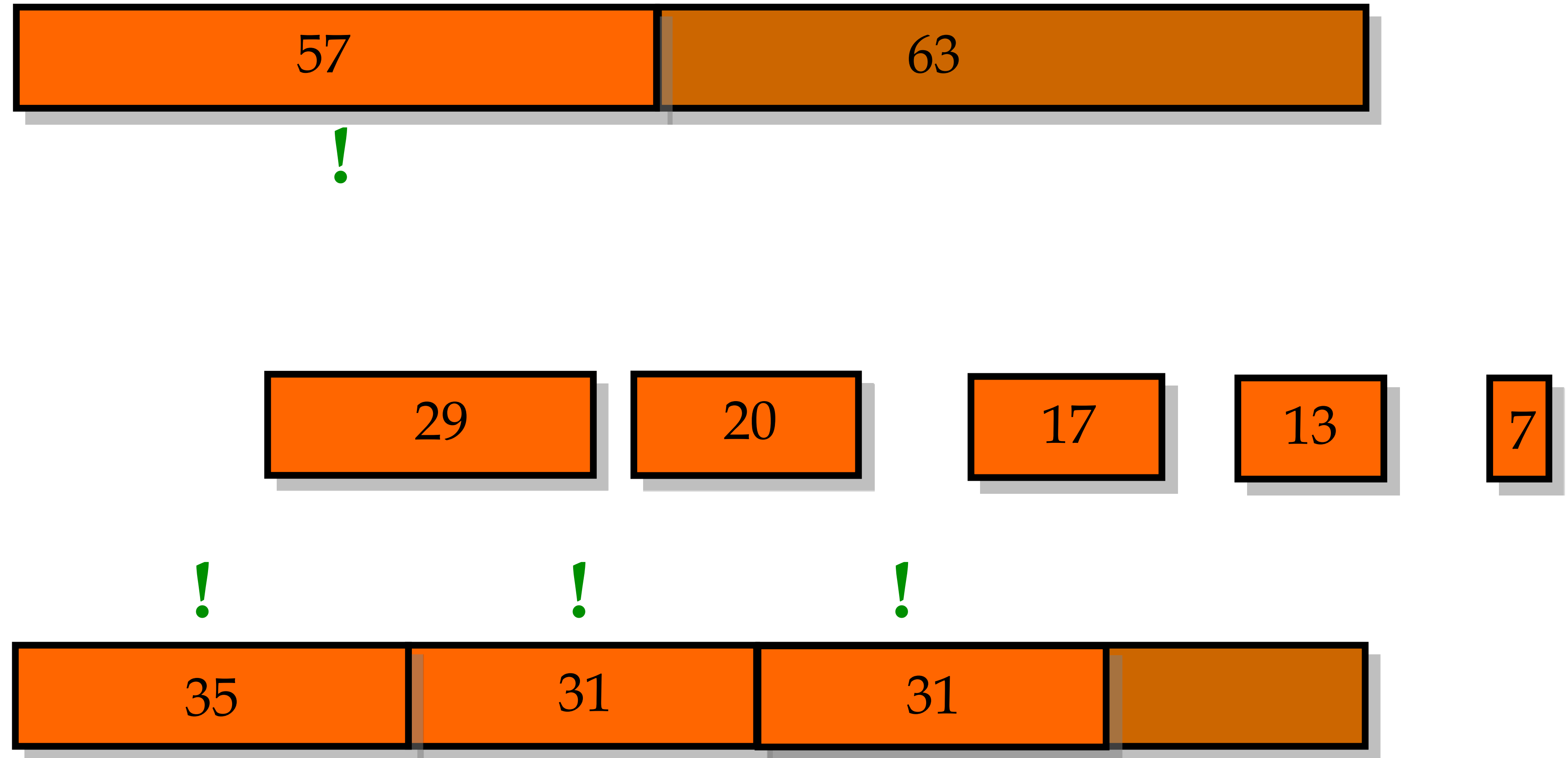
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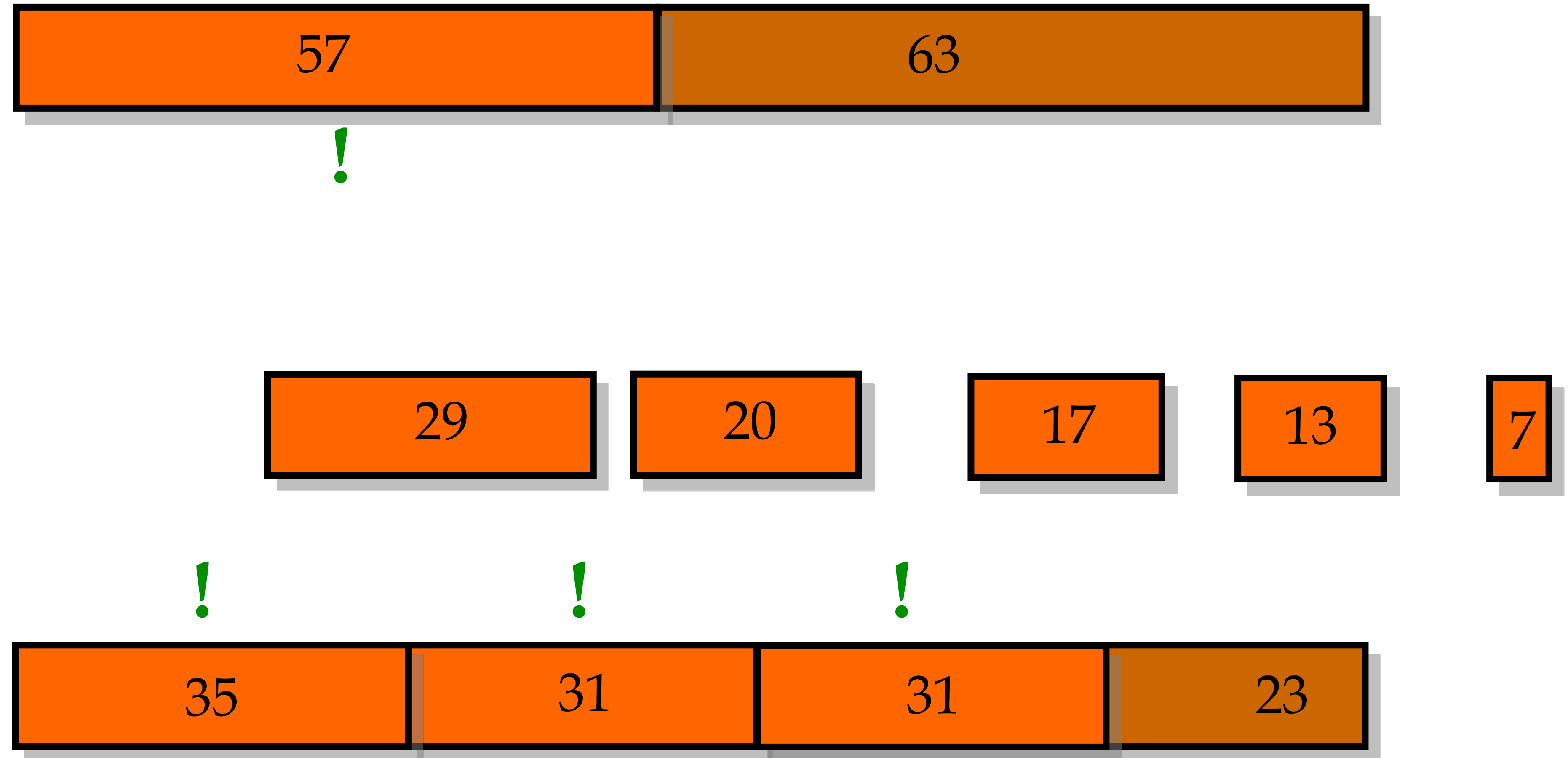
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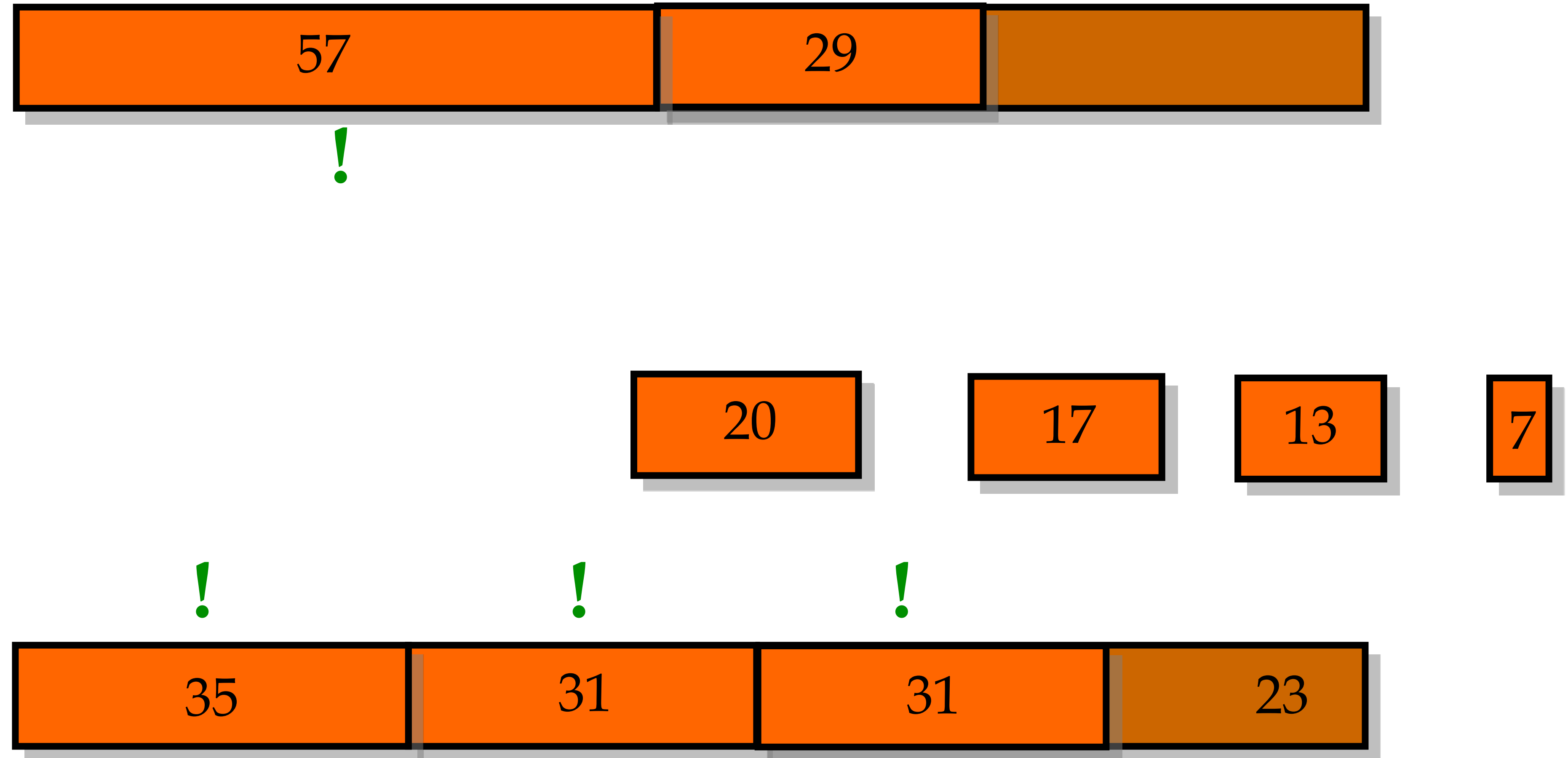
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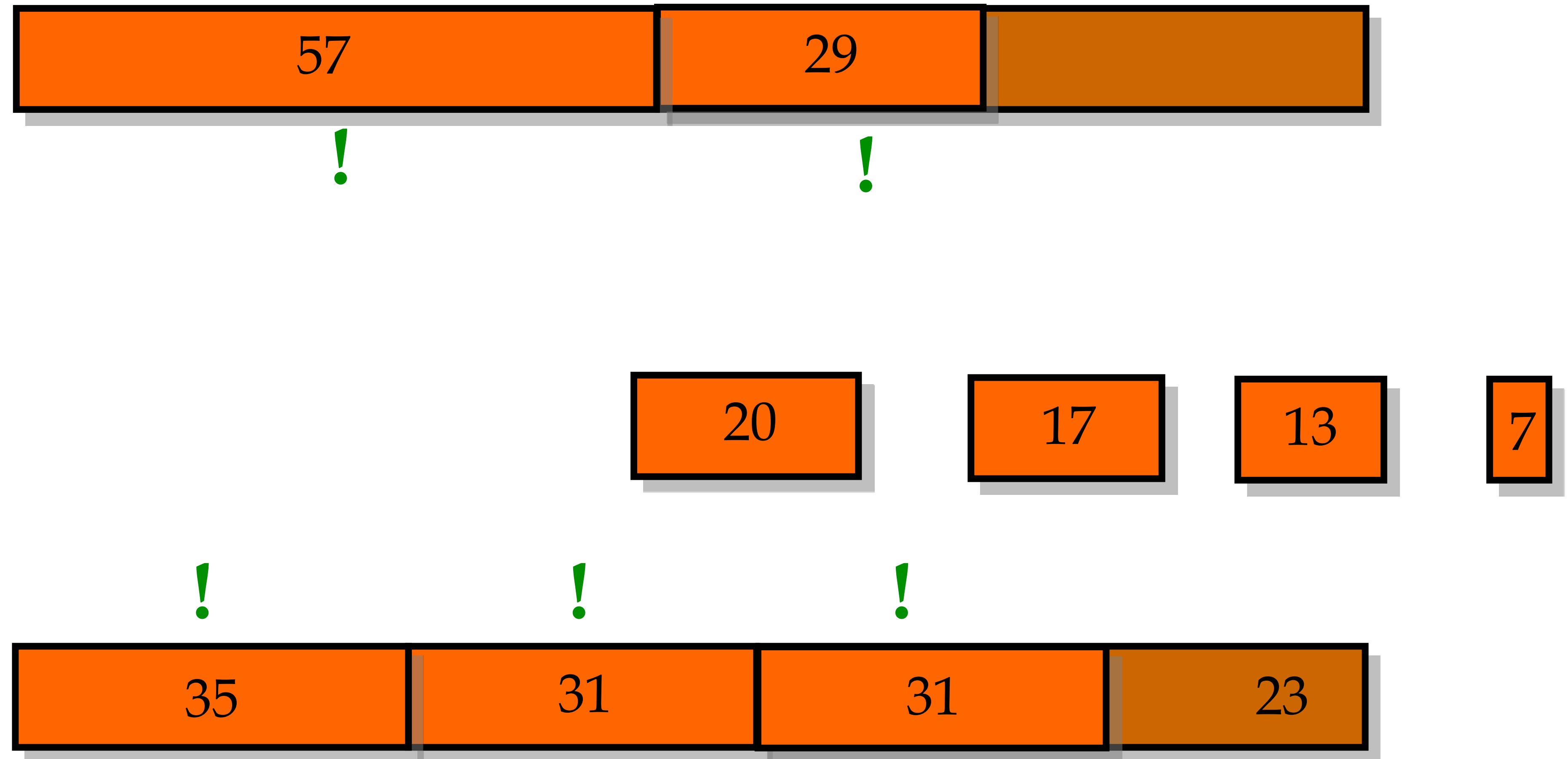
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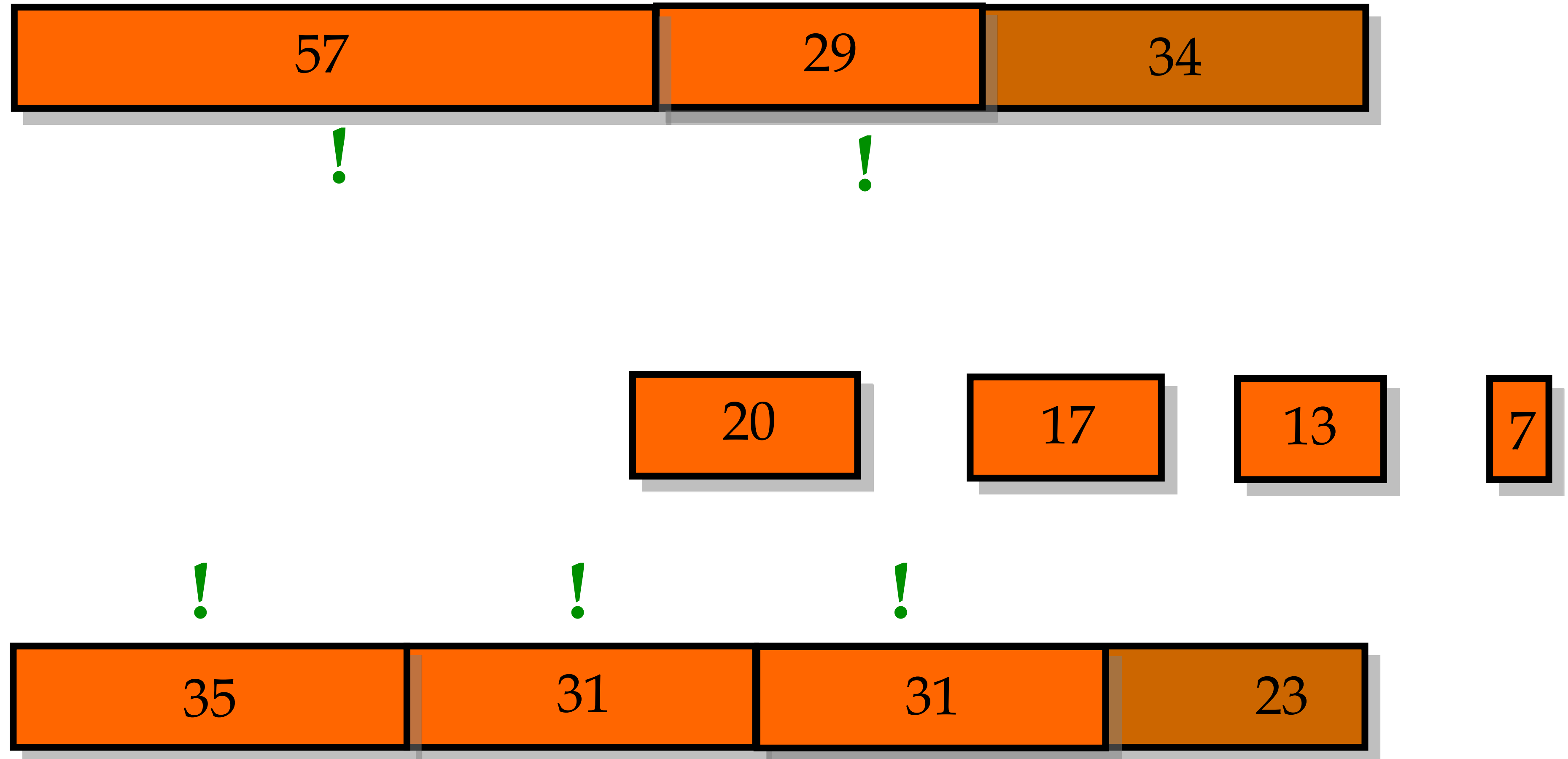
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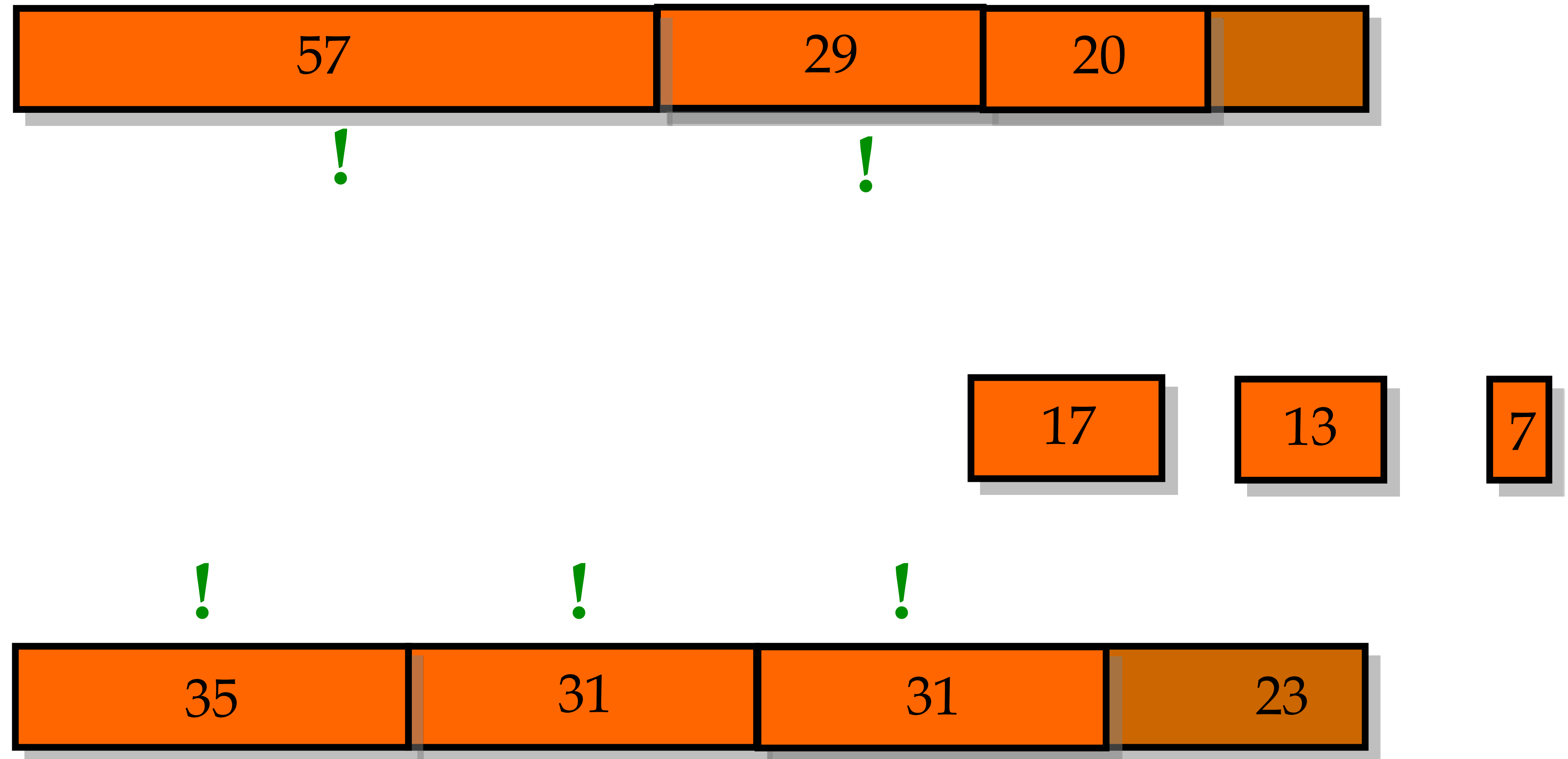
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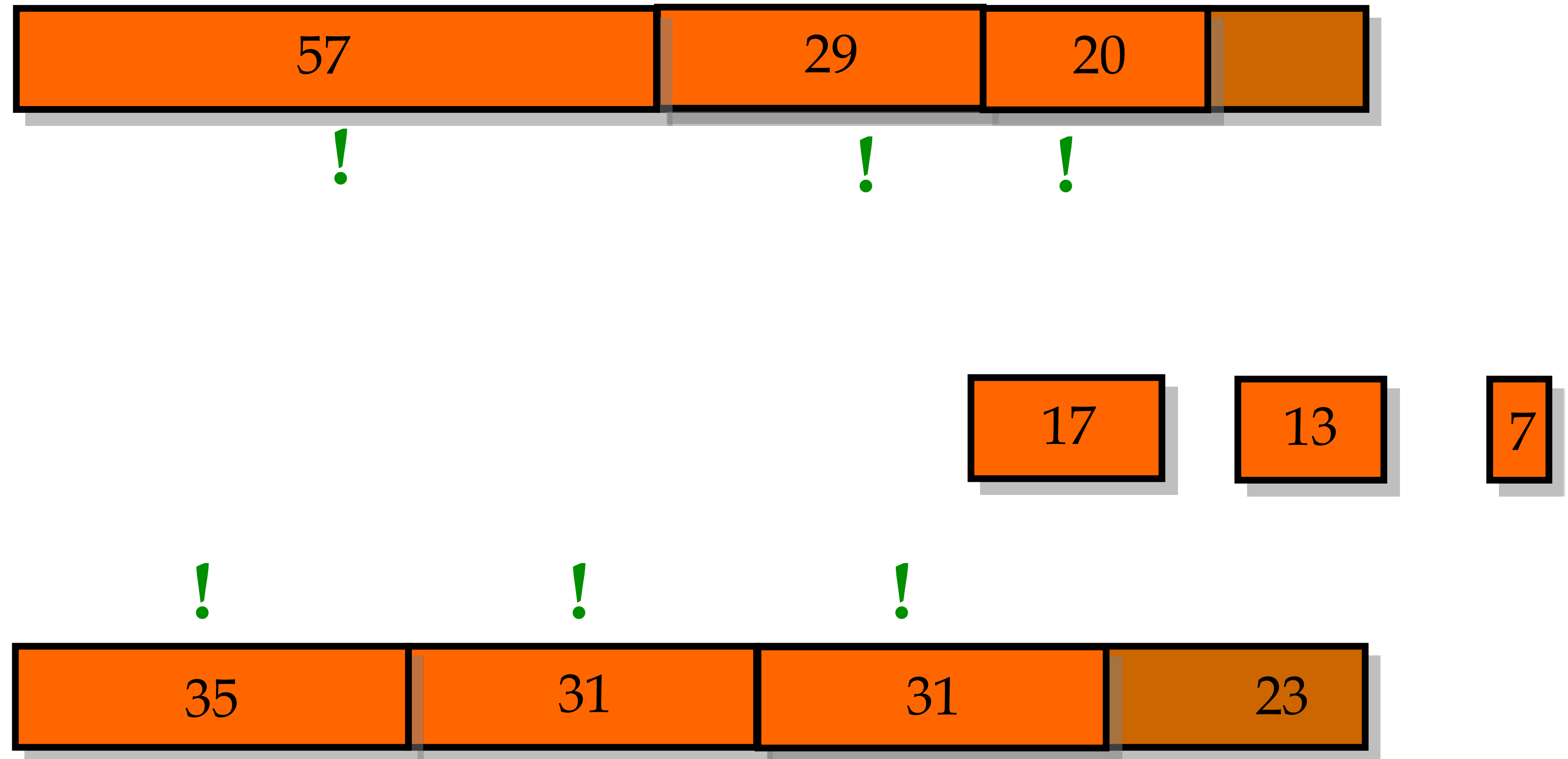
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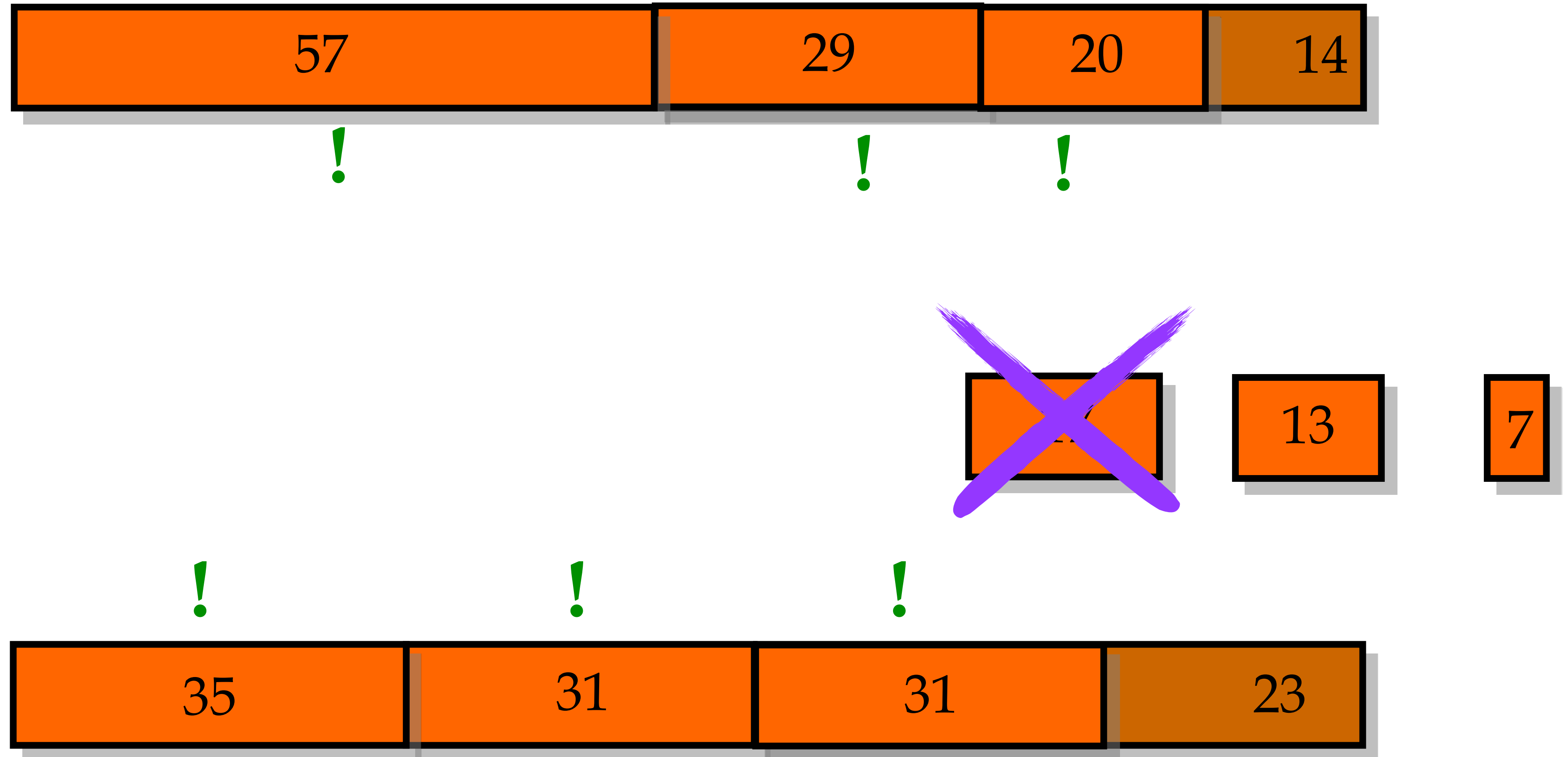
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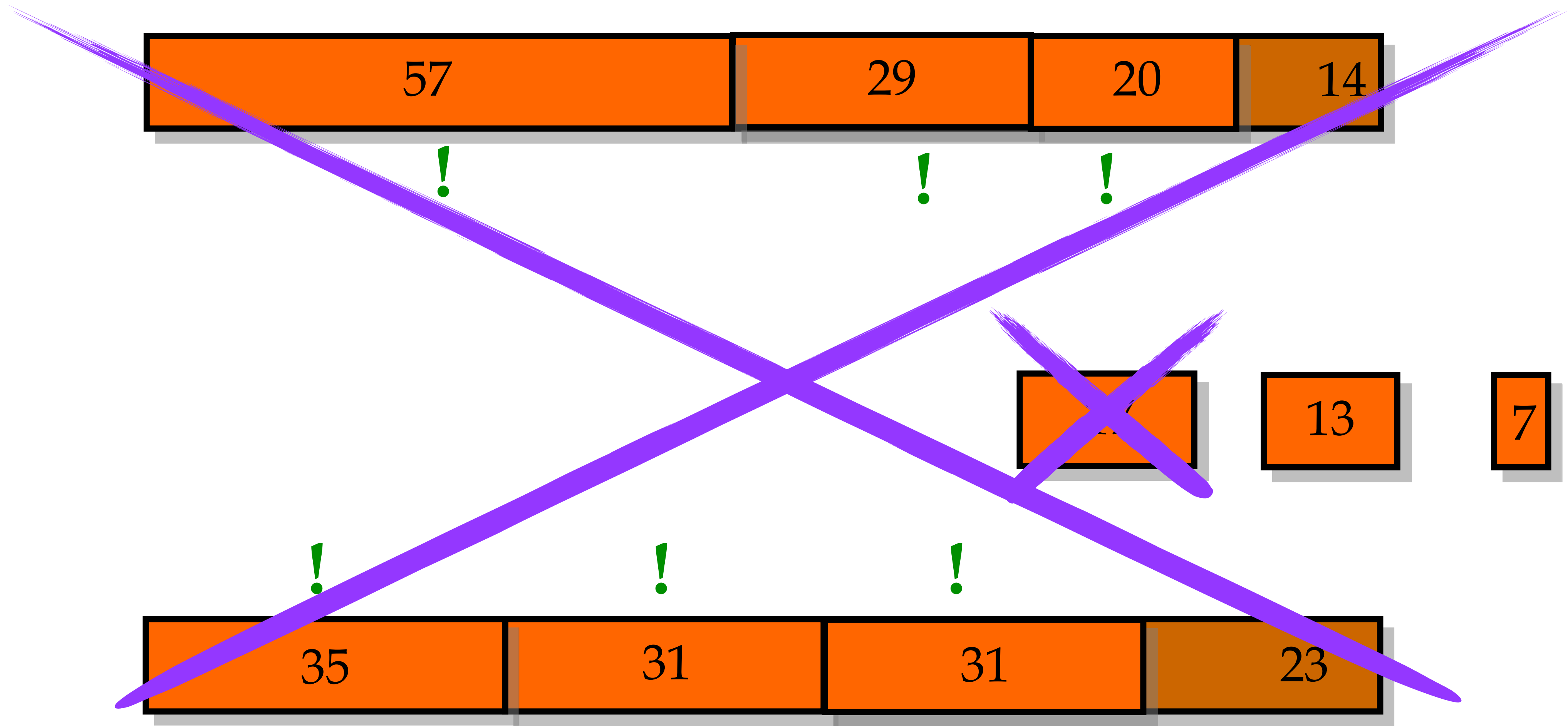
Zur Erinnerung: Subset Sum



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Dynamic Programming

Satz 2.6 Algorithmus 2.5 berechnet besten Lösungswert für das Rucksackproblem in einer Laufzeit $O(nZ)$.

Algorithmus 2.5 (Dynamic Programming für Knapsack)

Eingabe: $z_1, \dots, z_n, Z, p_1, \dots, p_n$
Ausgabe: Funktion $P : \{1, \dots, Z\} \times \{1, \dots, n\} \rightarrow \mathbb{R}; (x, i) \mapsto P(x, i)$
 mit $P(x, i) = \max \sum_{j=1}^i p_j y_j$ mit $\sum_{j=1}^i z_j y_j \leq x$, für $y_j \in \{0, 1\}$

```

1: for (x = 0) to Z do
2:   P(x, 0) := 0
3: for (i = 1) to n do
4:   for (x = 0) to (z_i - 1) do
5:     P(x, i) := P(x, i - 1)
6:   for (x = z_i) to Z do
7:     if ((P(x - z_i, i - 1) + p_i) > P(x, i - 1)) then
8:       P(x, i) := P(x - z_i, i - 1) + p_i
9:     else
0:       P(x, i) := P(x, i - 1)
    
```

$O(nZ)$ {

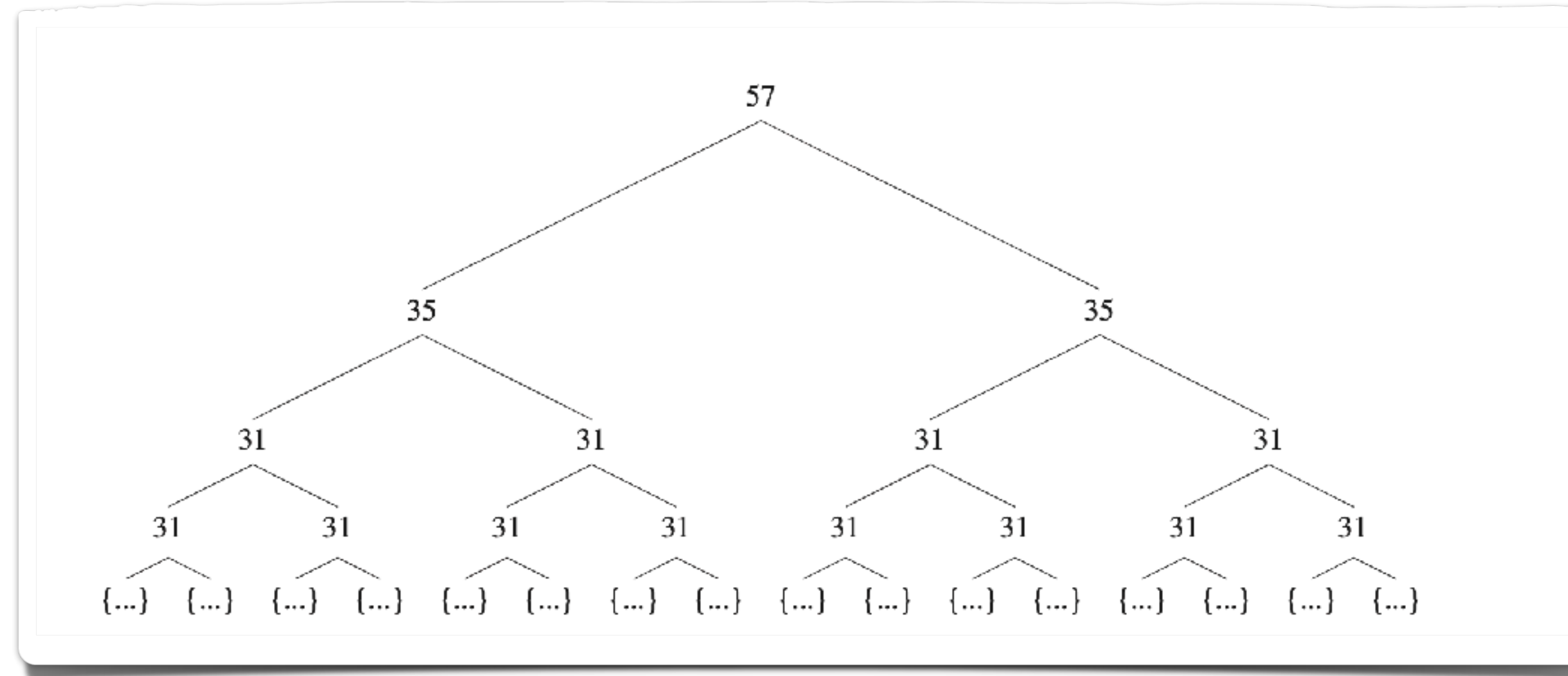
$x \backslash i$	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	6	6	6	6	6	6	6	6
2	0	0	6	6	6	11	11	11	11	11
3	0	0	6	6	6	11	11	11	14	14
4	0	0	6	6	6	11	11	11	14	15
5	0	0	6	6	6	11	11	12	14	15
6	0	0	6	6	6	11	11	12	14	15
7	0	0	6	6	6	11	11	12	14	15

Induktion!

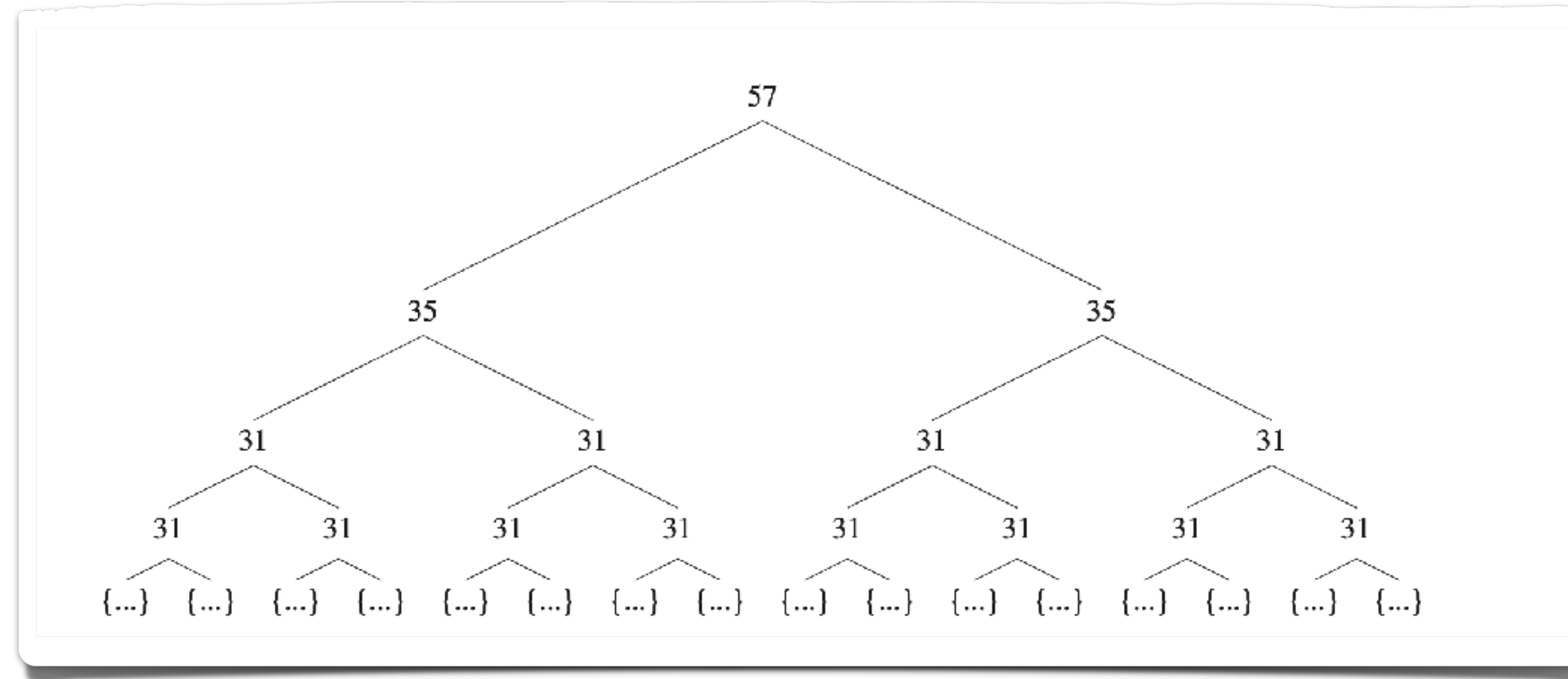
- 1. Objekt i ist zu teuer für Kostenschranke x , falls $z_i > x$
- 2. Objekt i ist nicht zu teuer, bringt aber keine Verbesserung
- 3. Objekt i ist nicht zu teuer und bringt eine Verbesserung

Branch-and-Bound

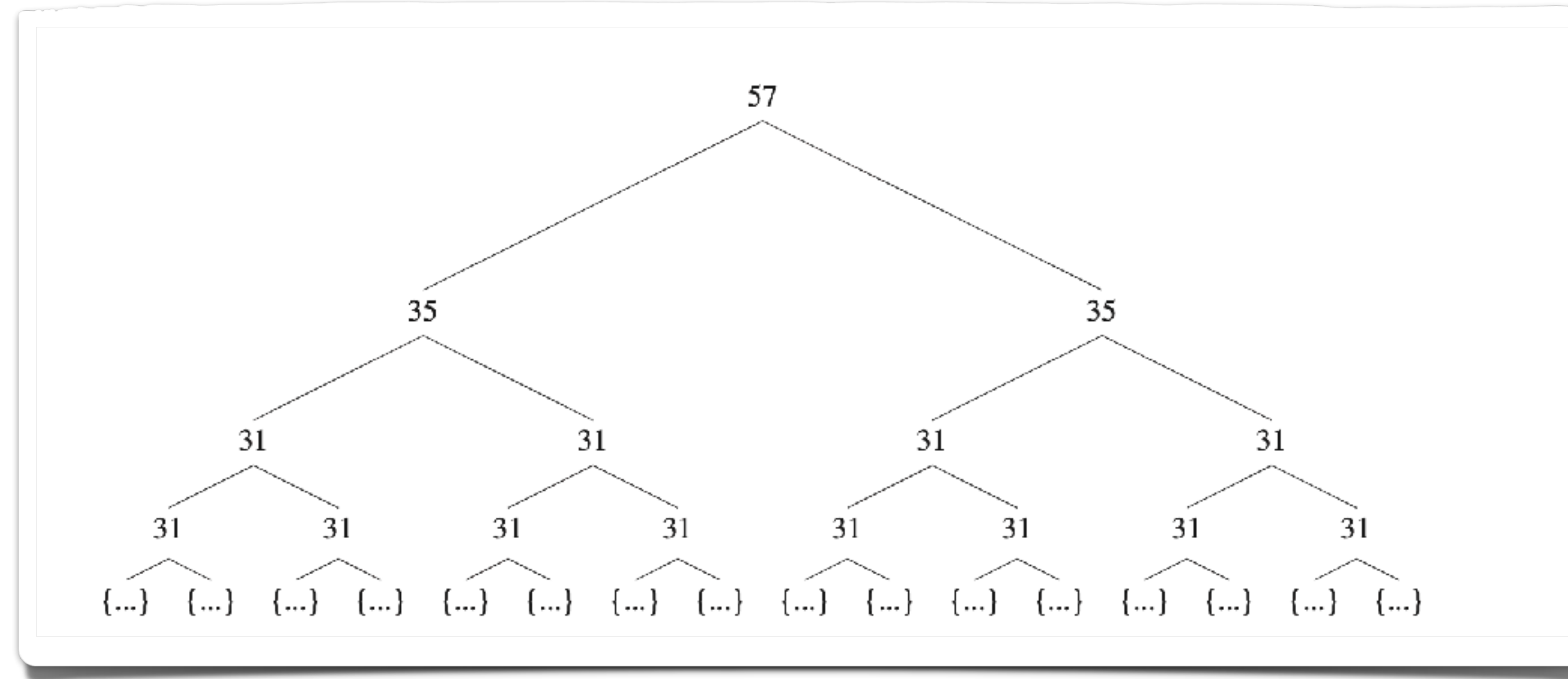
Branch-and-Bound



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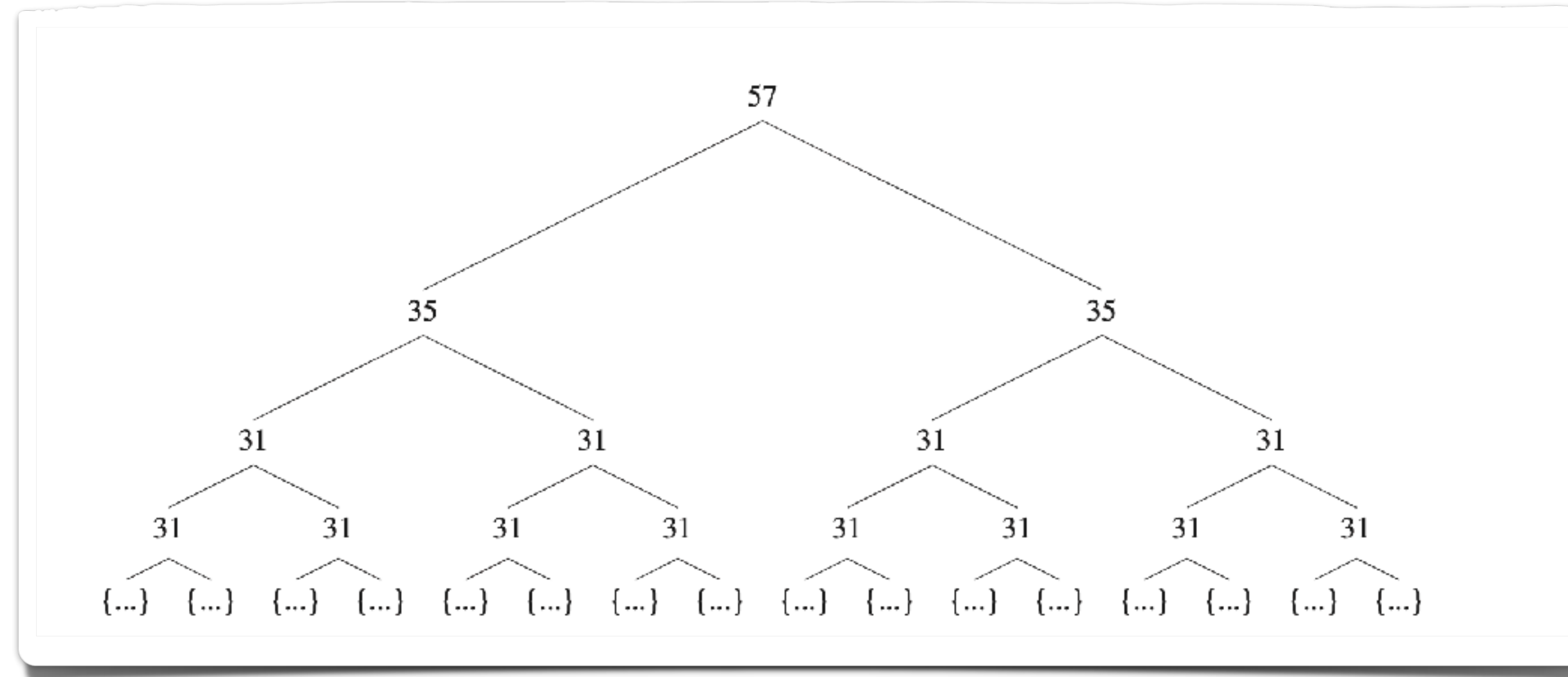
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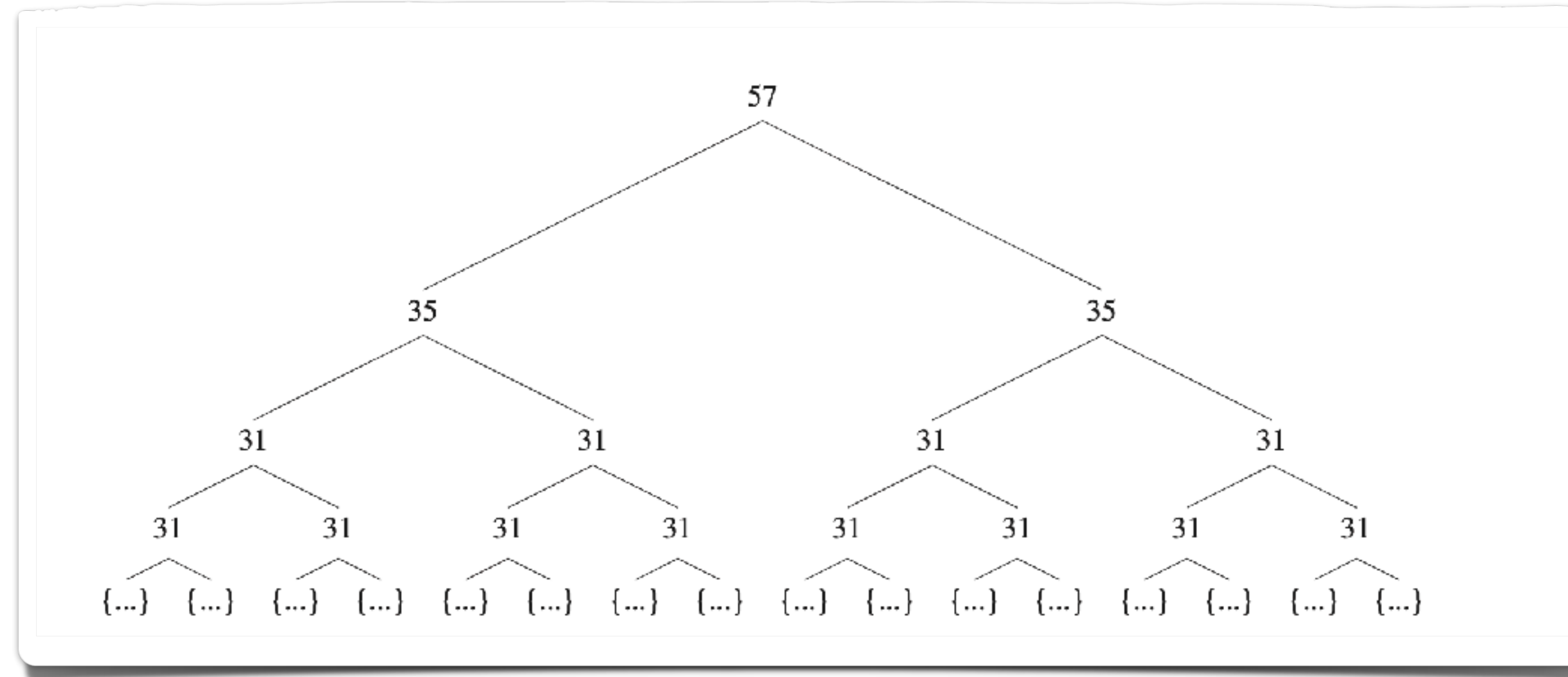
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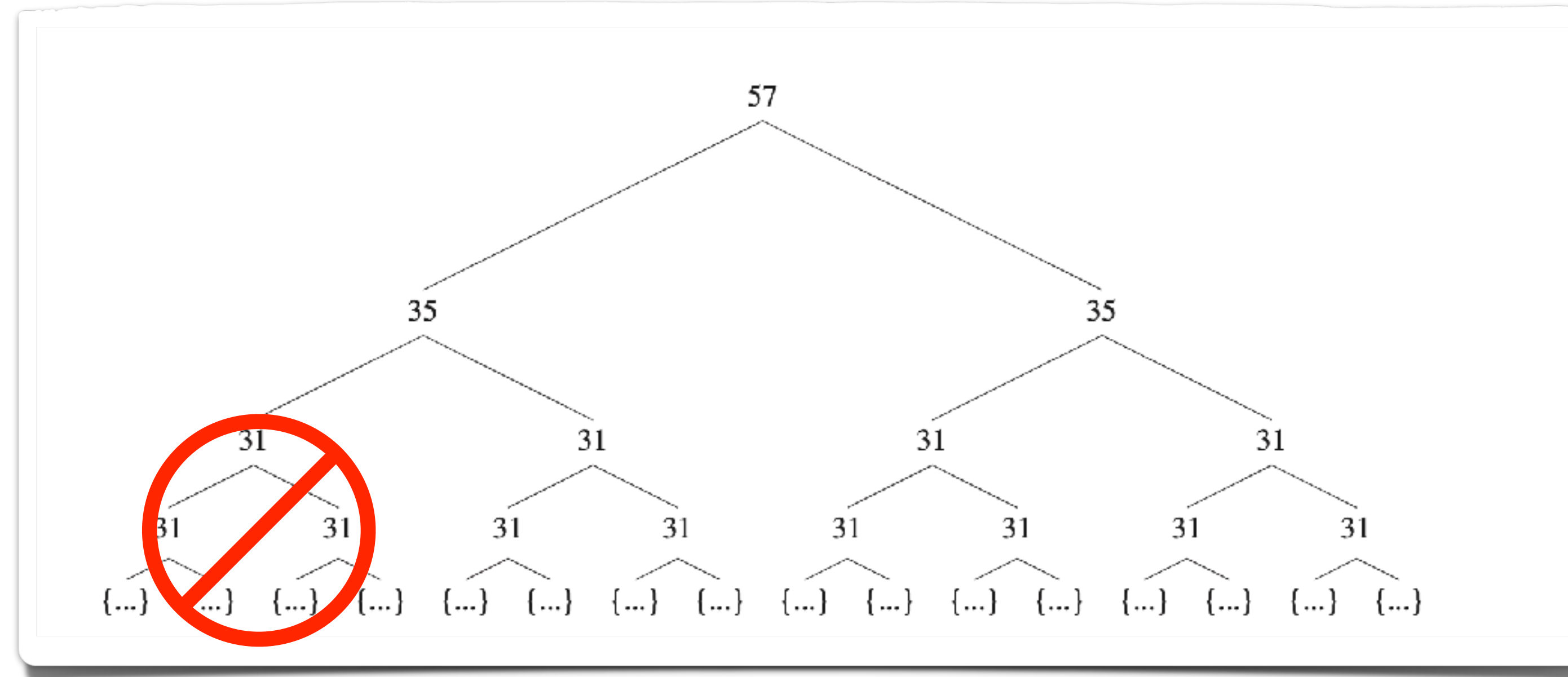
- Beschneide Enumerationsbaum

Branch-and-Bound



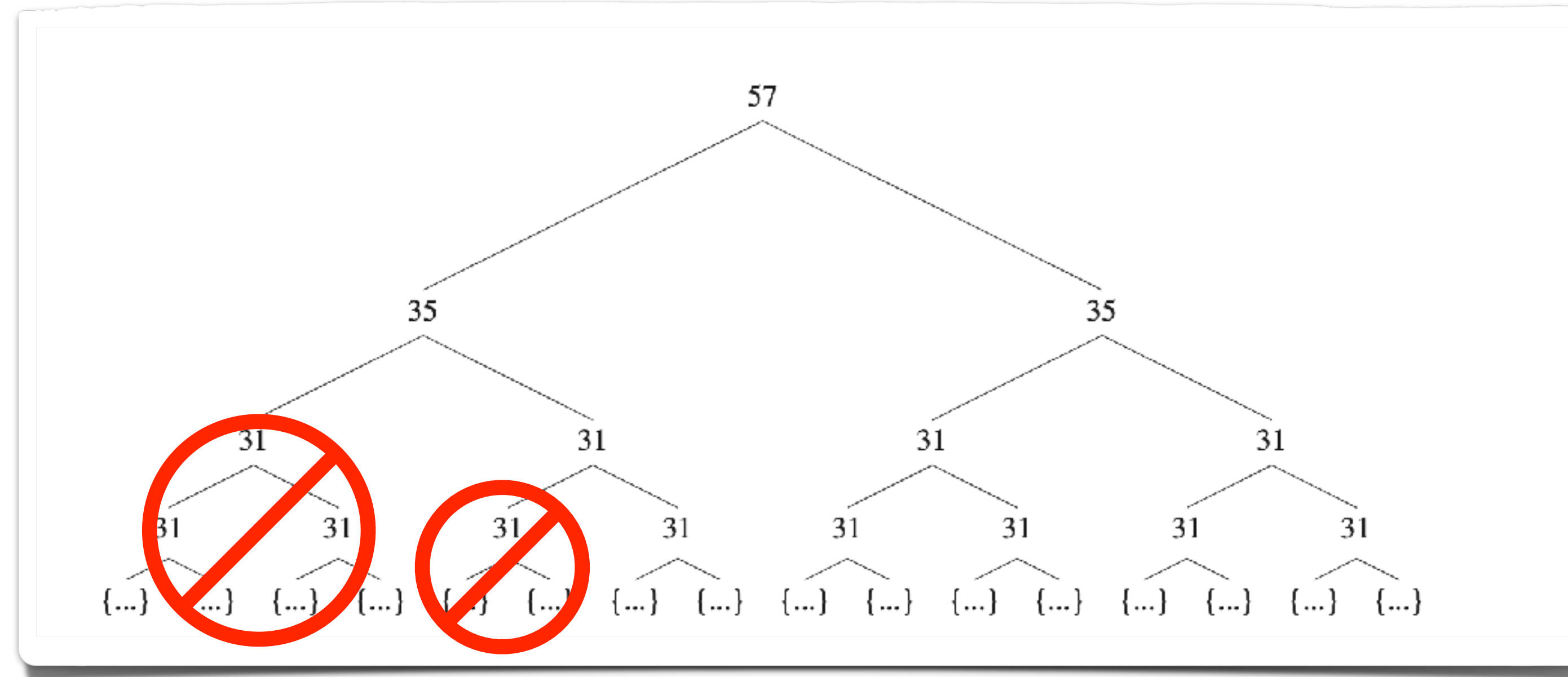
- Beschneide Enumerationsbaum
- Verfolge Mindest- und Maximalwerte

Branch-and-Bound



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- Verfolge Mindest- und Maximalwerte

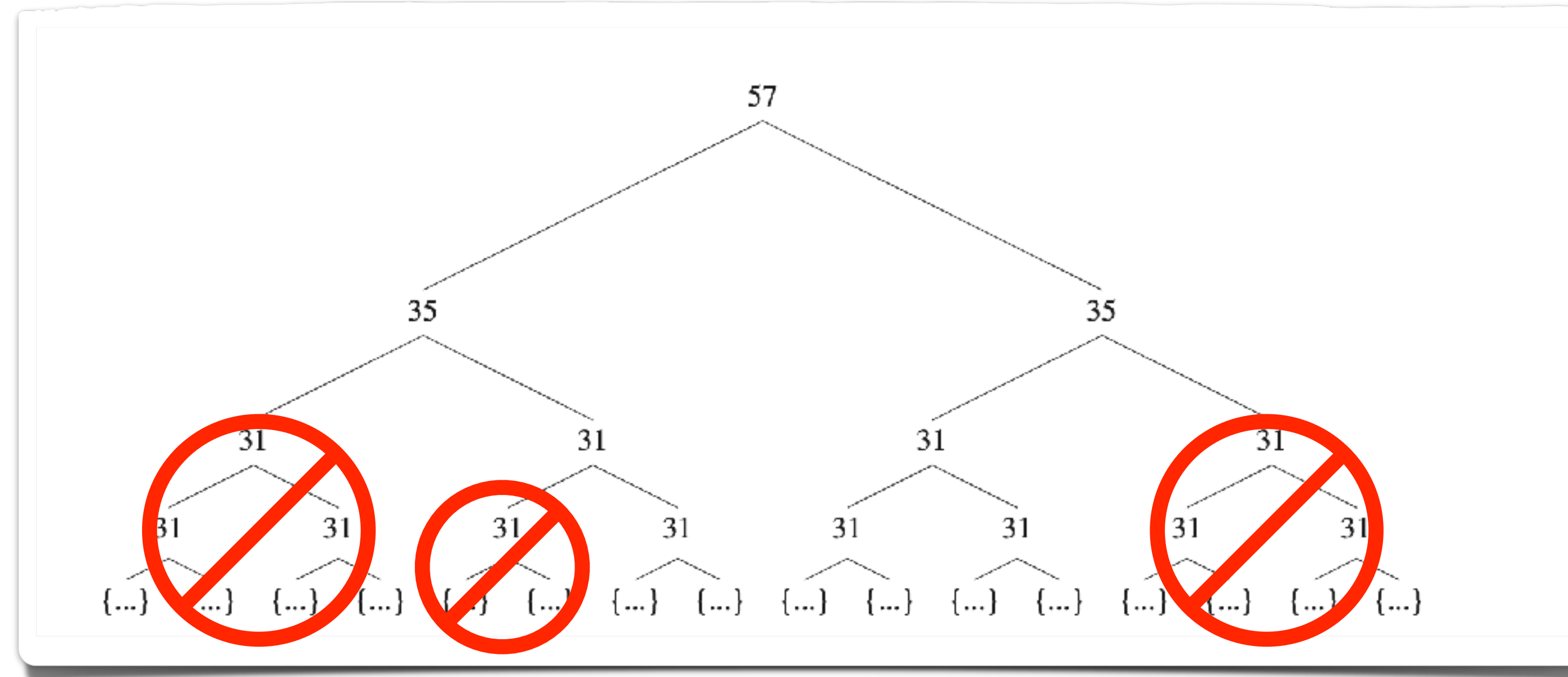
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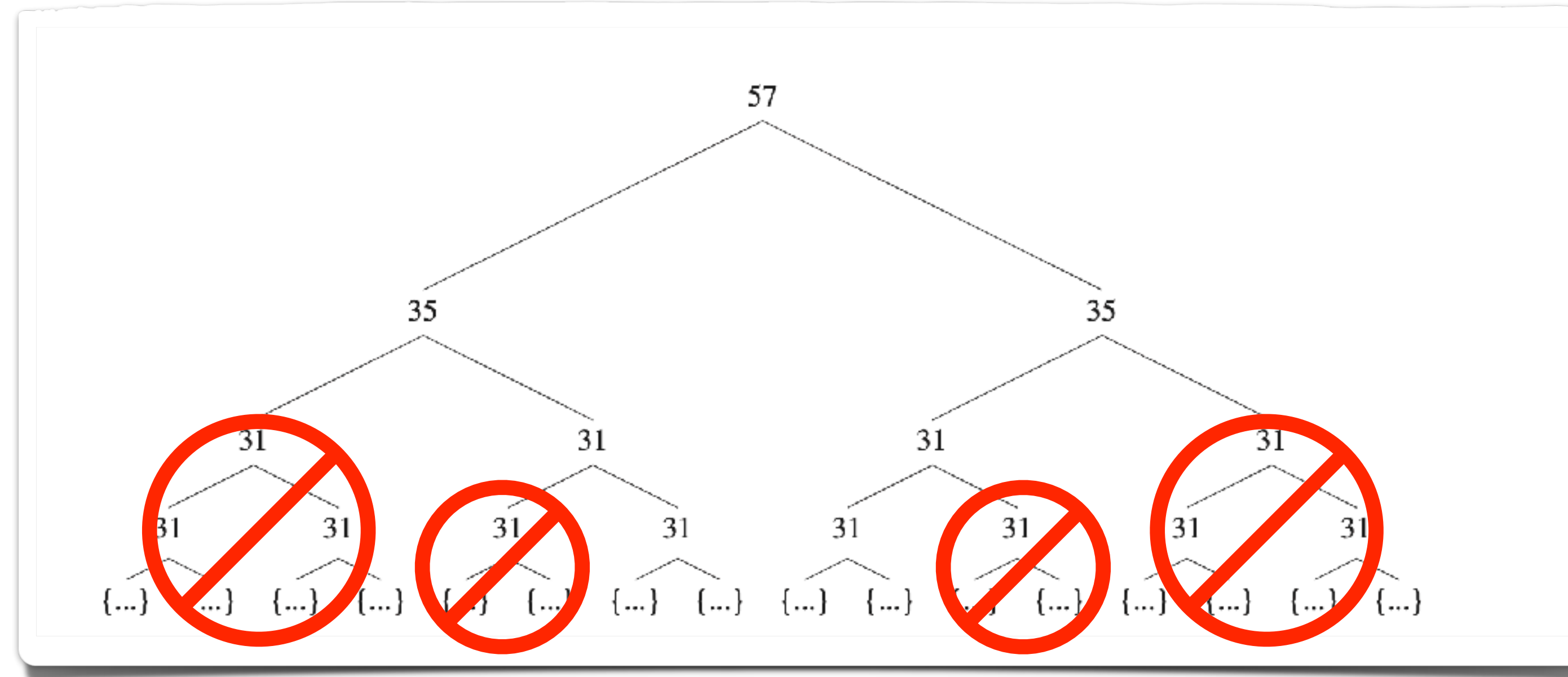
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July, 1960

NUMBER 3

AN AUTOMATIC METHOD OF SOLVING DISCRETE PROGRAMMING PROBLEMS

BY A. H. LAND AND A. G. DOIG

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1. INTRODUCTION

THERE IS A growing literature [1, 3, 5, 6] about optimization problems which could be formulated as linear programming problems with additional constraints that some or all of the variables may take only integral values. This form of linear programming arises whenever there are indivisibilities. It is not meaningful, for instance, to schedule 3-7/10 flights between two cities, or to undertake only 1/4 of the necessary setting up operation for running a job through a machine shop. Yet it is basic to linear programming that the variables are free to take on any positive value,¹ and this sort of answer is very likely to turn up.

In some cases, notably those which can be expressed as transport problems, the linear programming solution will itself yield discrete values of the variables. In other cases the percentage change in the maximand² from common sense rounding of the variables is sufficiently small to be neglected. But there remain many problems where the discrete variable constraints are significant and costly.

Until recently there was no general automatic routine for solving such problems, as opposed to procedures for proving the optimality of conjectured solutions, and the work reported here is intended to fill the gap. About the time of its completion an alternative method was proposed by Gomory [5] and subsequently extended by Beale [1]. Gomory's method

¹ Or more generally, any value within a bounded interval.

² We shall speak throughout of maximisation, but of course an exactly analogous argument applies to minimisation.

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497



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497



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BY A. H. LAND AND A. G. DOIG

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1. INTRODUCTION

THERE IS A growing literature [1, 3, 5, 6] about optimization problems which could be formulated as linear programming problems with additional constraints that some or all of the variables may take only integral values. This form of linear programming arises whenever there are indivisibilities. It is not meaningful, for instance, to schedule 3-7/10 flights between two cities, or to undertake only 1/4 of the necessary setting up operation for running a job through a machine shop. Yet it is basic to linear programming that the variables are free to take on any positive value,¹ and this sort of answer is very likely to turn up.

In some cases, notably those which can be expressed as transport problems, the linear programming solution will itself yield discrete values of the variables. In other cases the percentage change in the maximand² from common sense rounding of the variables is sufficiently small to be neglected. But there remain many problems where the discrete variable constraints are significant and costly.

Until recently there was no general automatic routine for solving such problems, as opposed to procedures for proving the optimality of conjectured solutions, and the work reported here is intended to fill the gap. About the time of its completion an alternative method was proposed by Gomory [5] and subsequently extended by Beale [1]. Gomory's method

¹ Or more generally, any value within a bounded interval.

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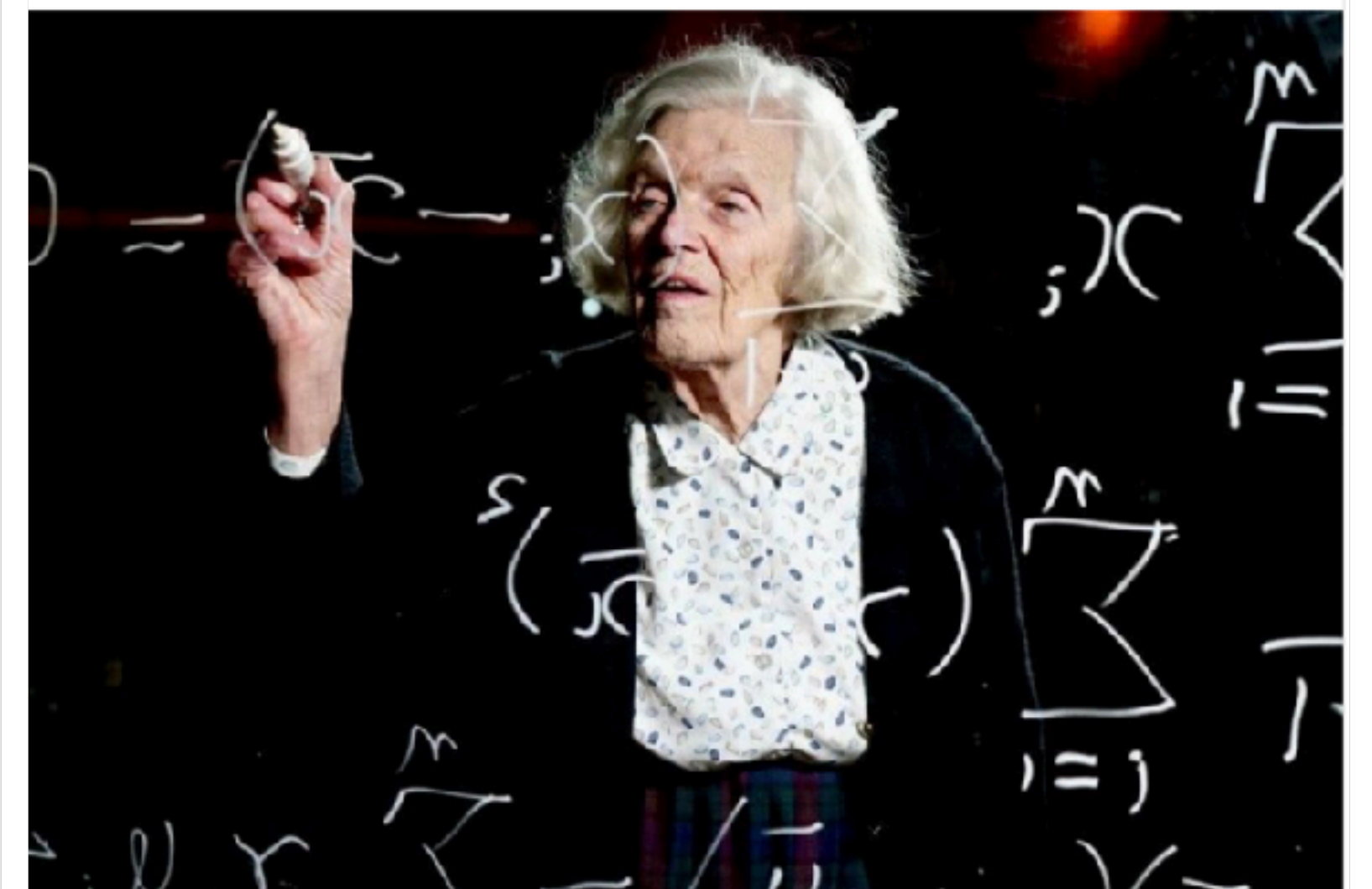


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7:30 / By Lauren Day

Posted 8 Oct 2018, updated 10 Oct 2018



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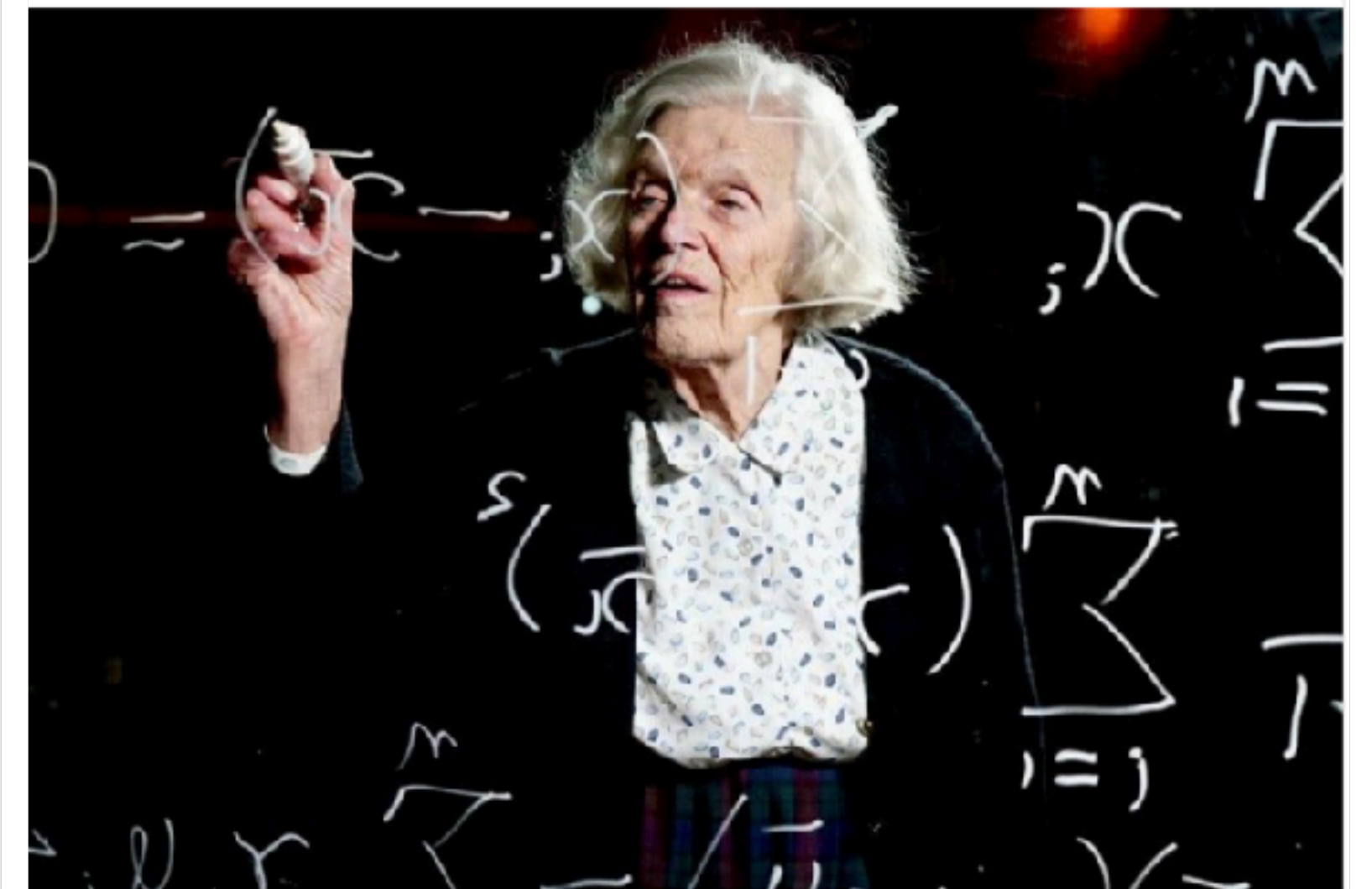


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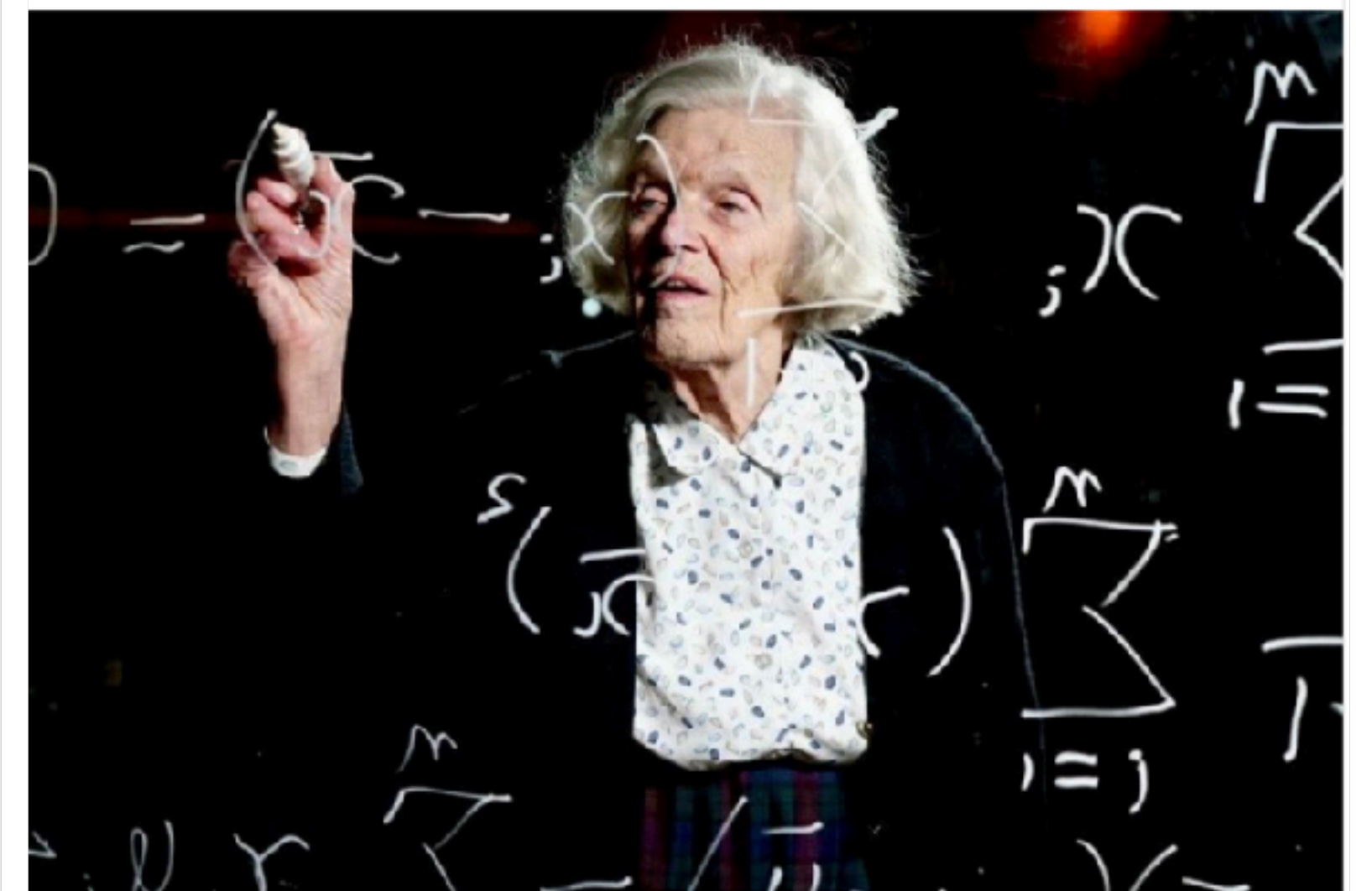


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In June 2019, Harcourt was made an **Officer of the Order of Australia**

3.2 Methodik

Ideen

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4. Wenn der erreichbare Wert in einem Teilbaum kleiner bleiben muss als der im ganzen Baum bereits erreichte, können wir den aktuellen Teilbaum abschneiden

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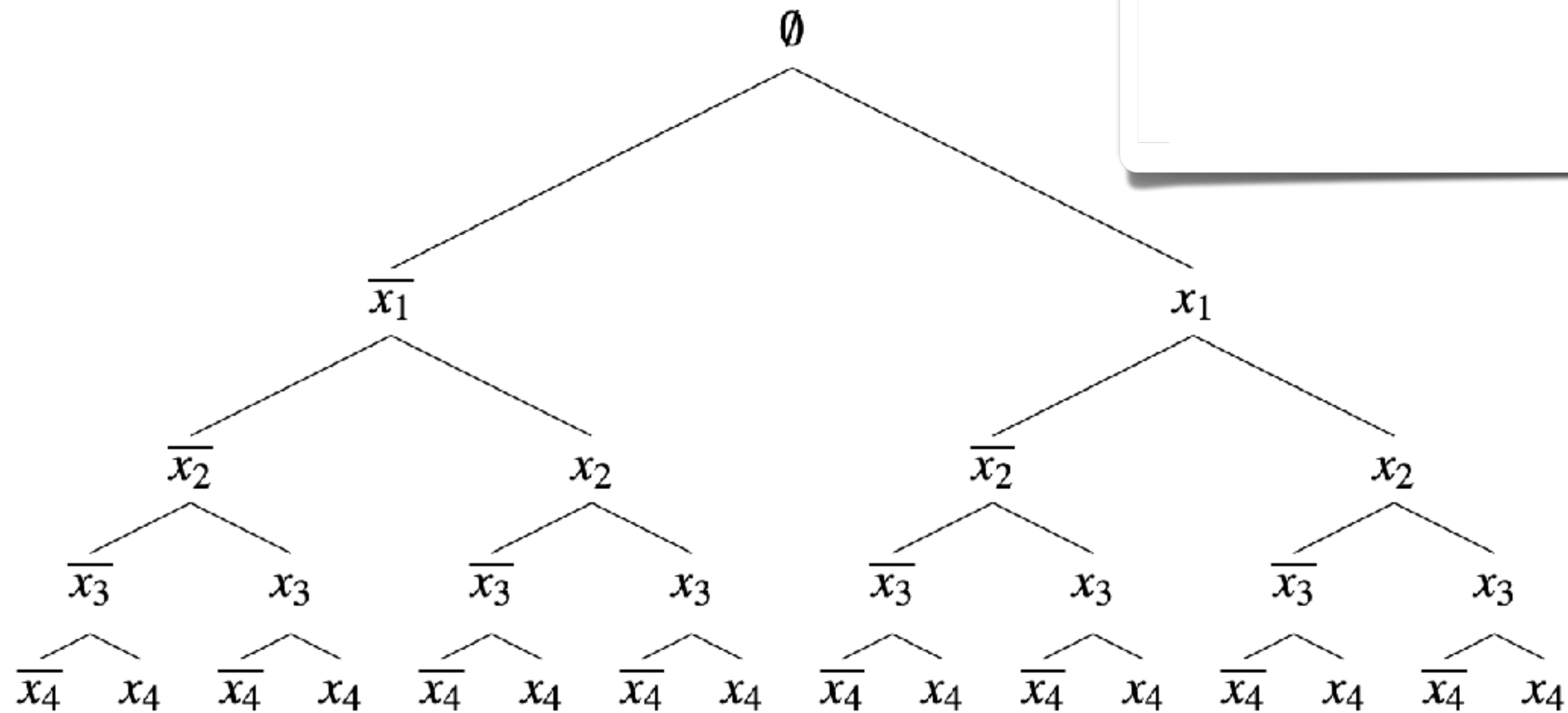


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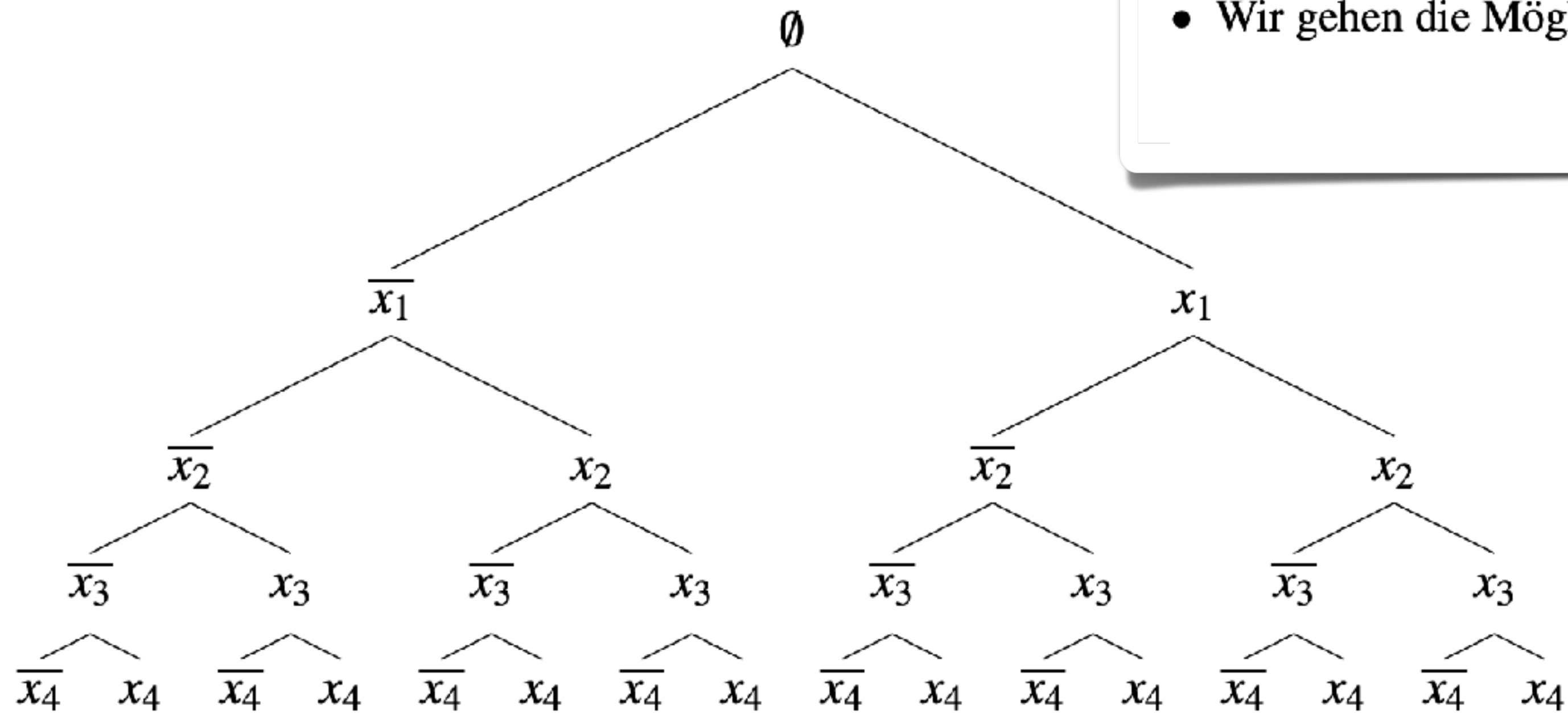
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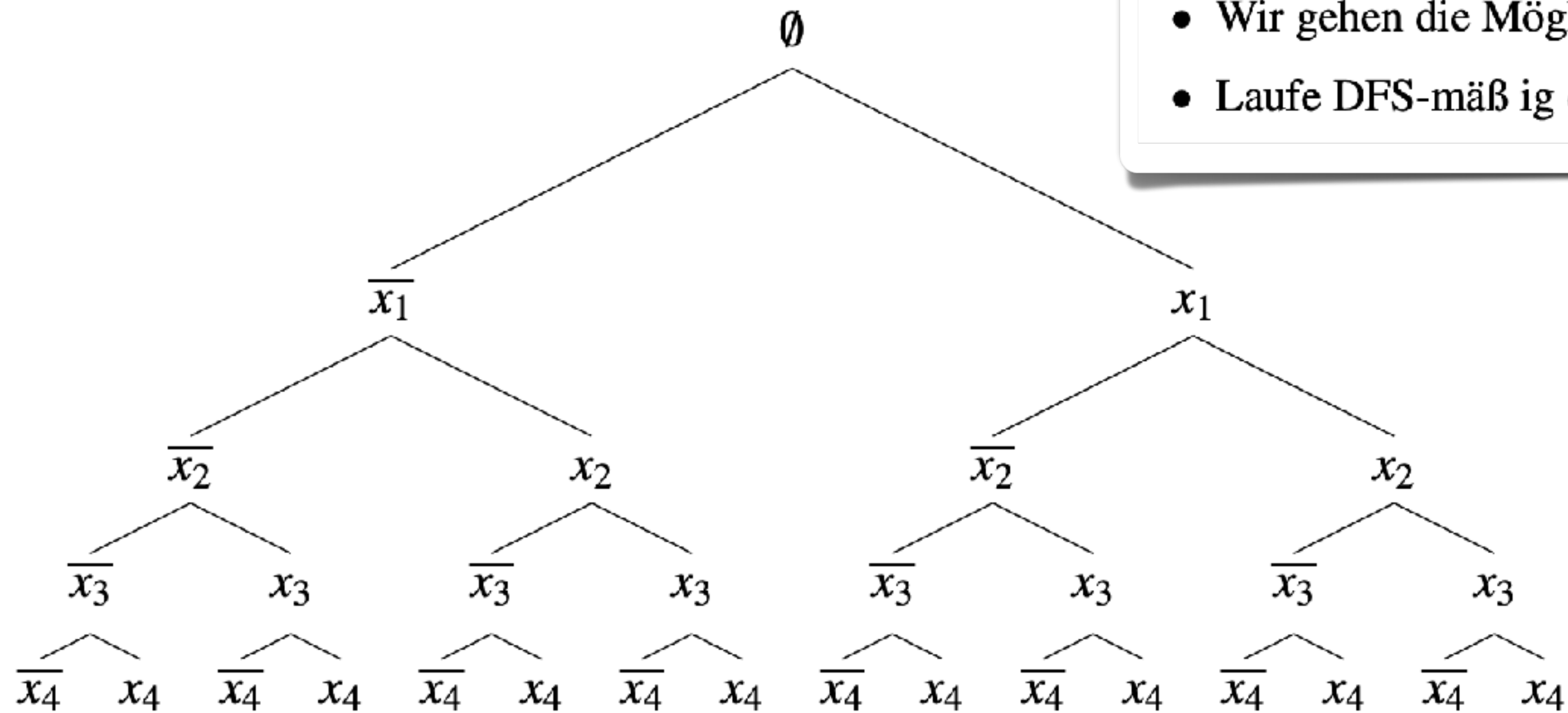
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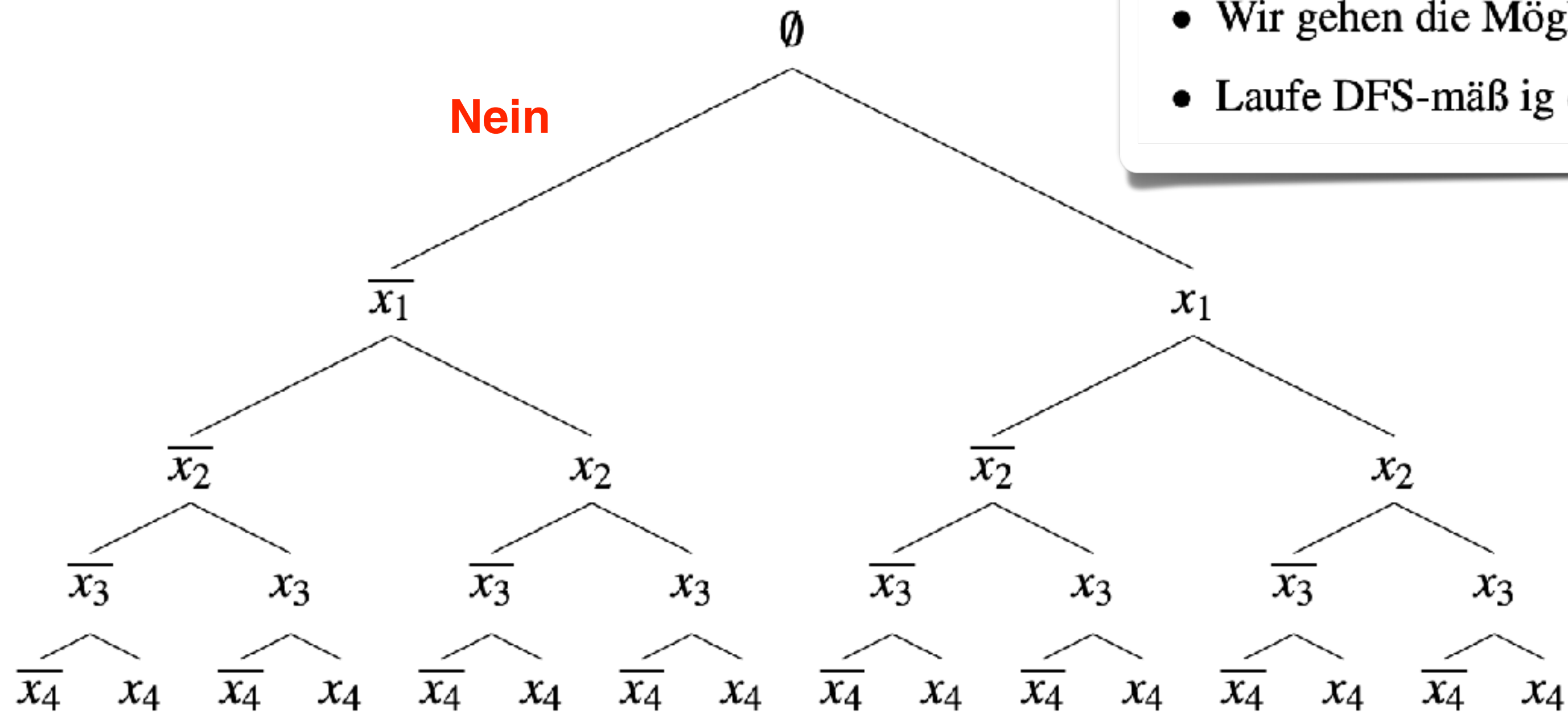
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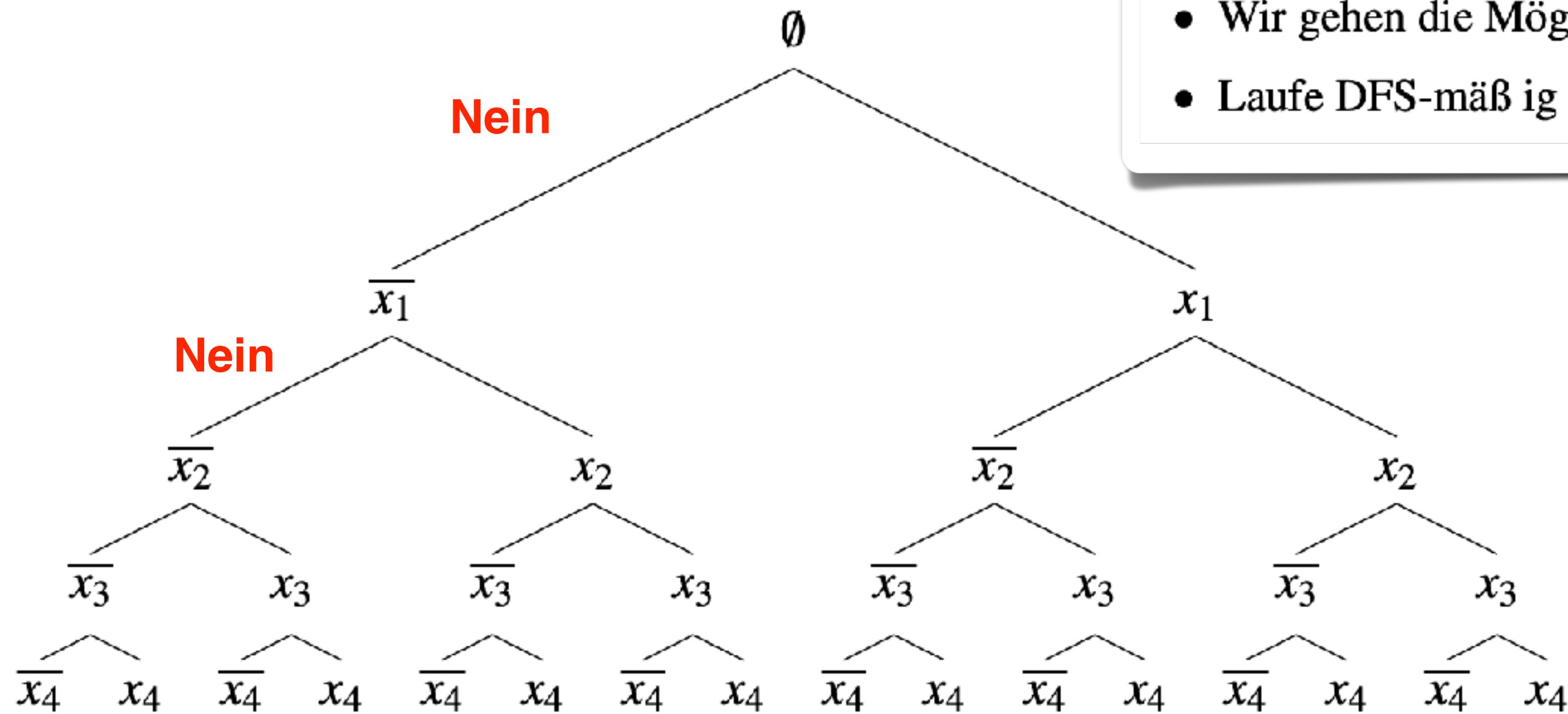
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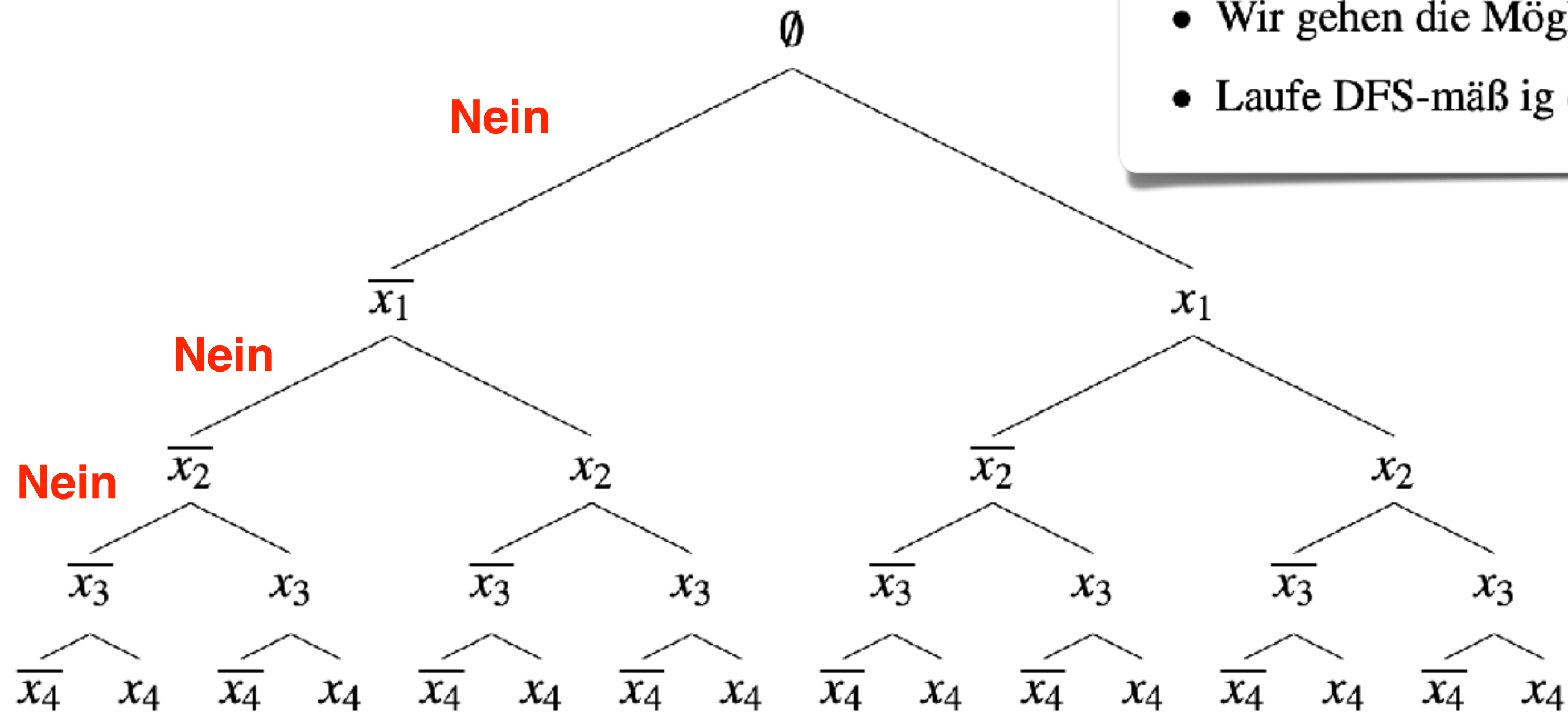
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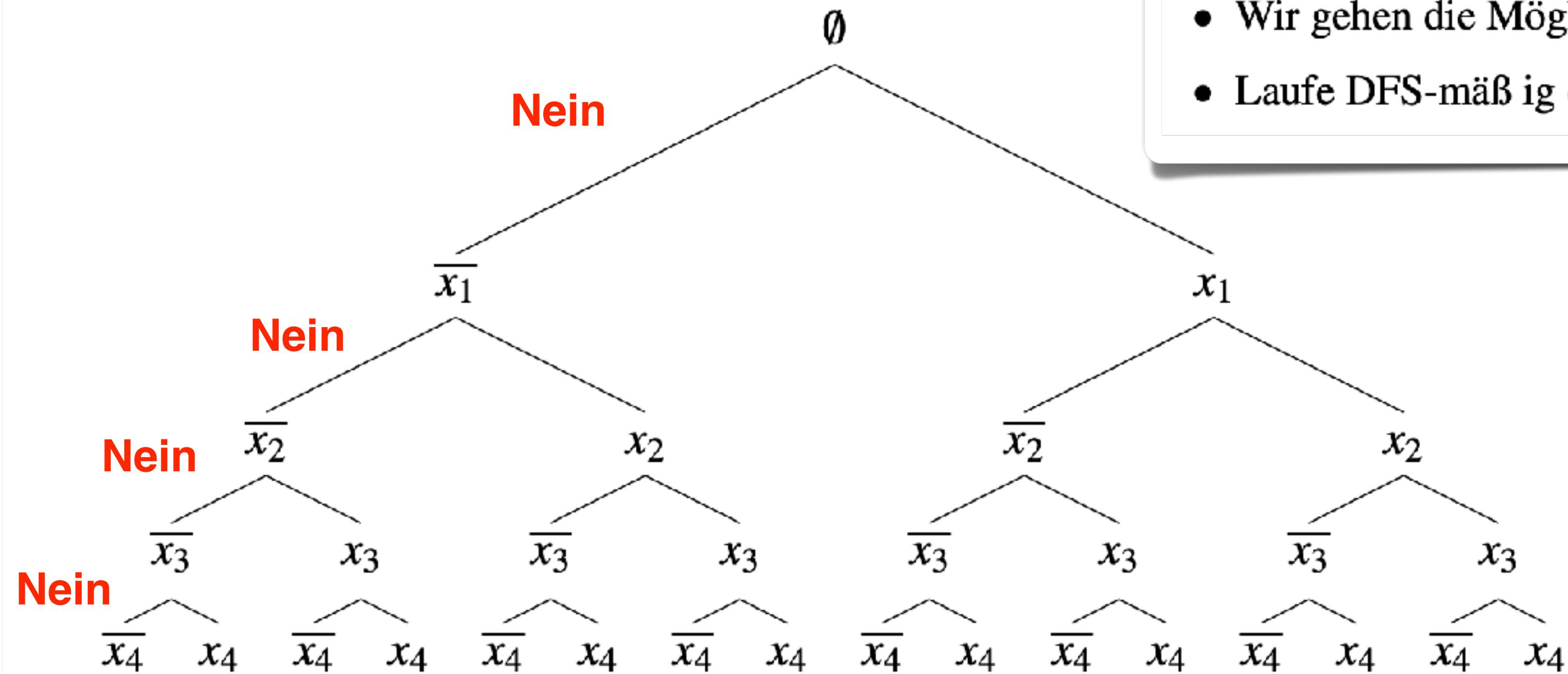
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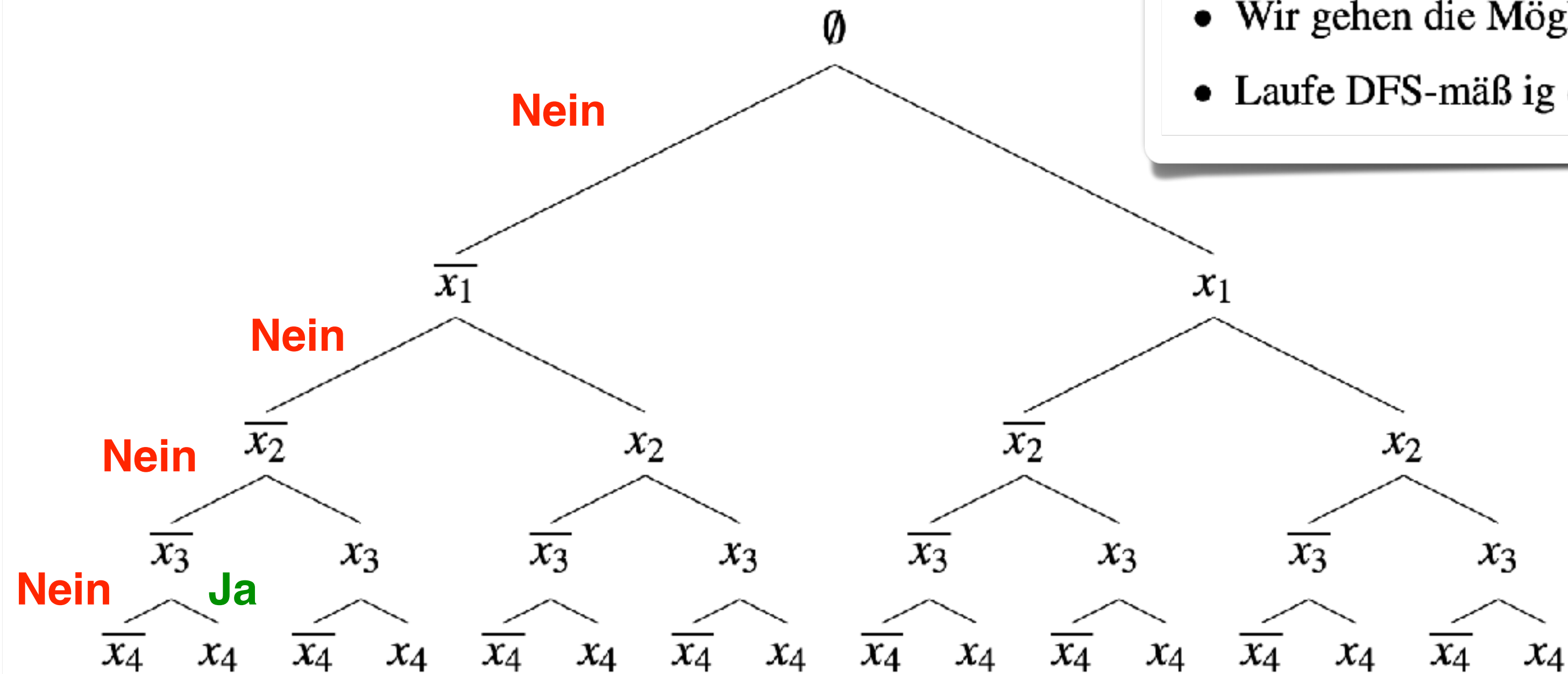
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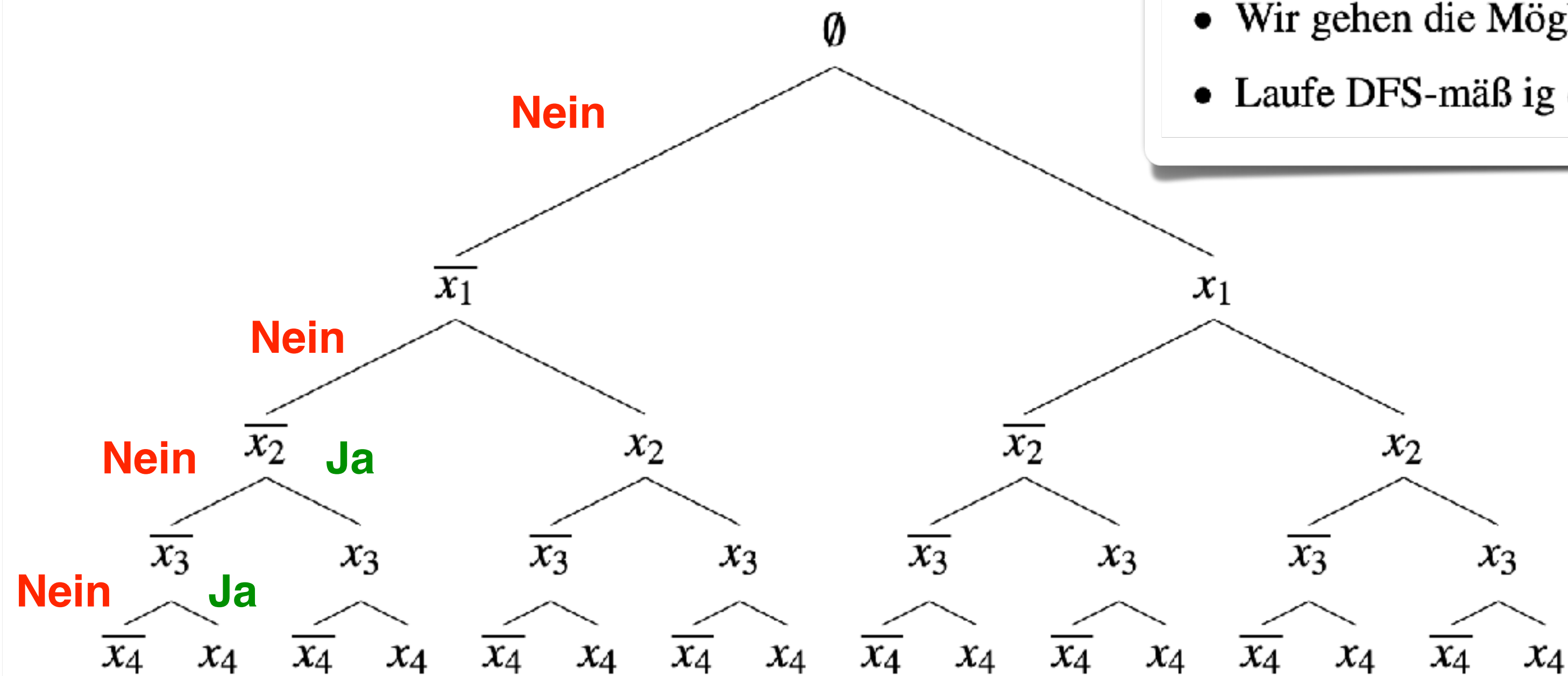
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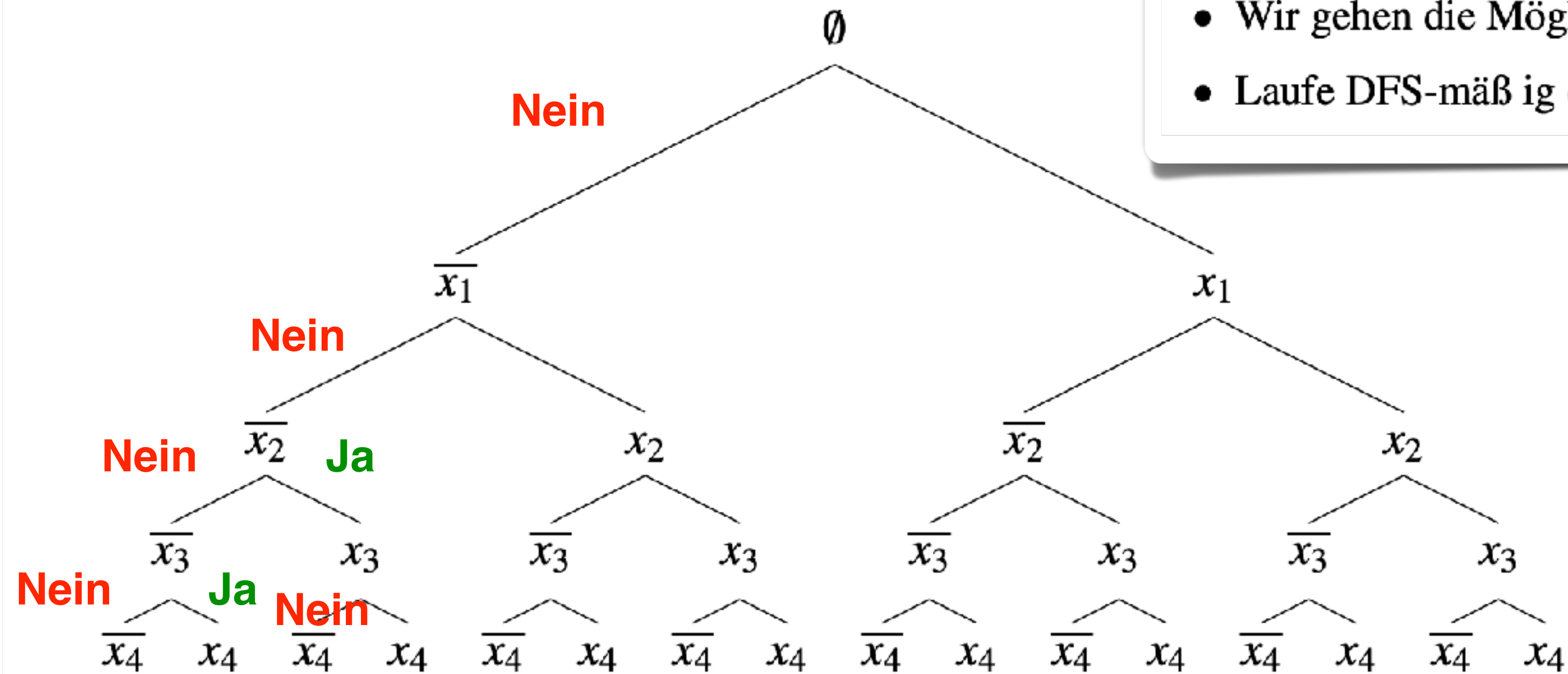
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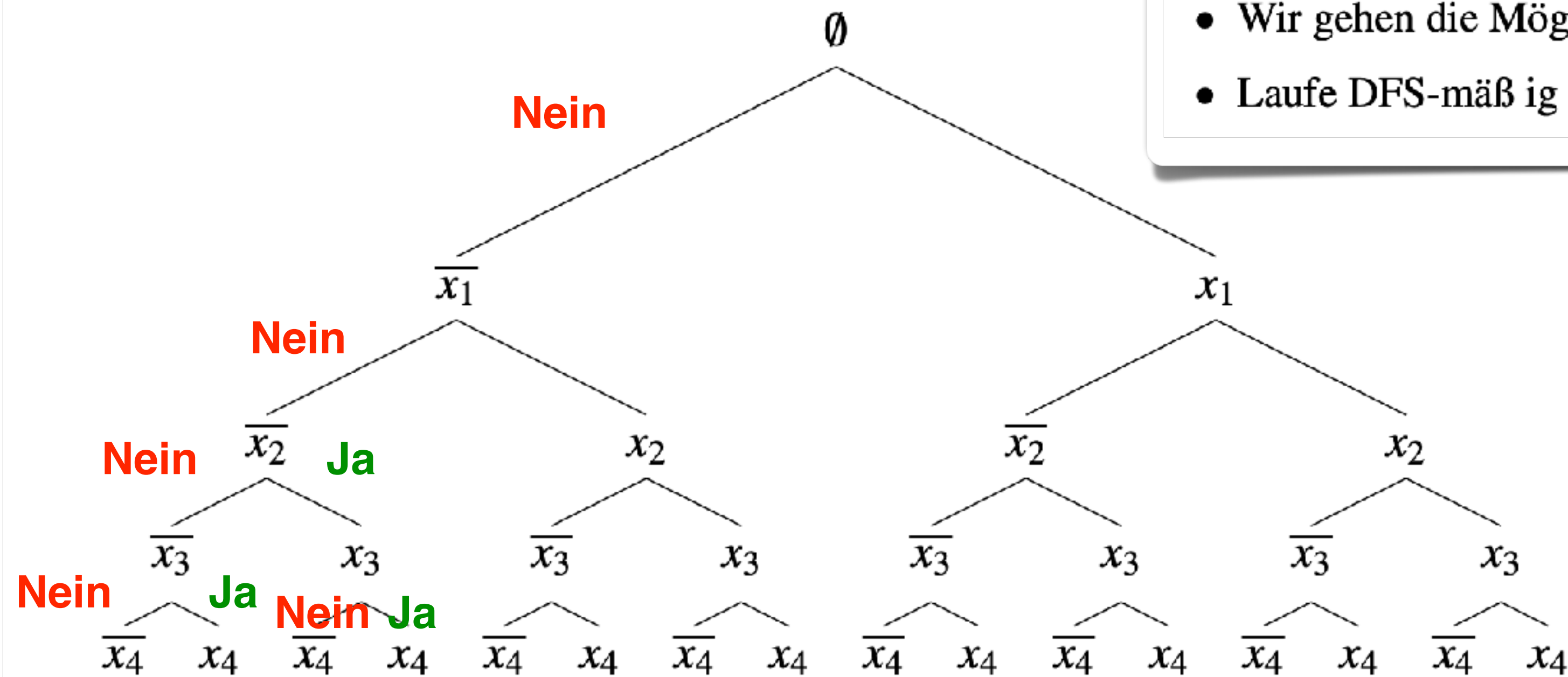
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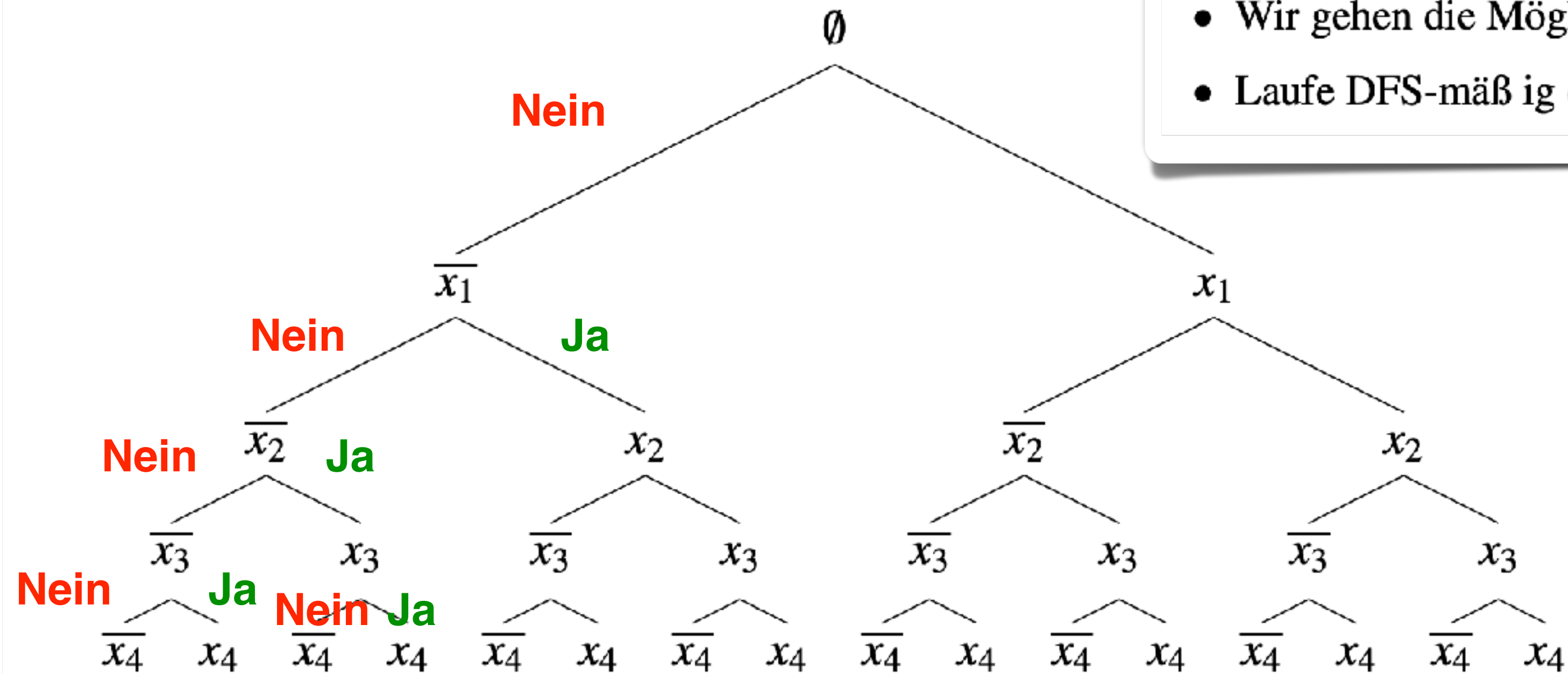
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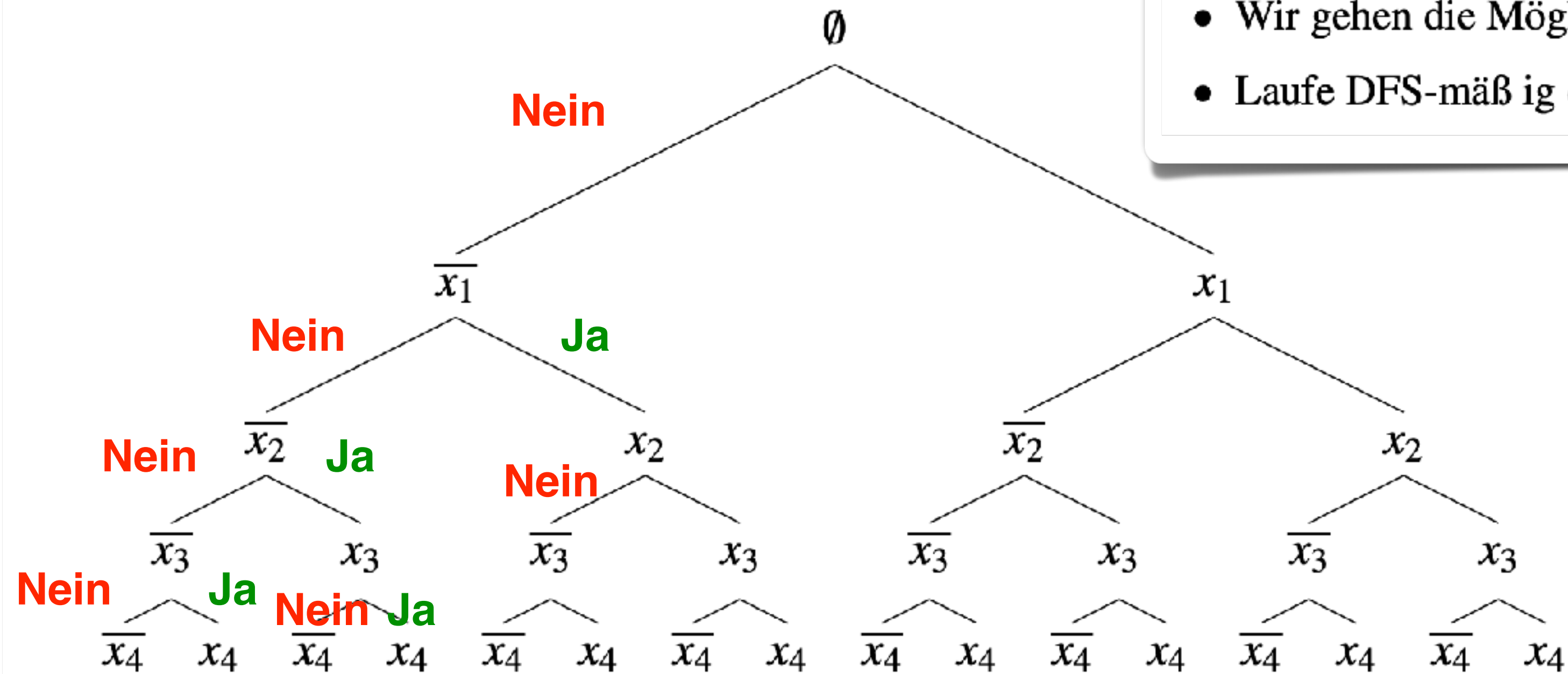
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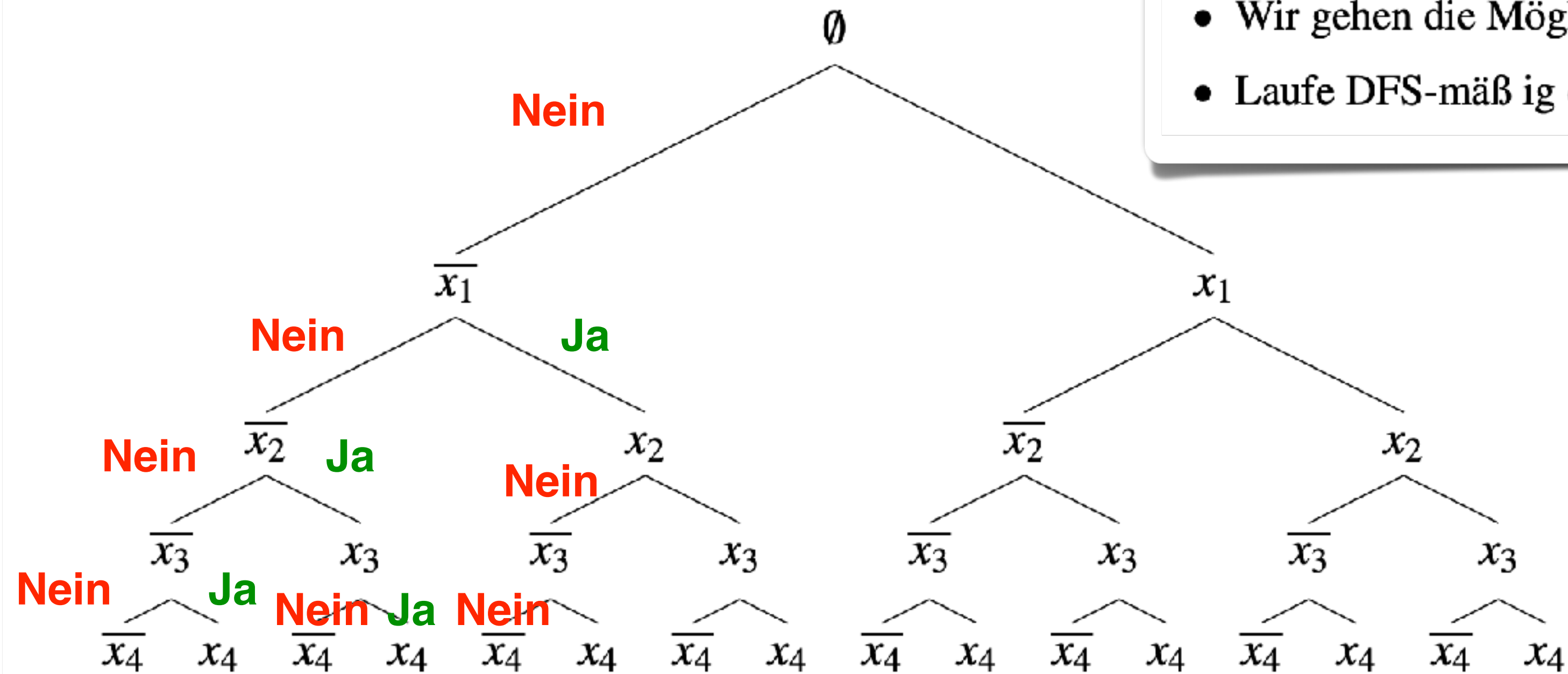
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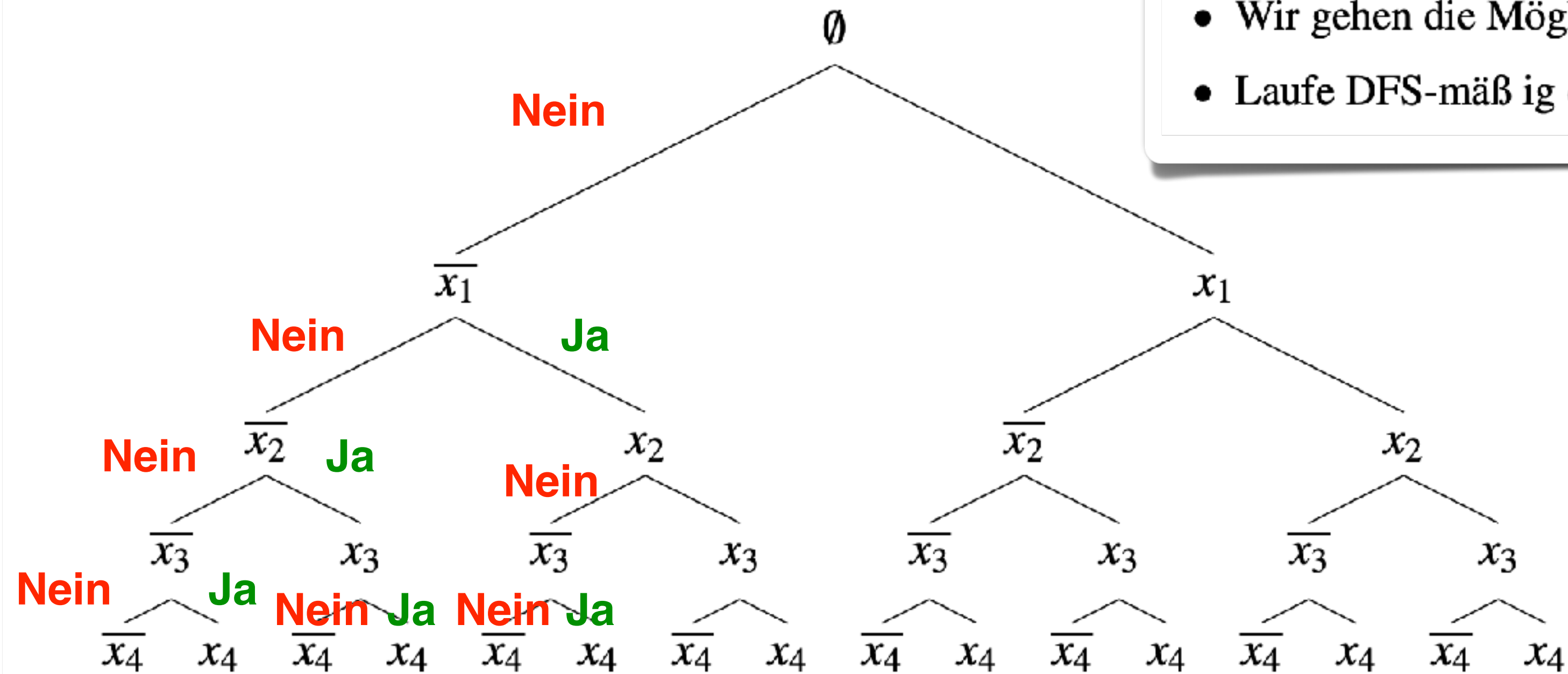
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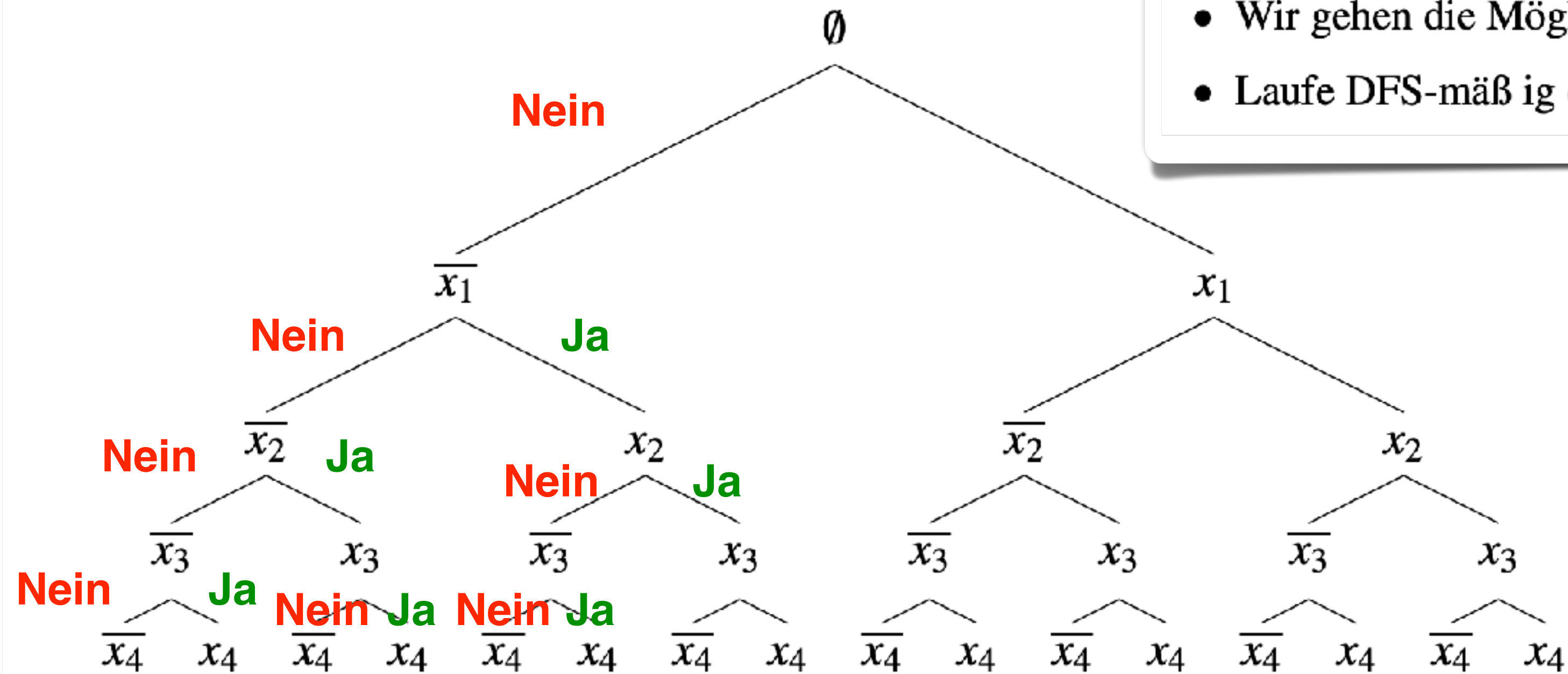
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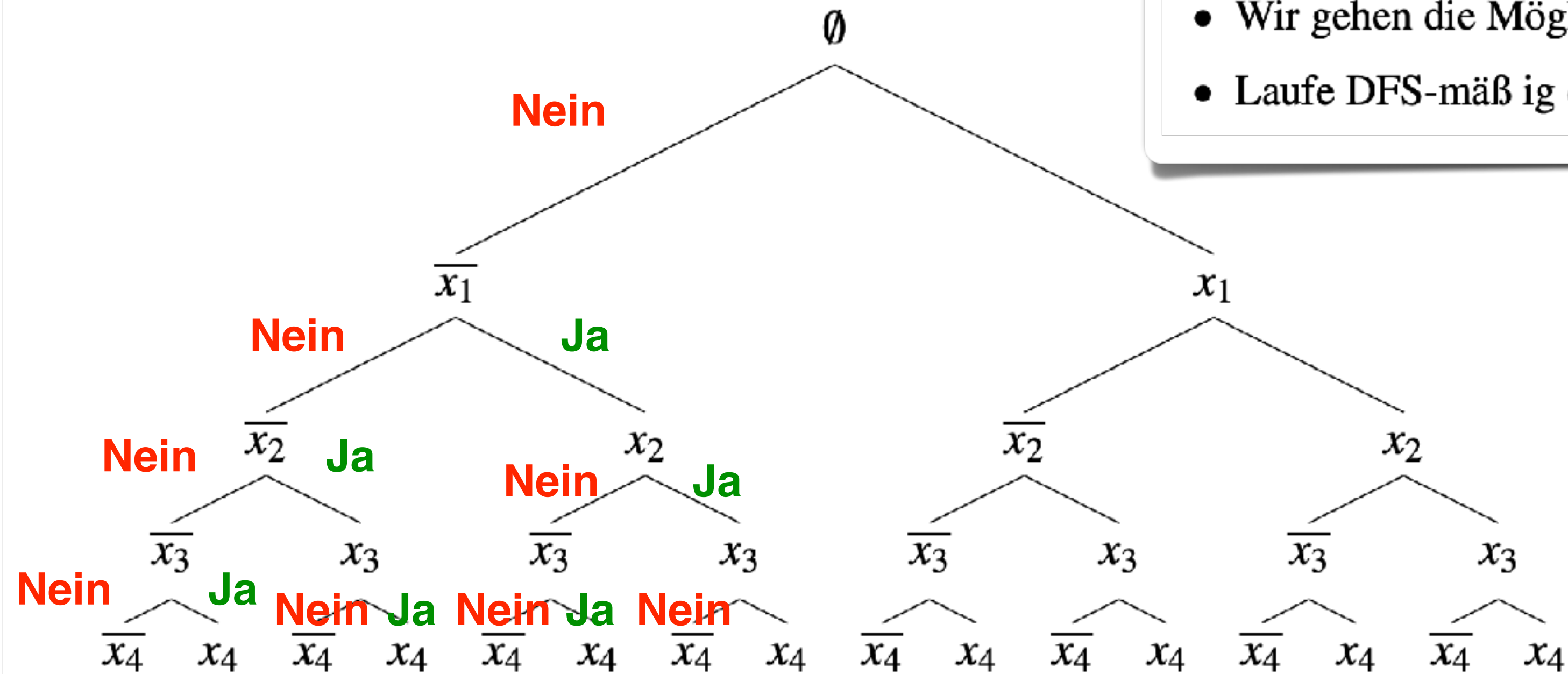
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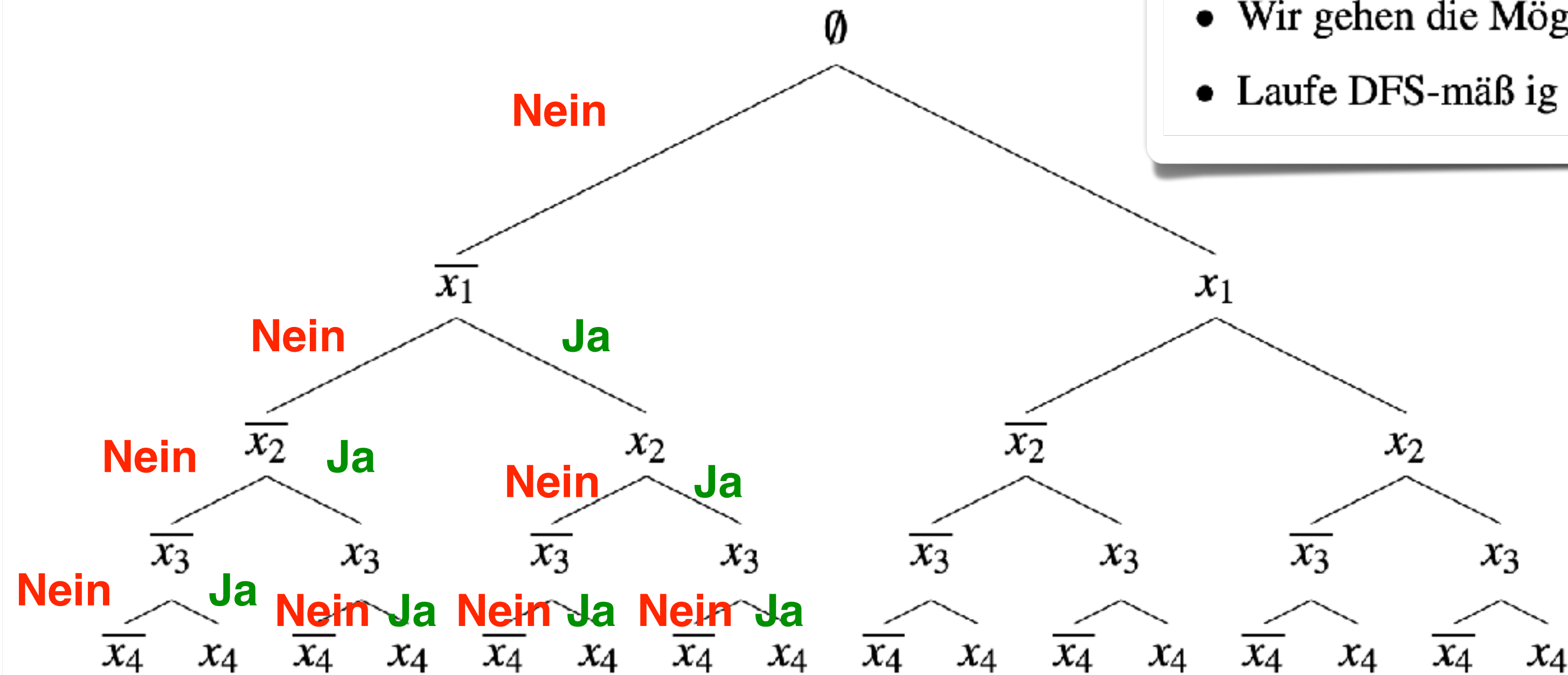
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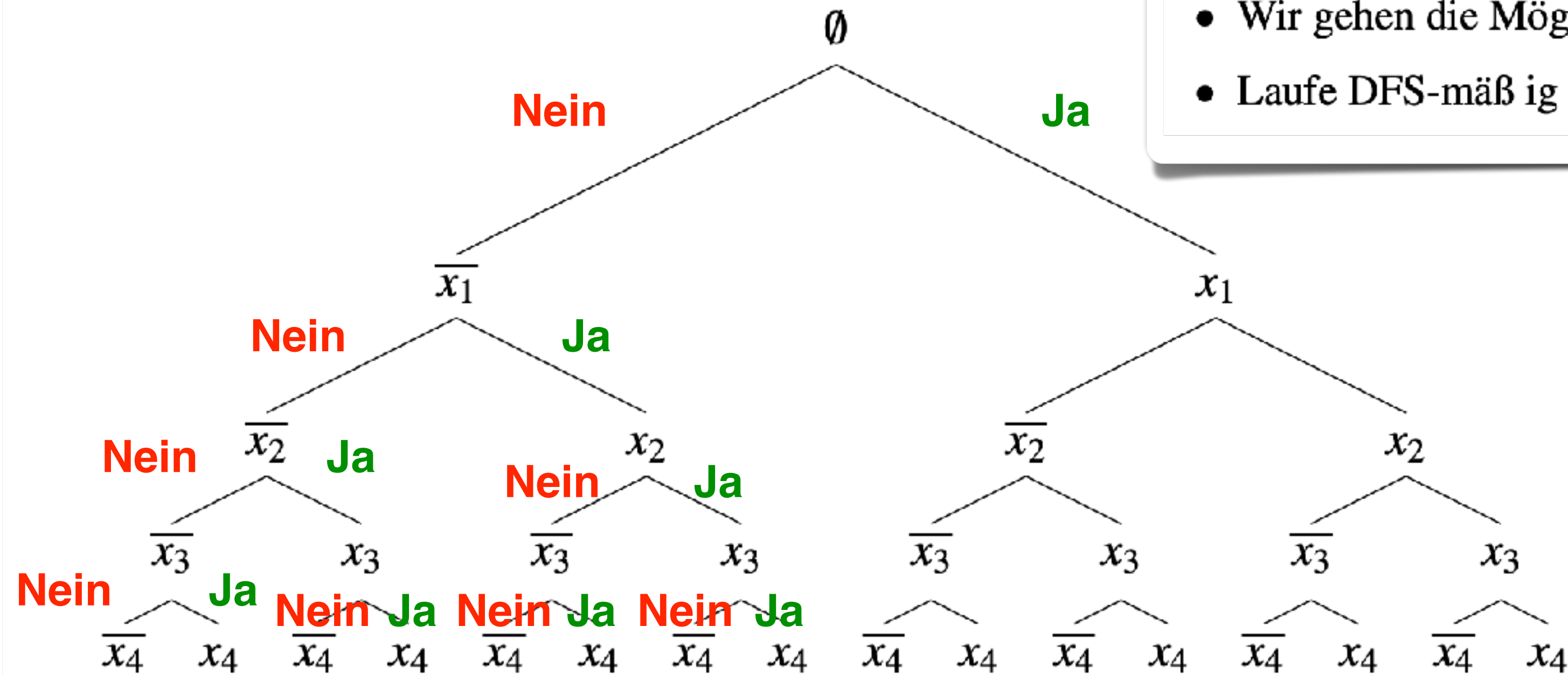
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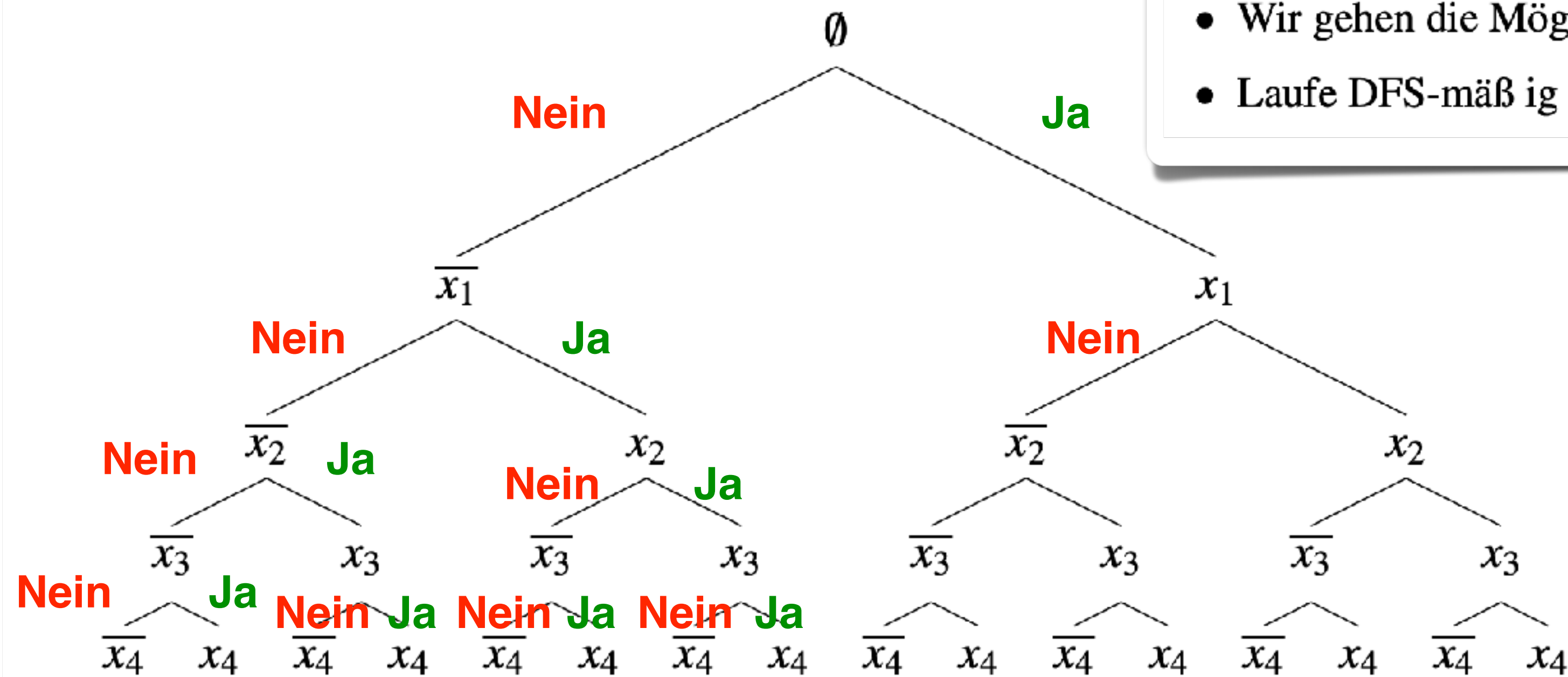
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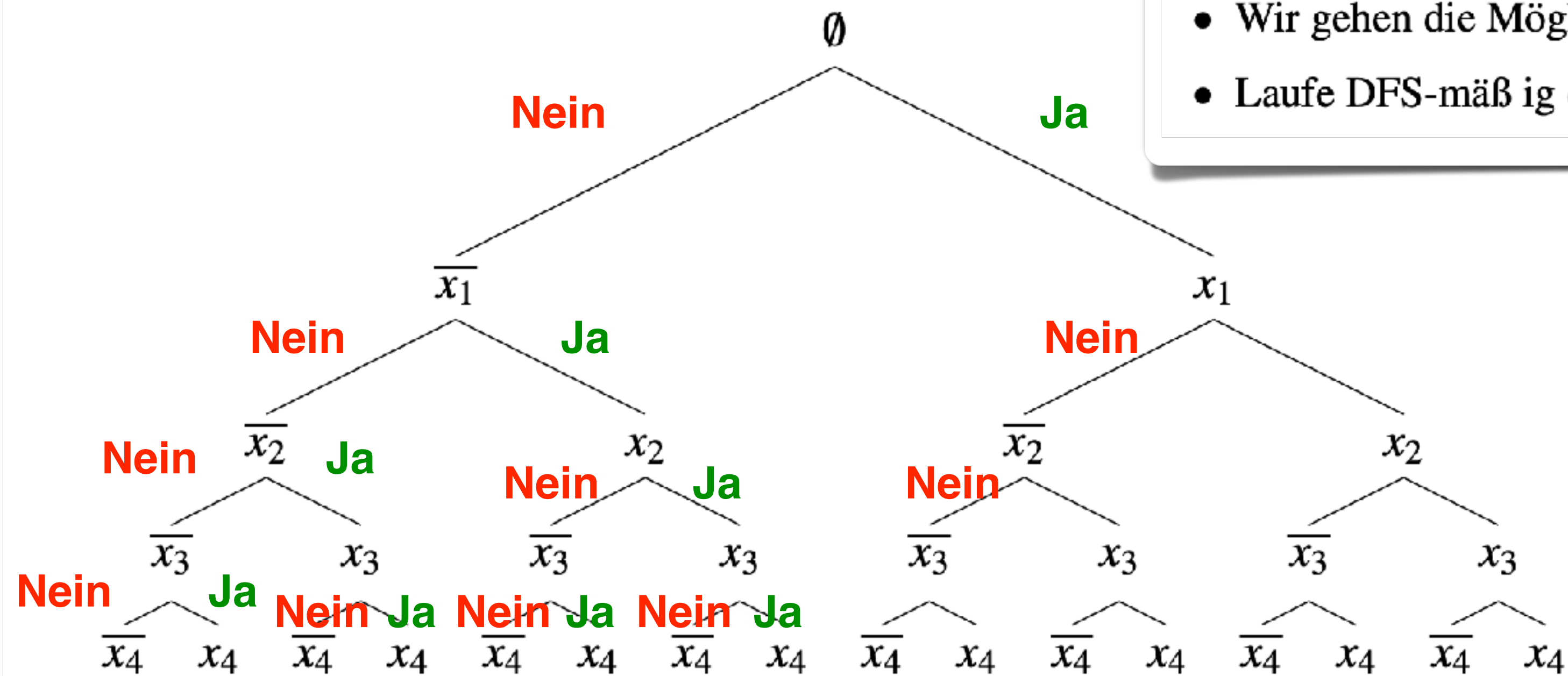
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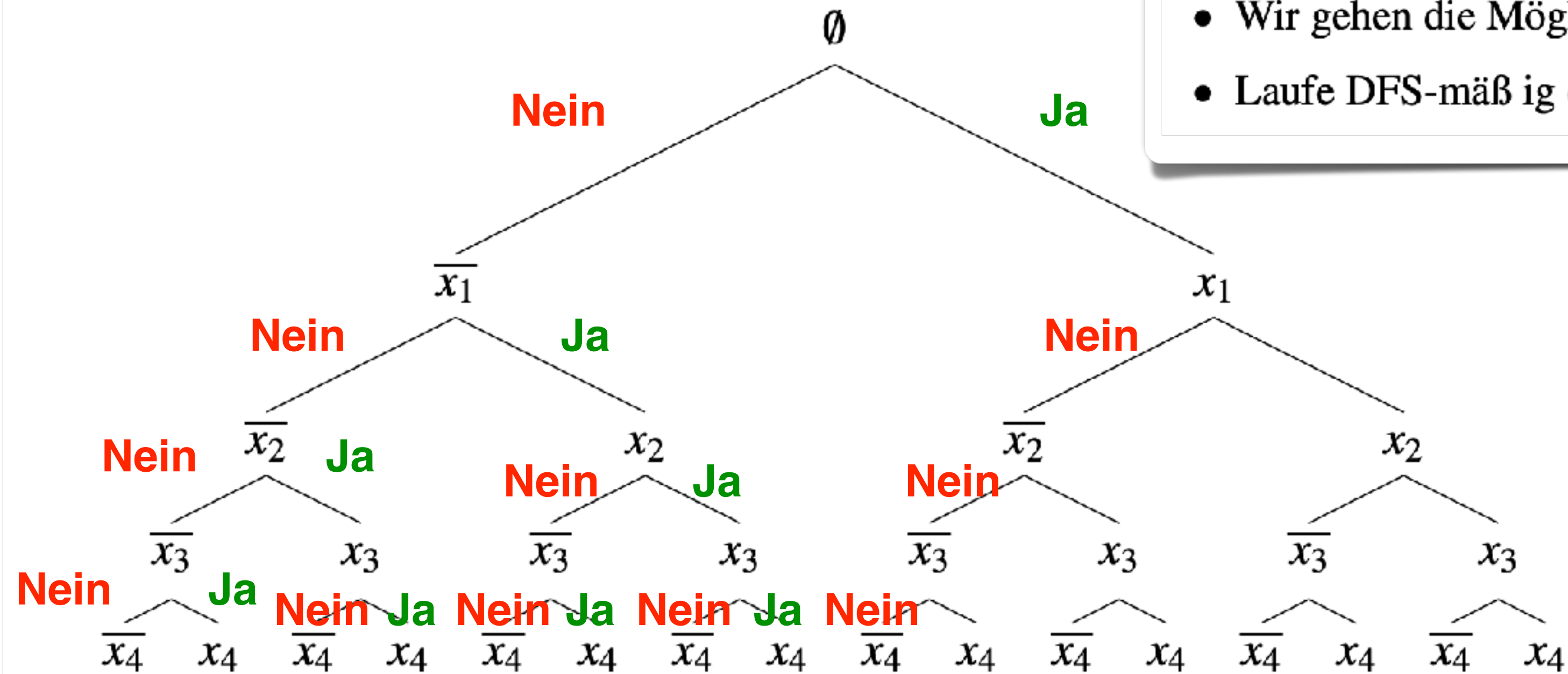
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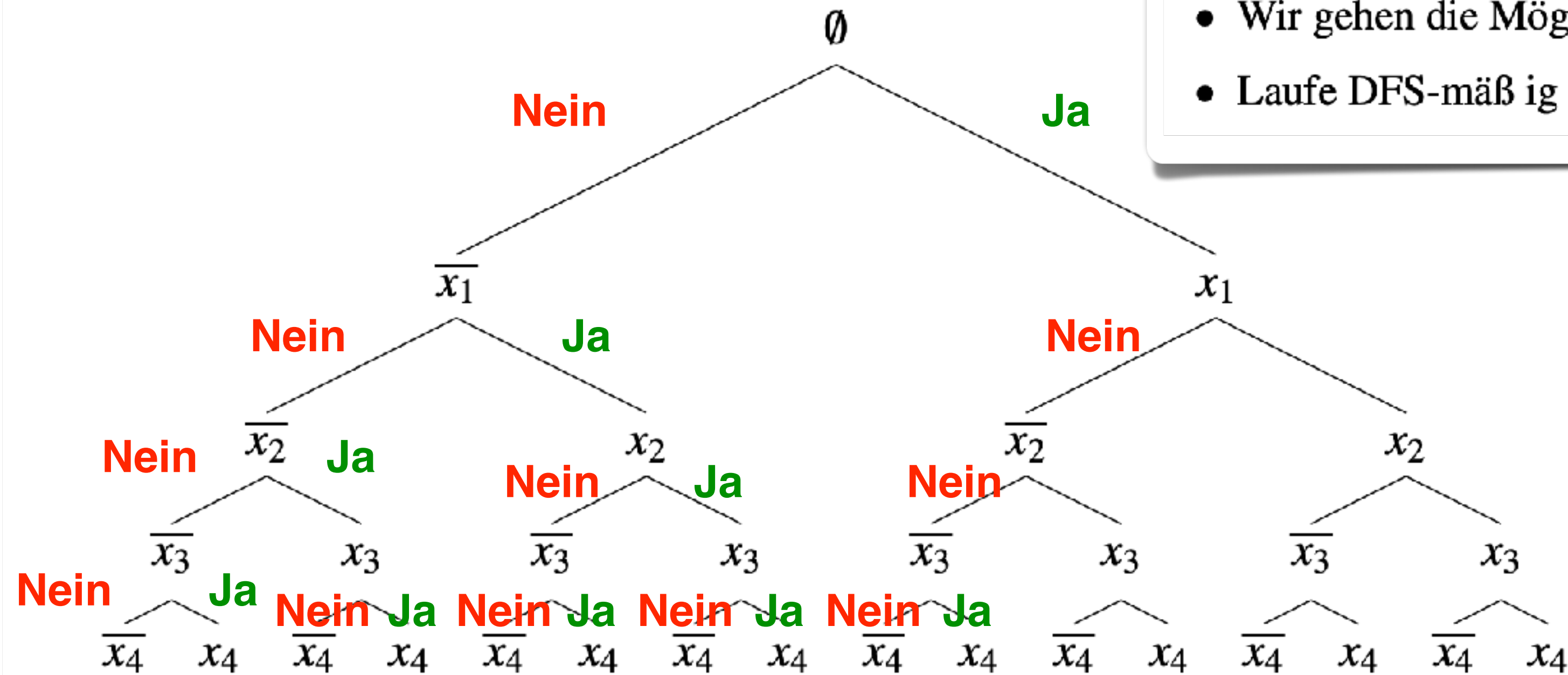
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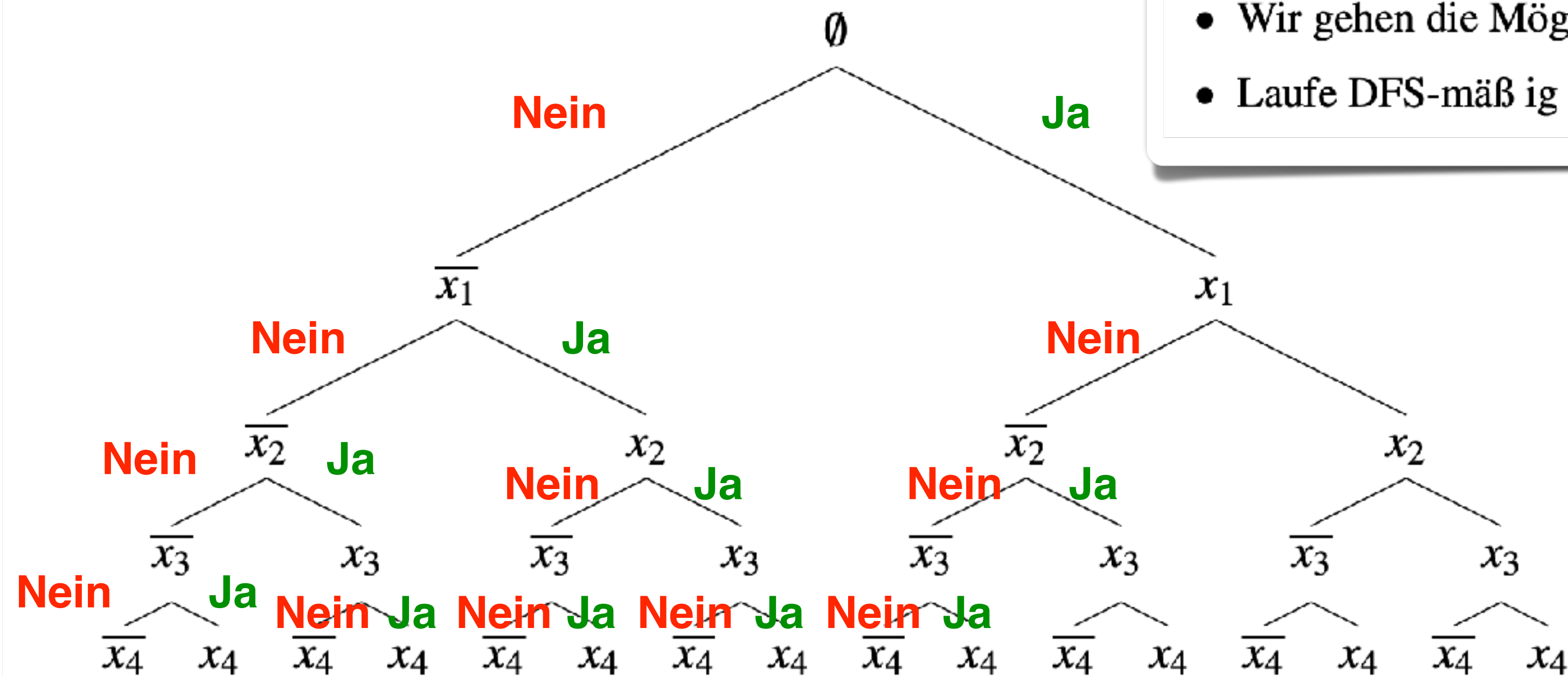
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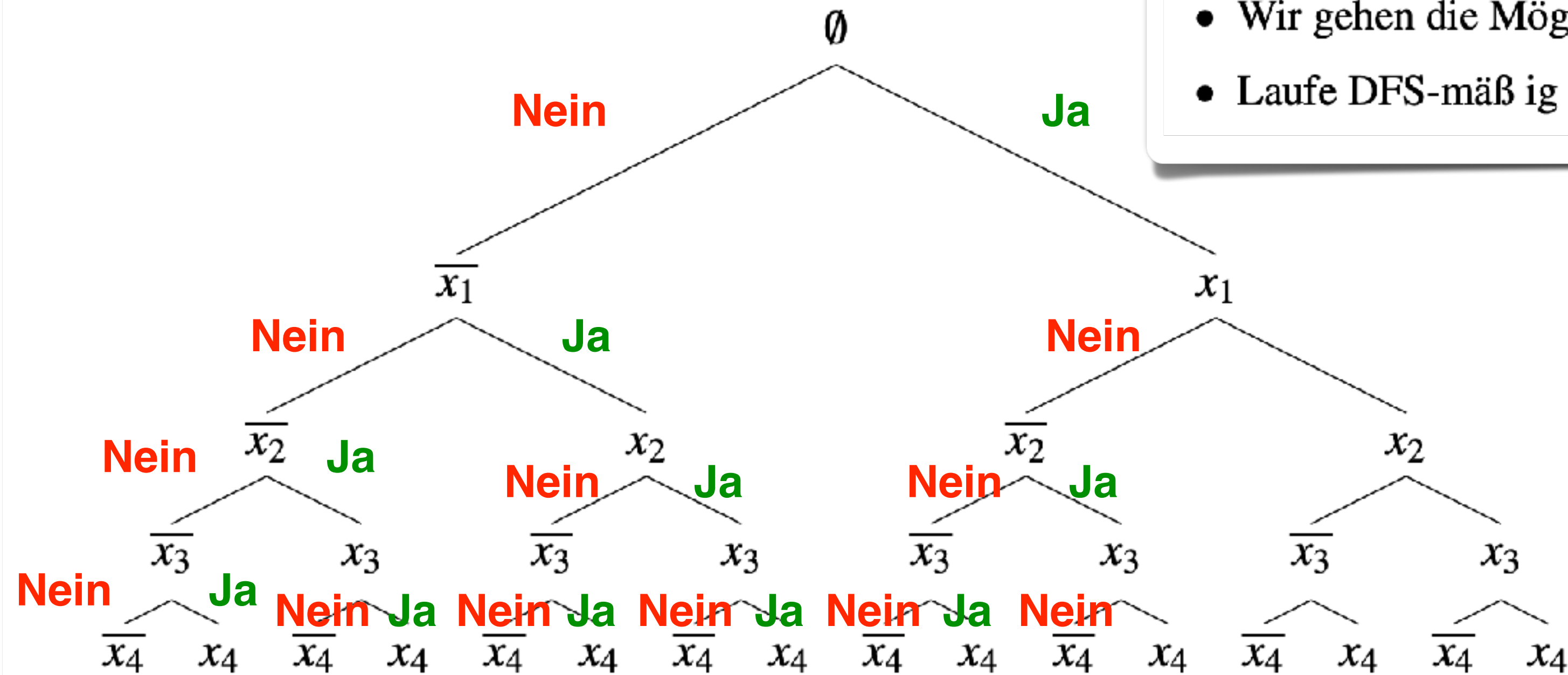
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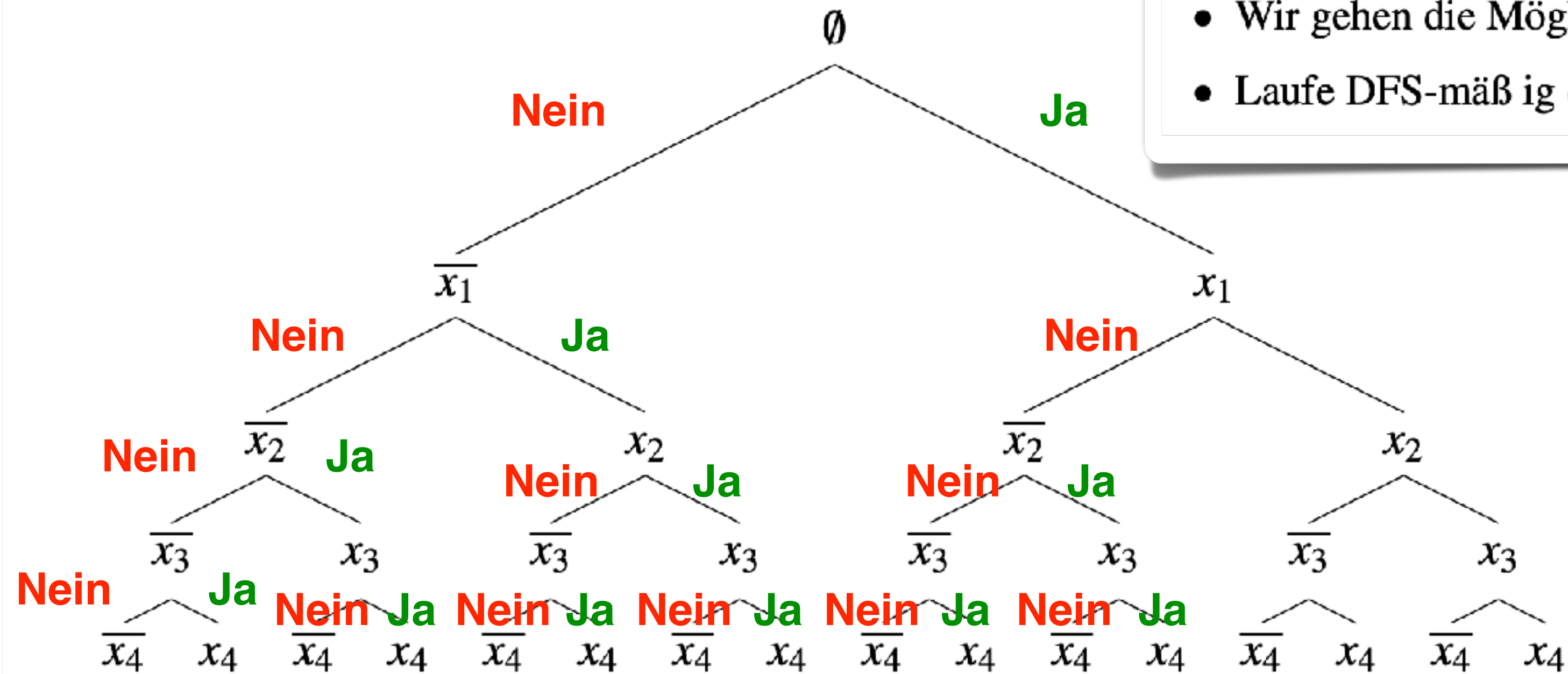
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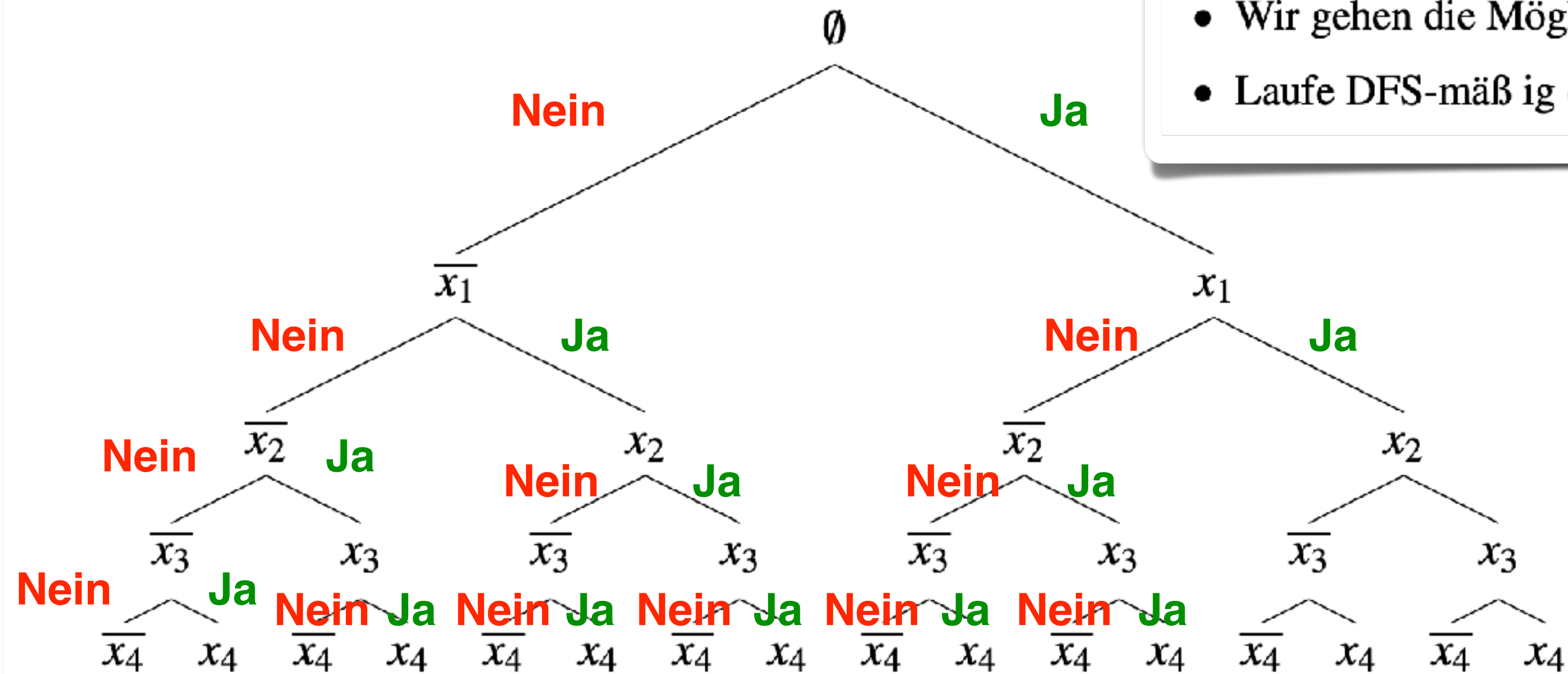
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- Laufe DFS-mäßig durch den Baum, d.h. arbeite die Entscheidungen im Stack ab.



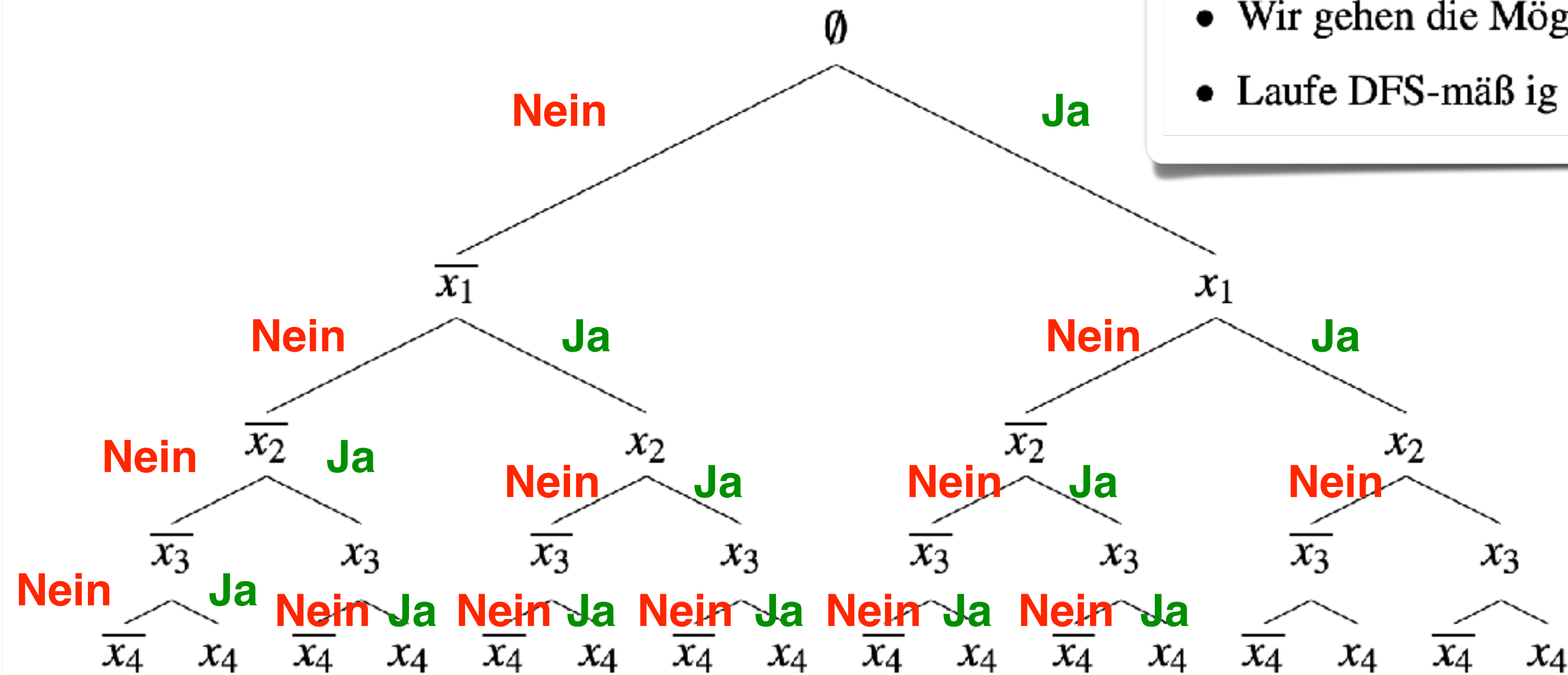
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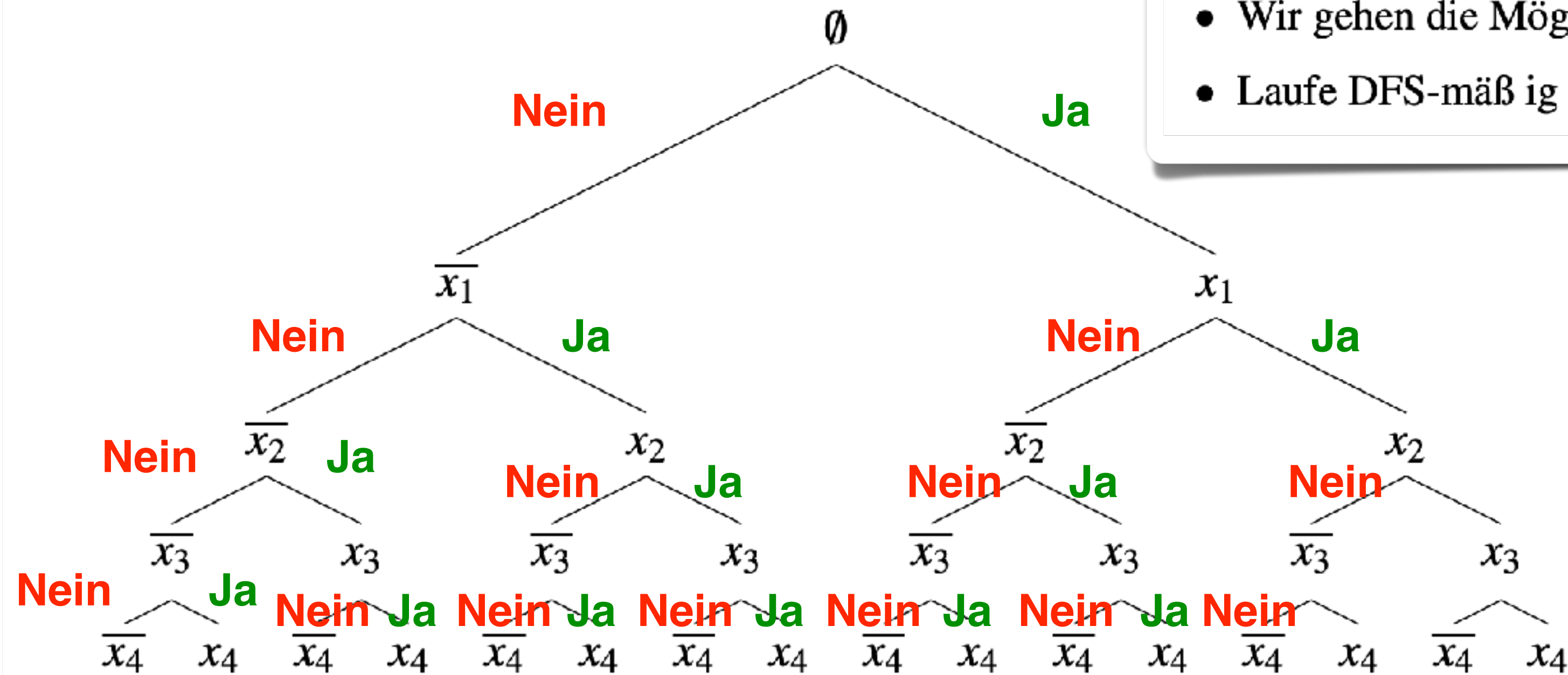
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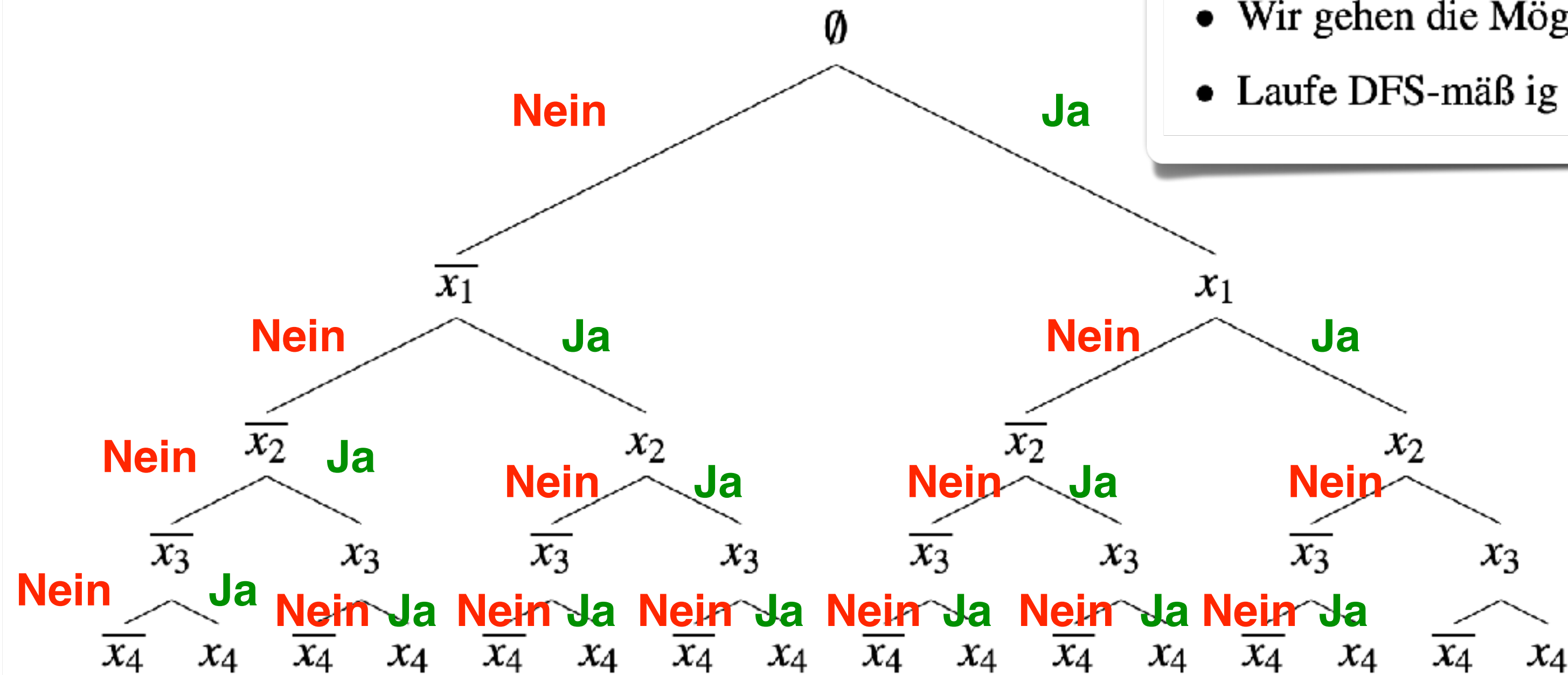
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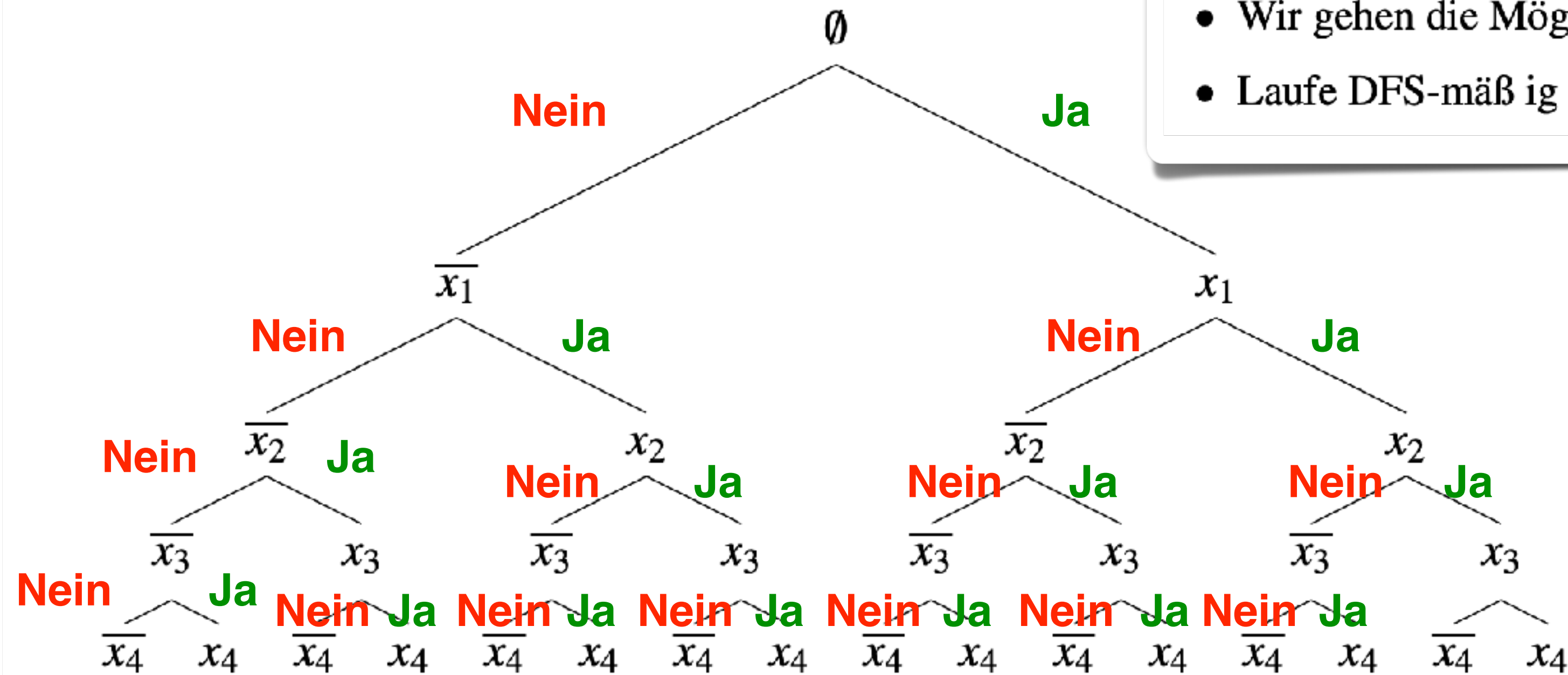
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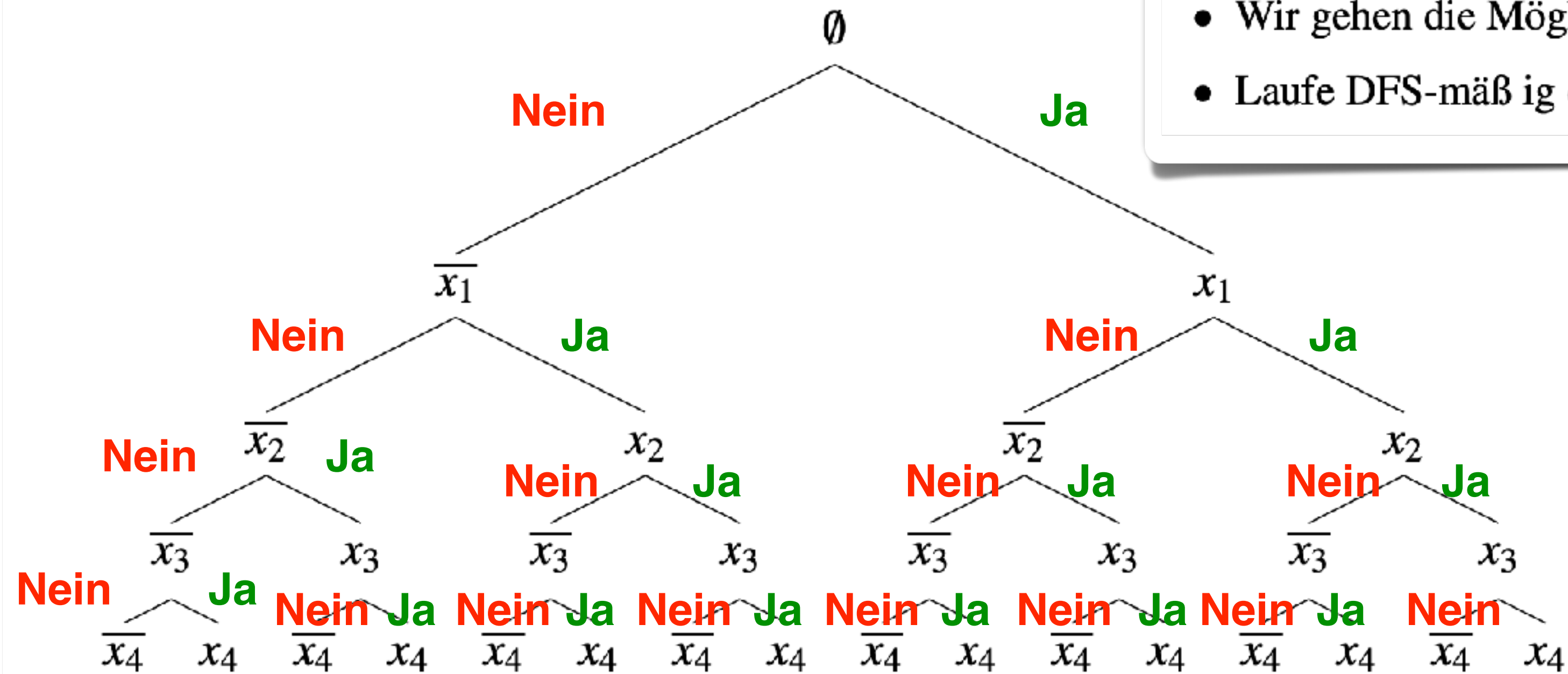
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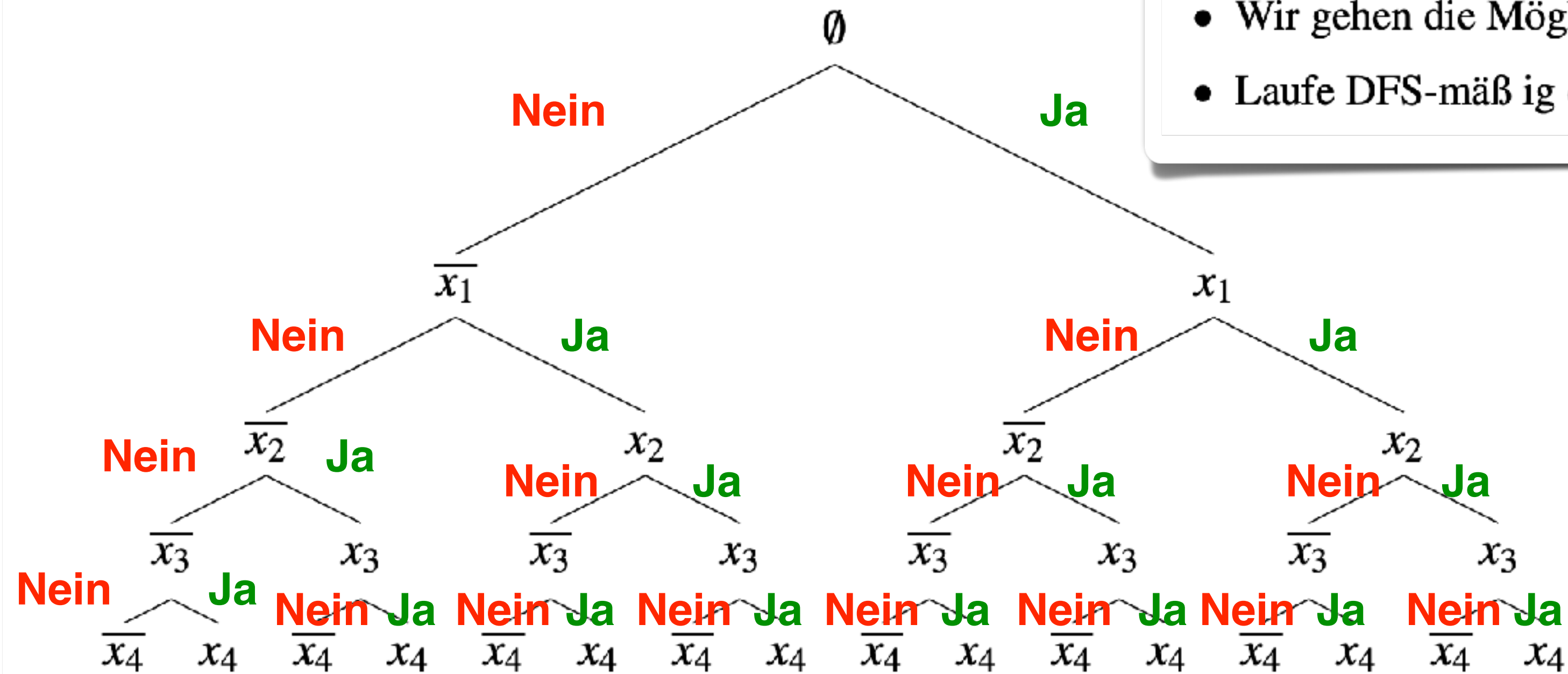
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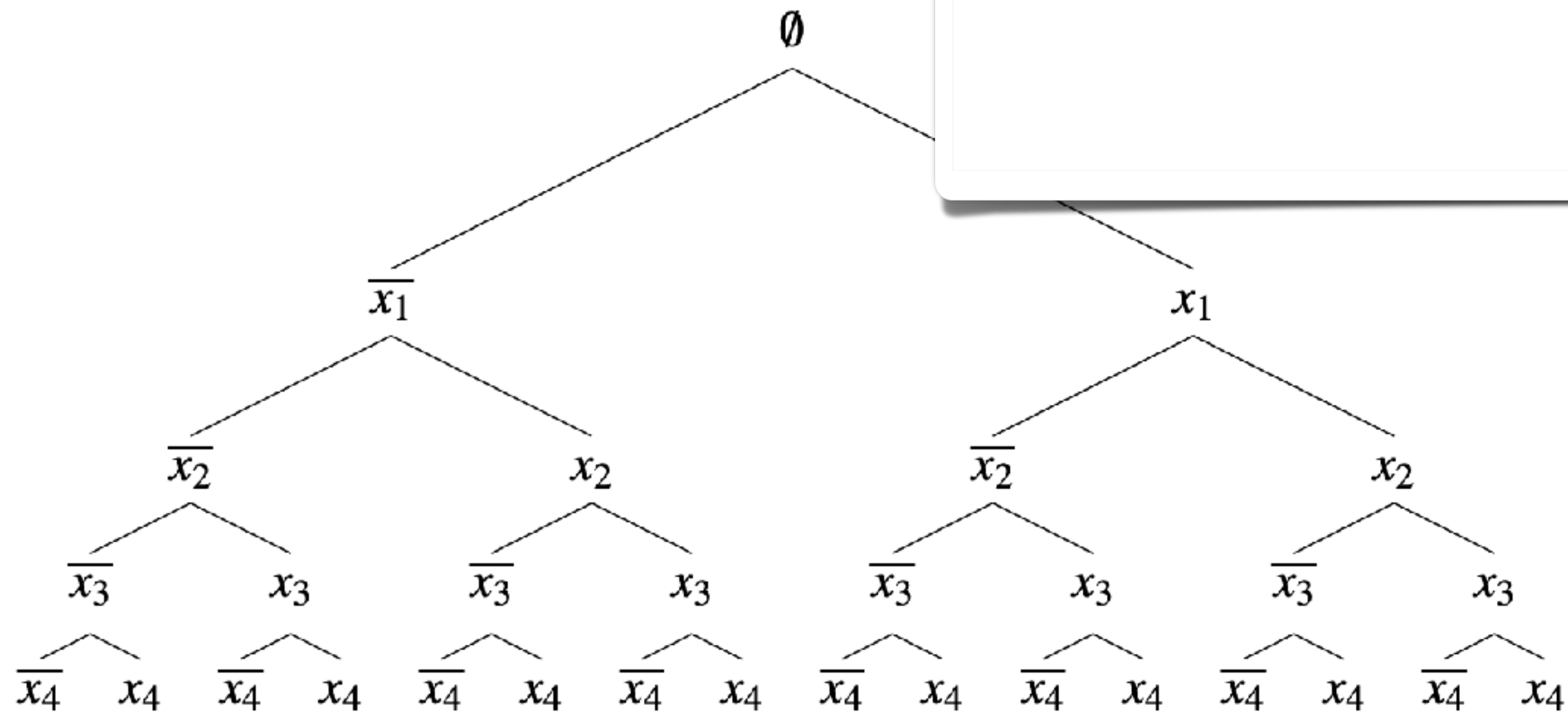


4. Einsatz von Schranken - Grundidee

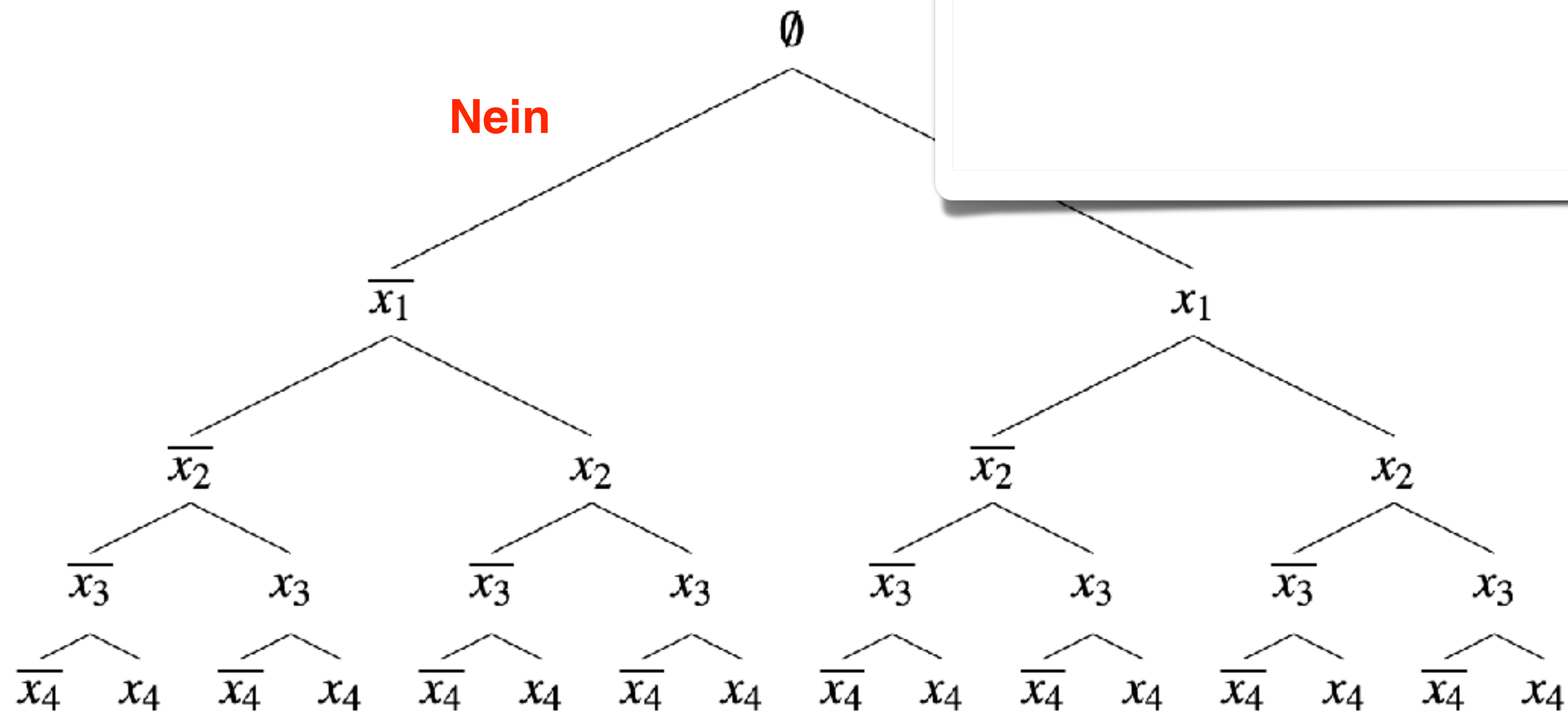
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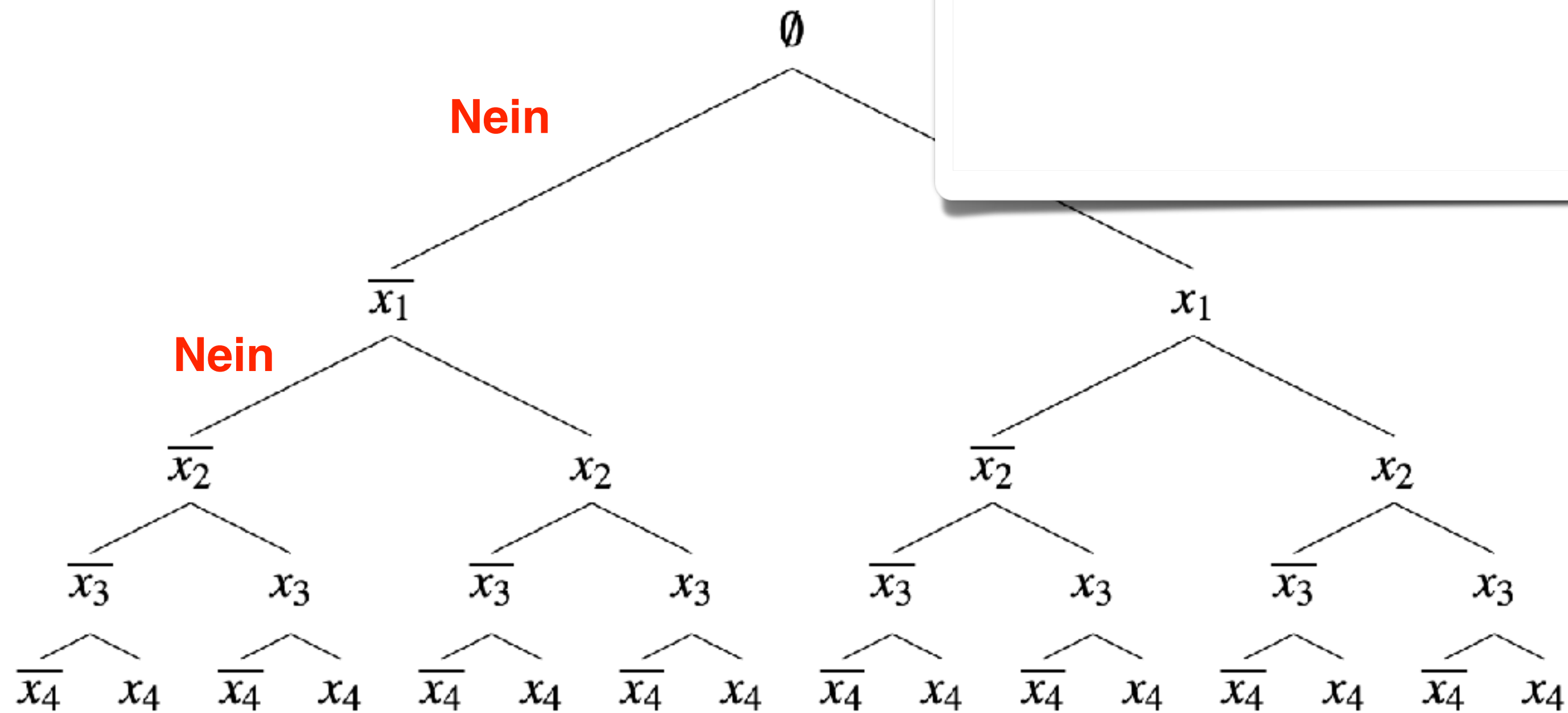
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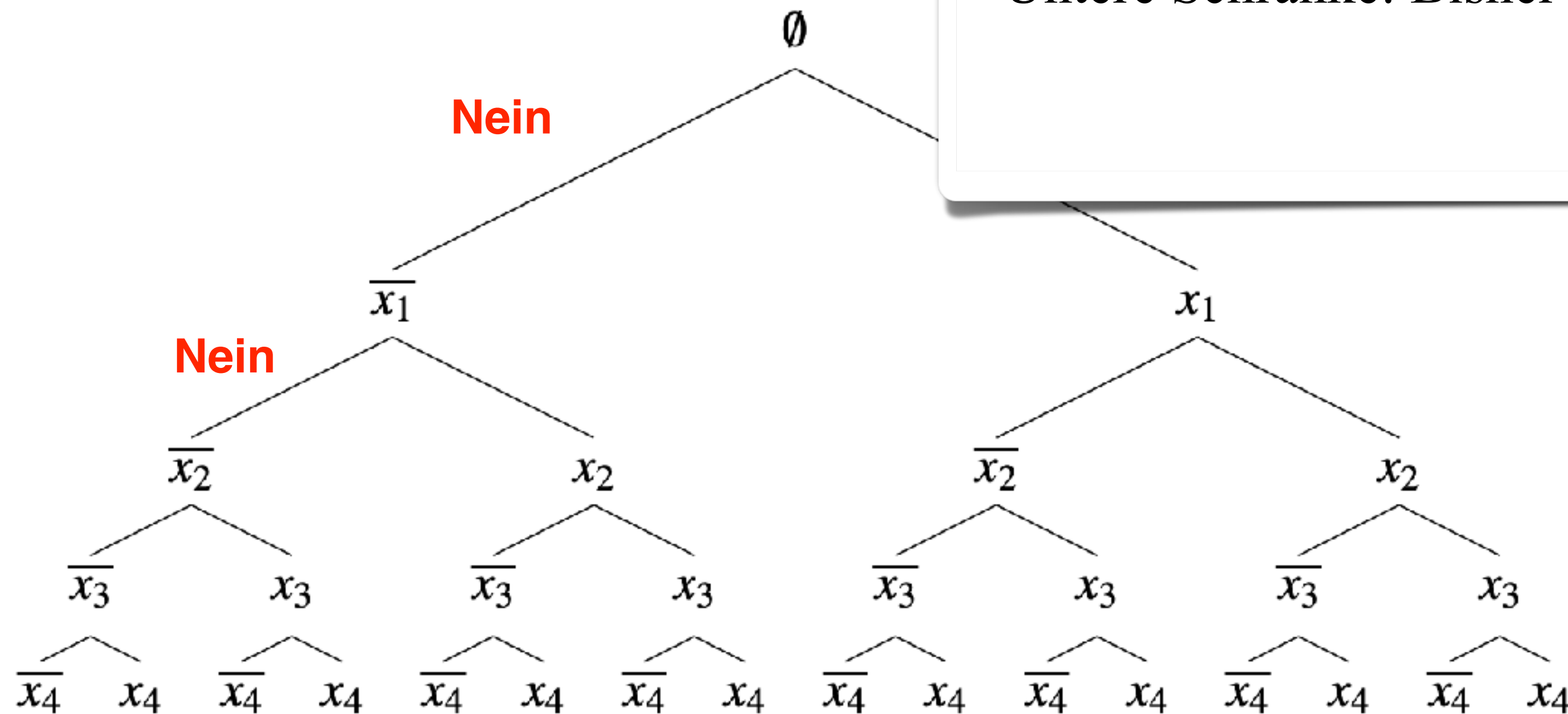


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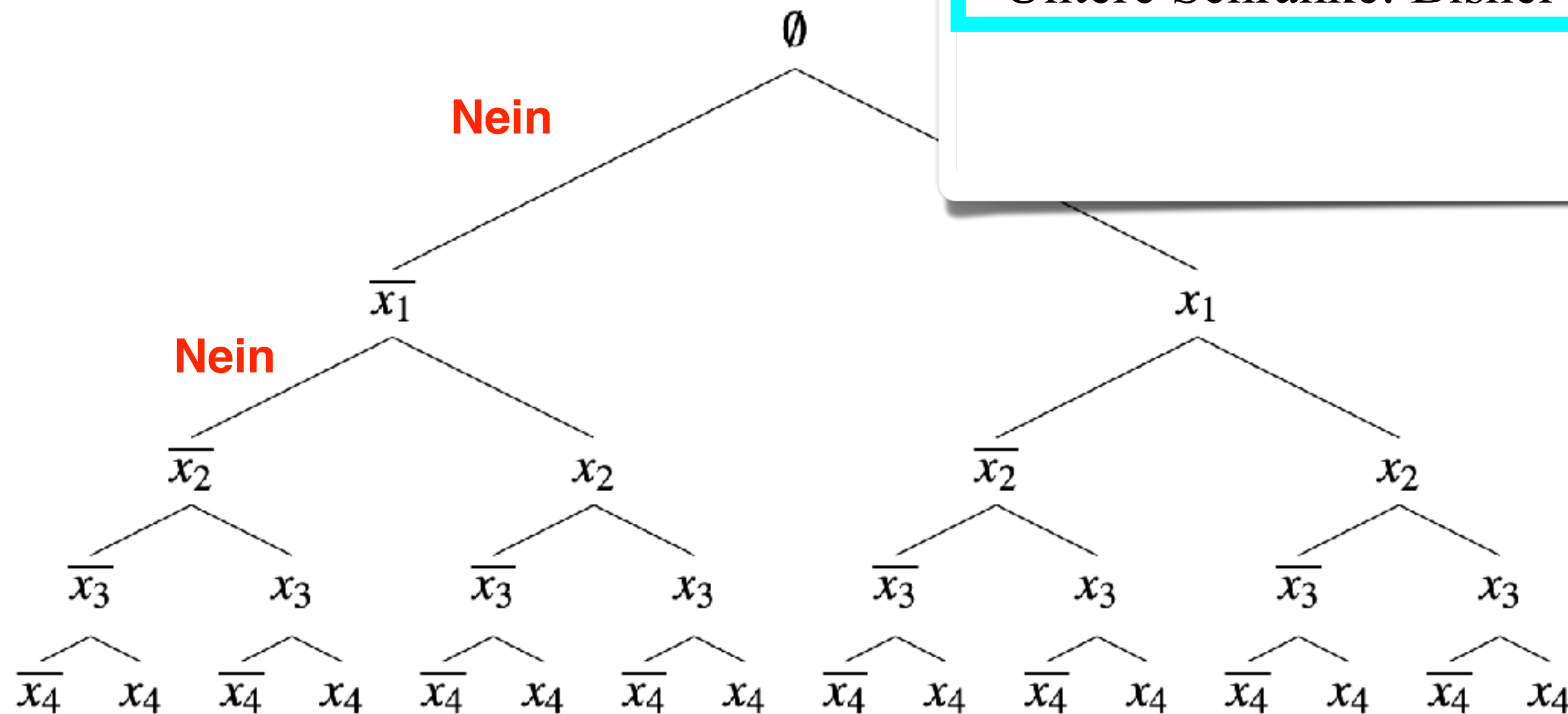
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- Untere Schranke: Bisher erreichter Zielfunktionswert im ganzen Baum



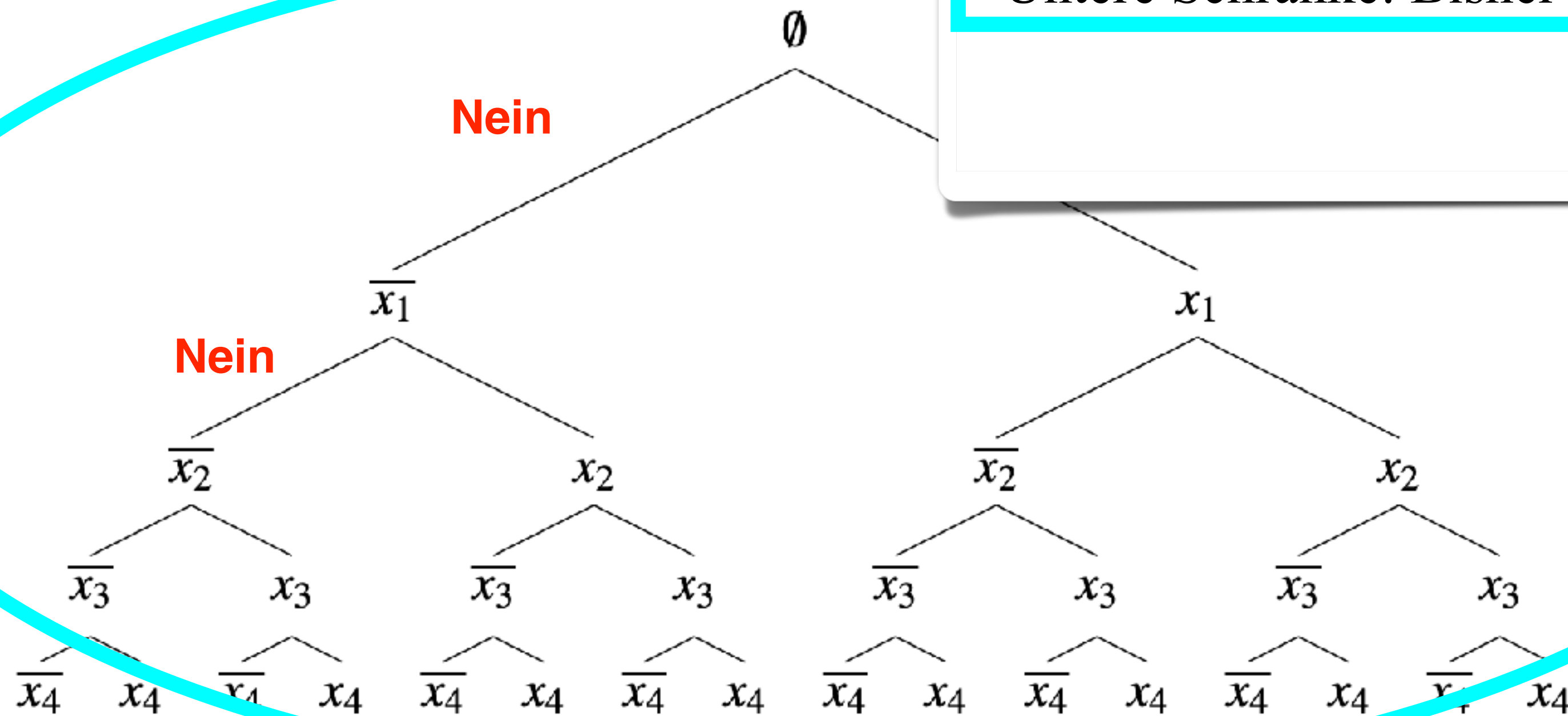
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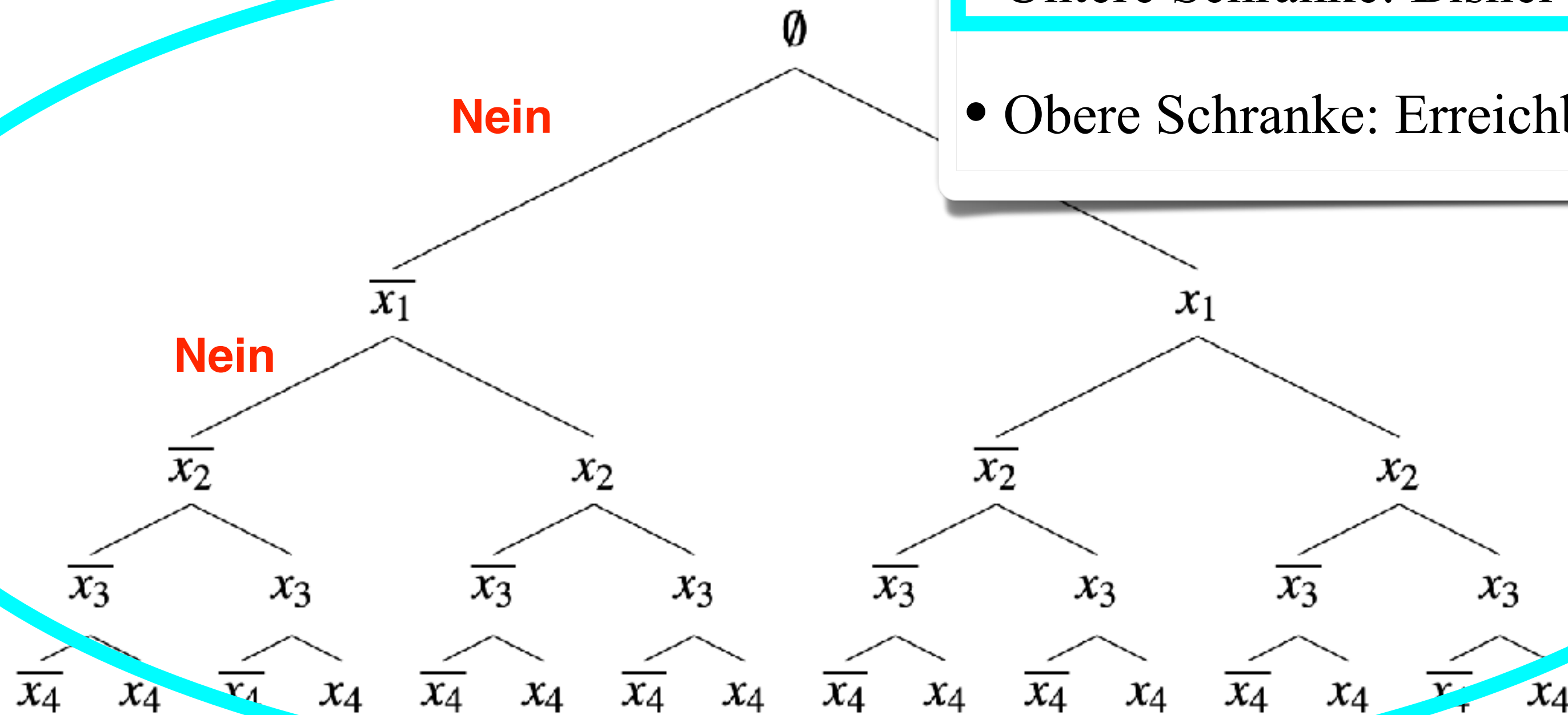
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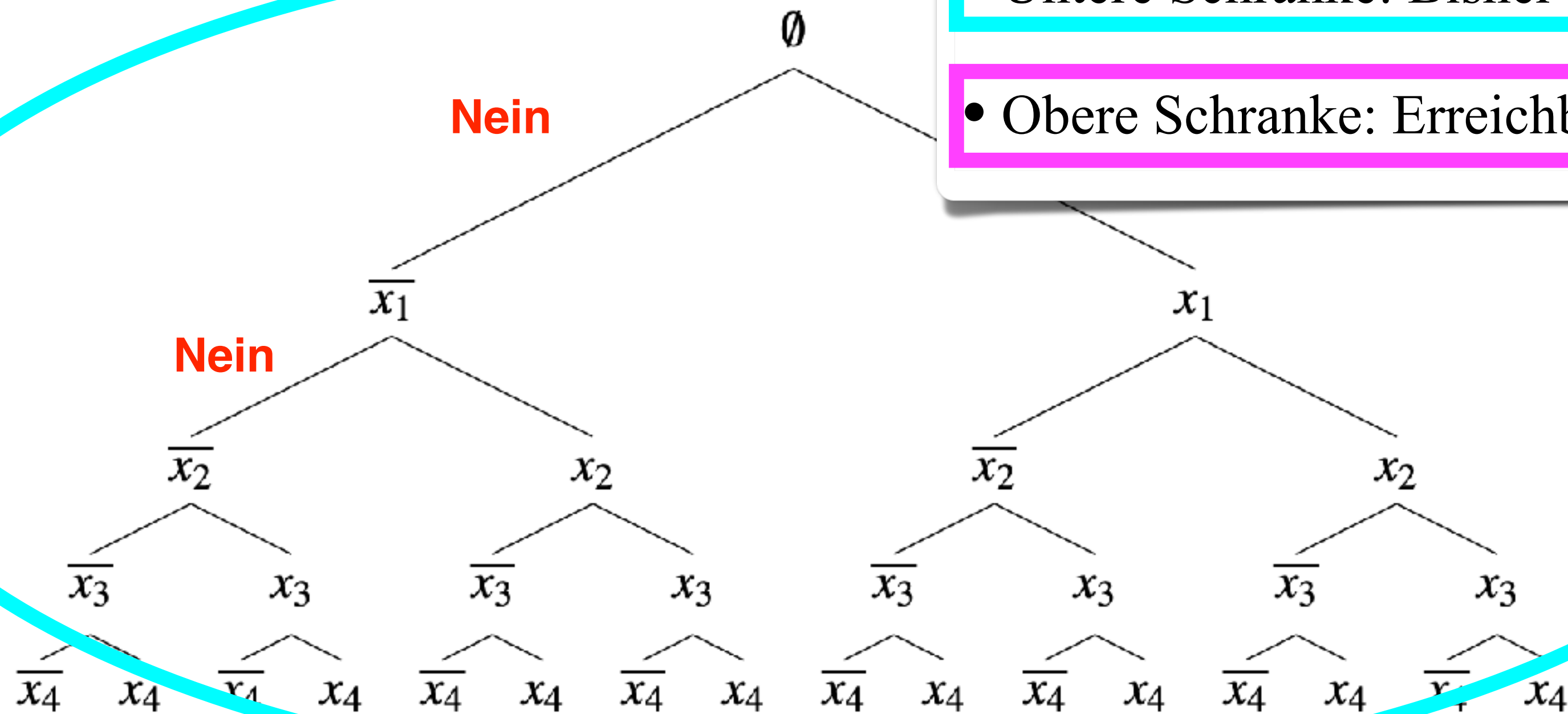
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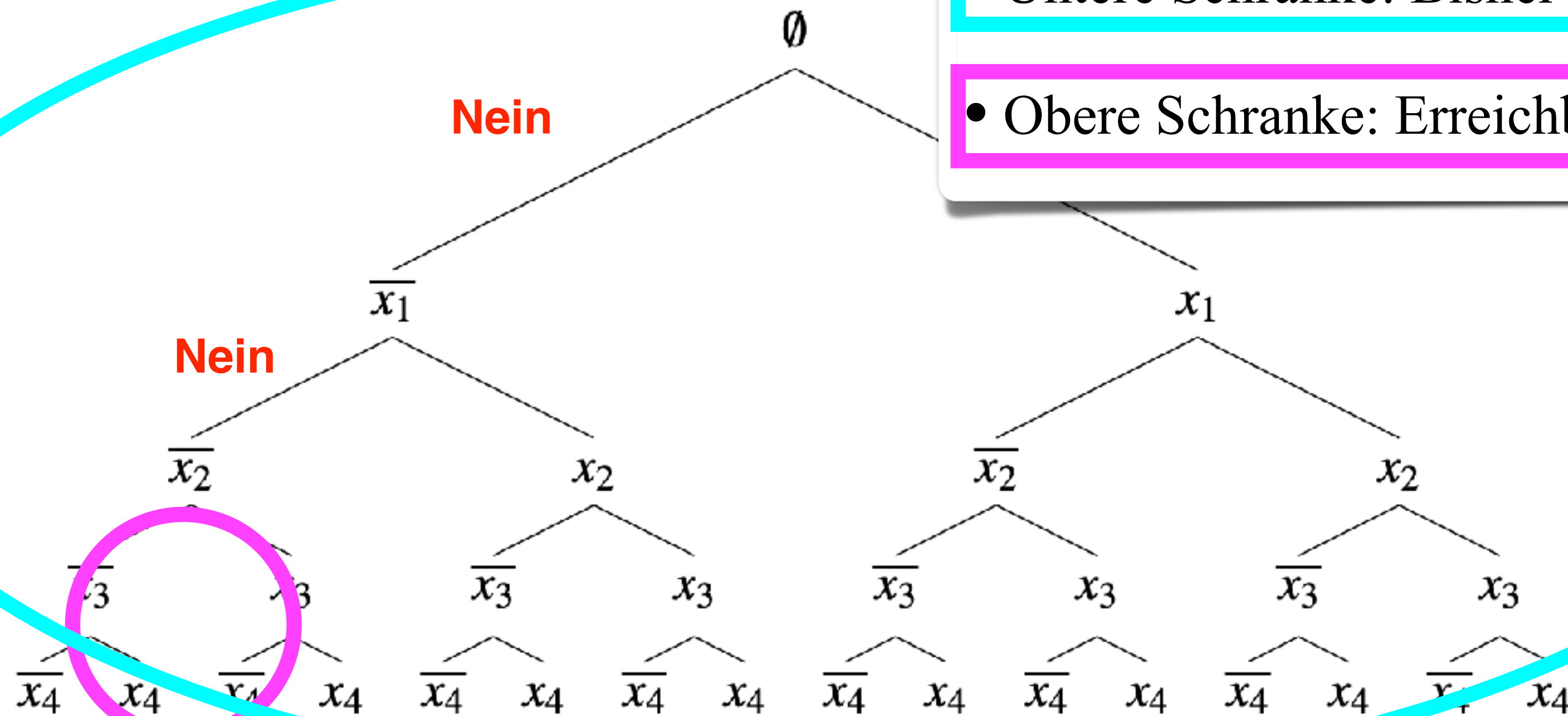
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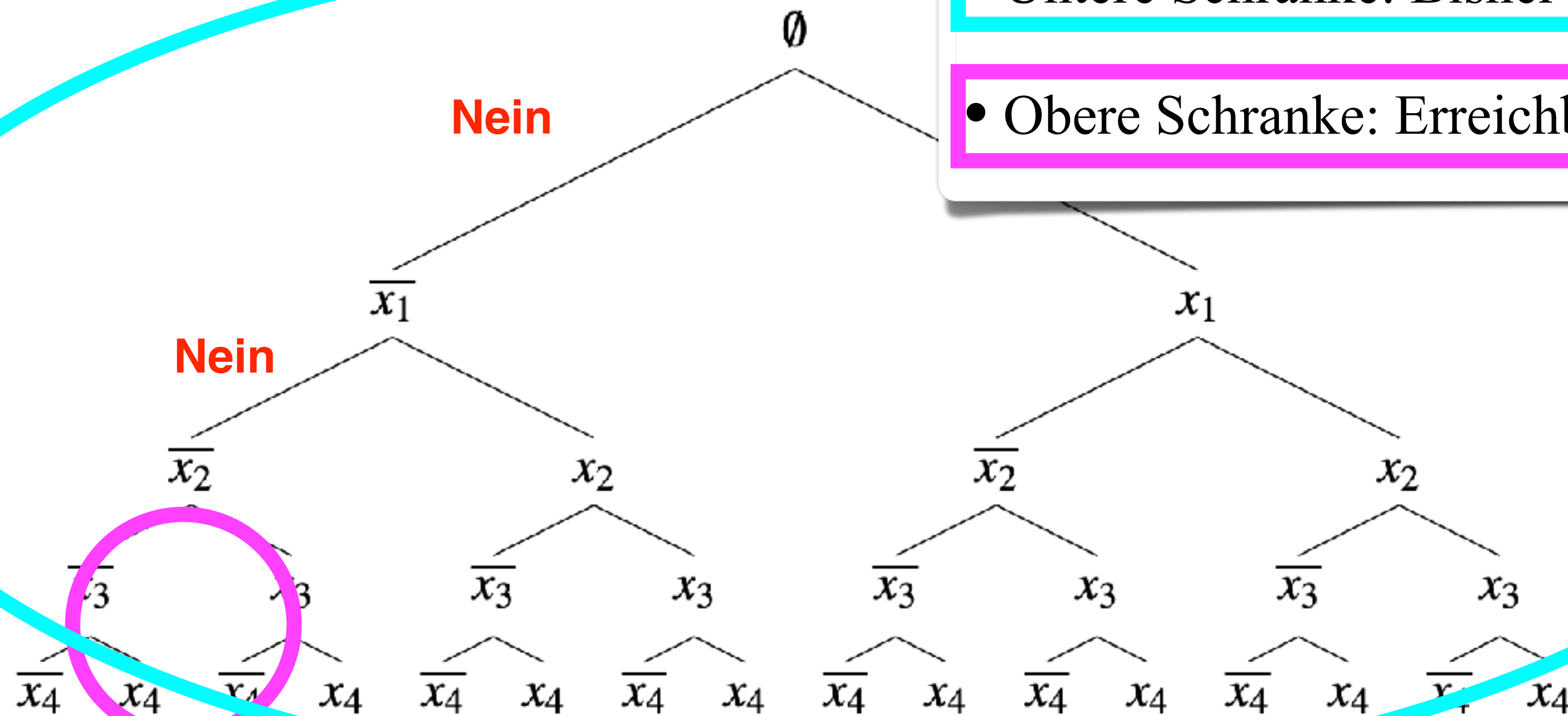
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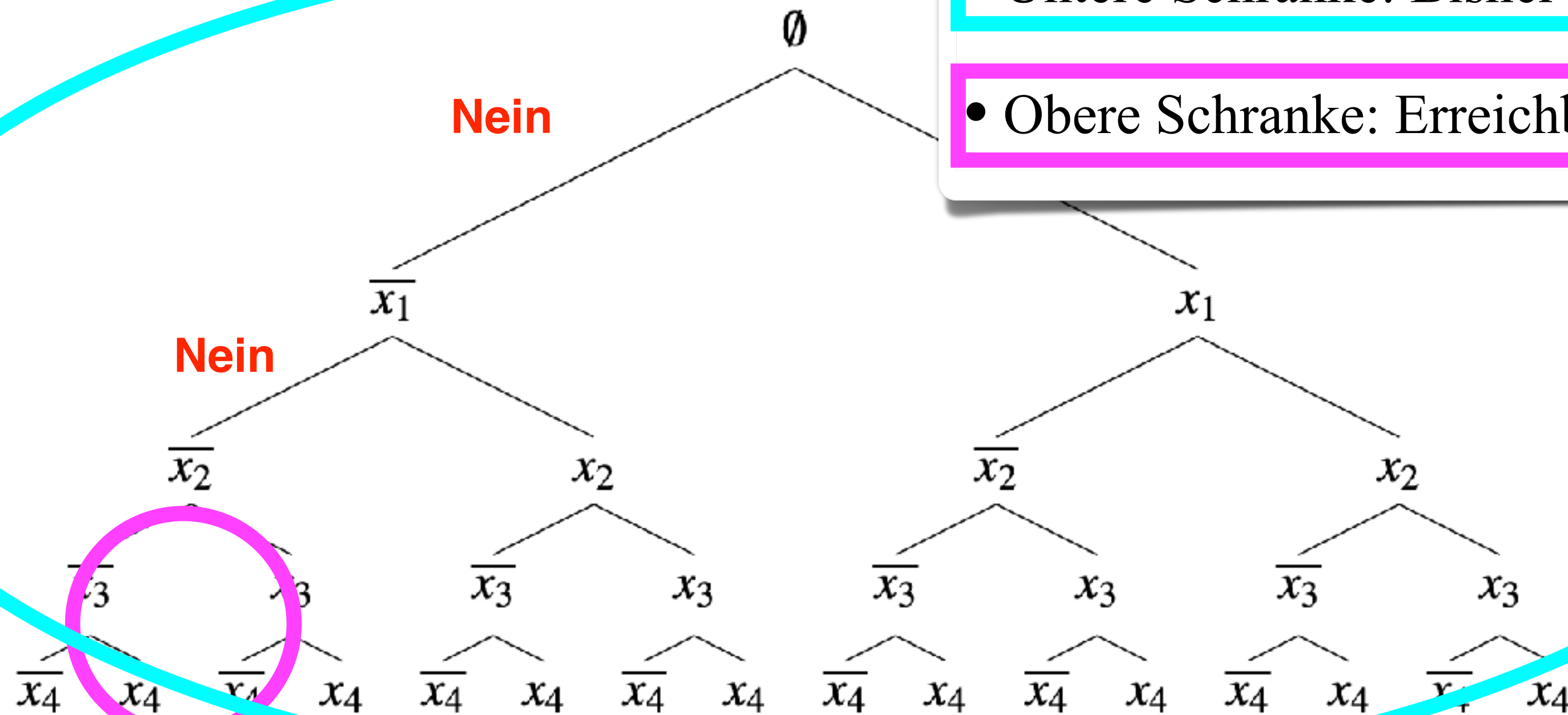
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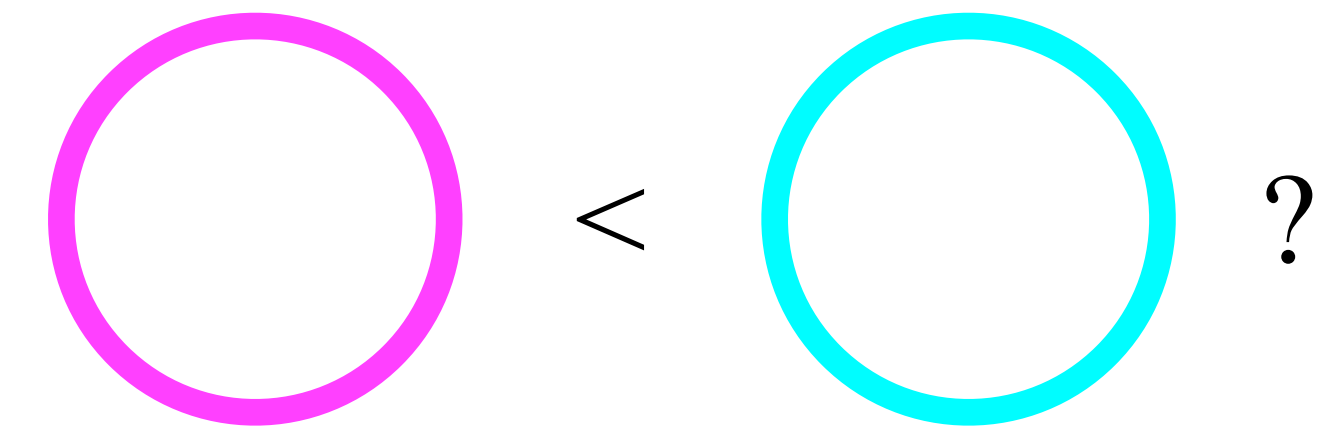
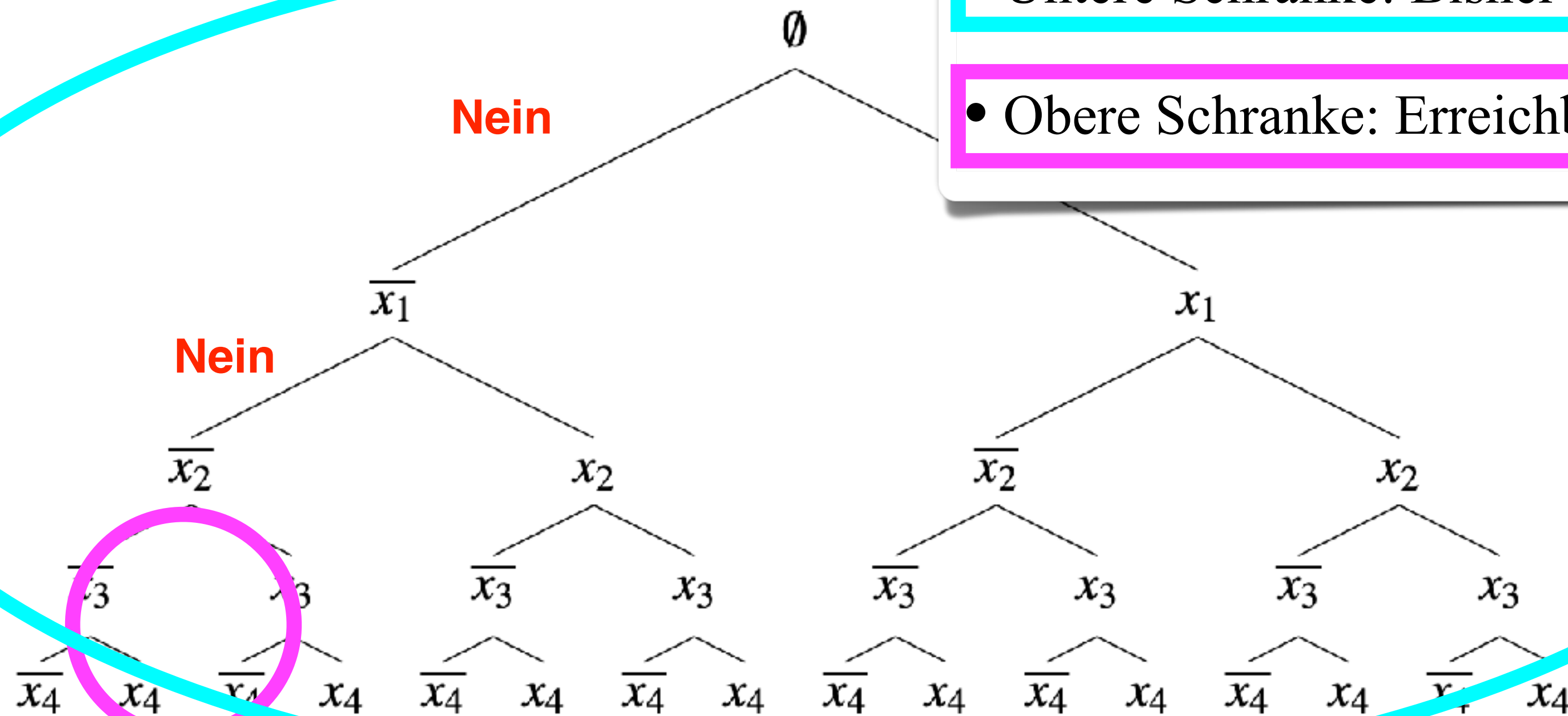
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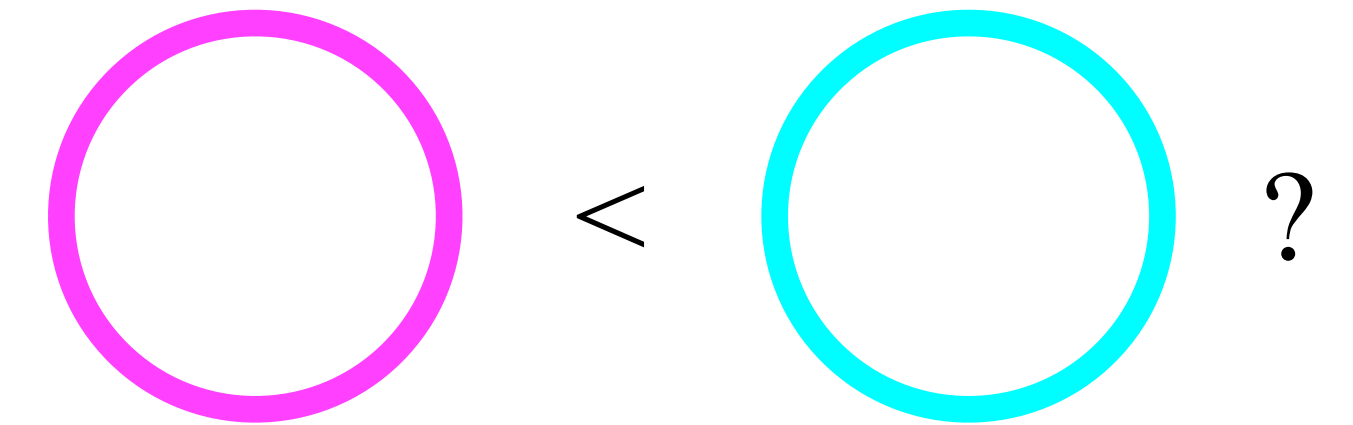
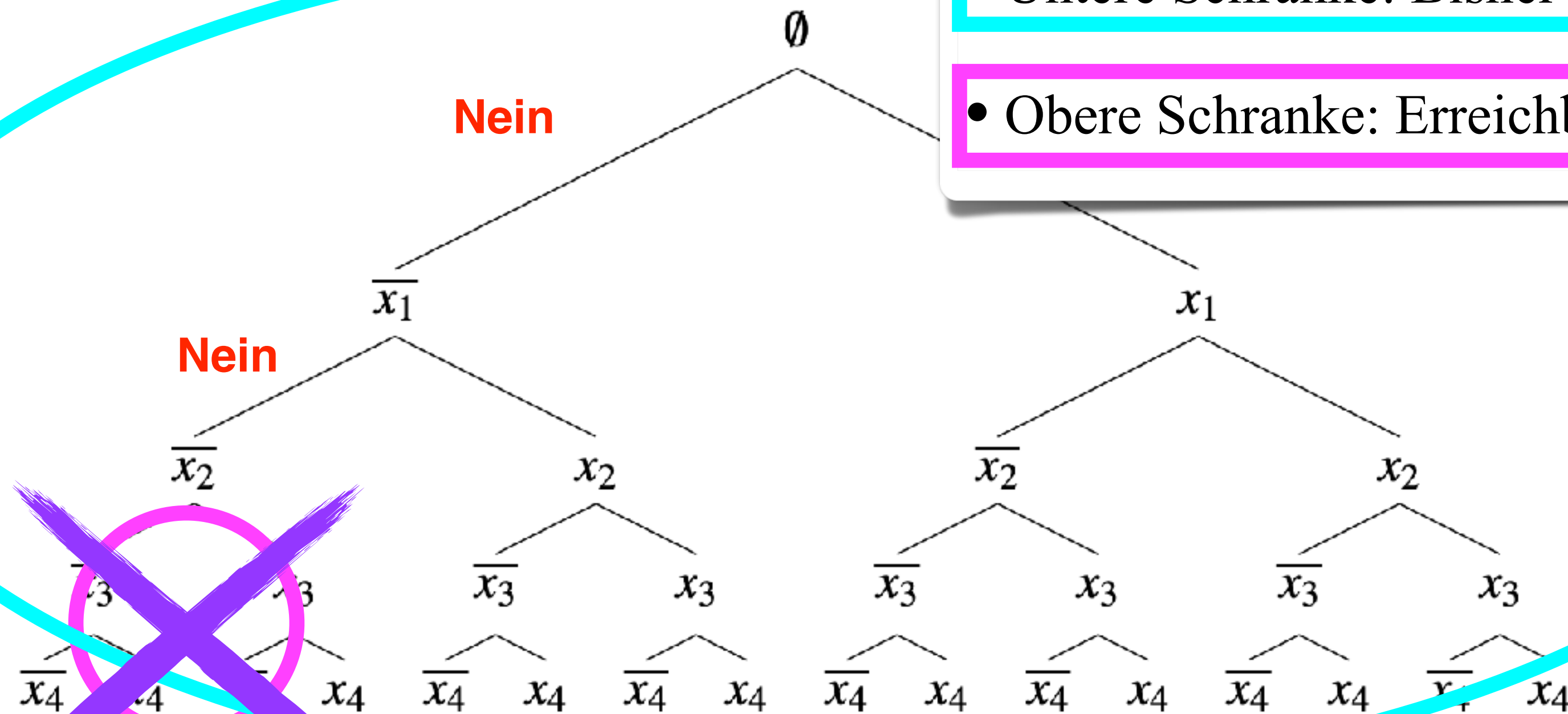
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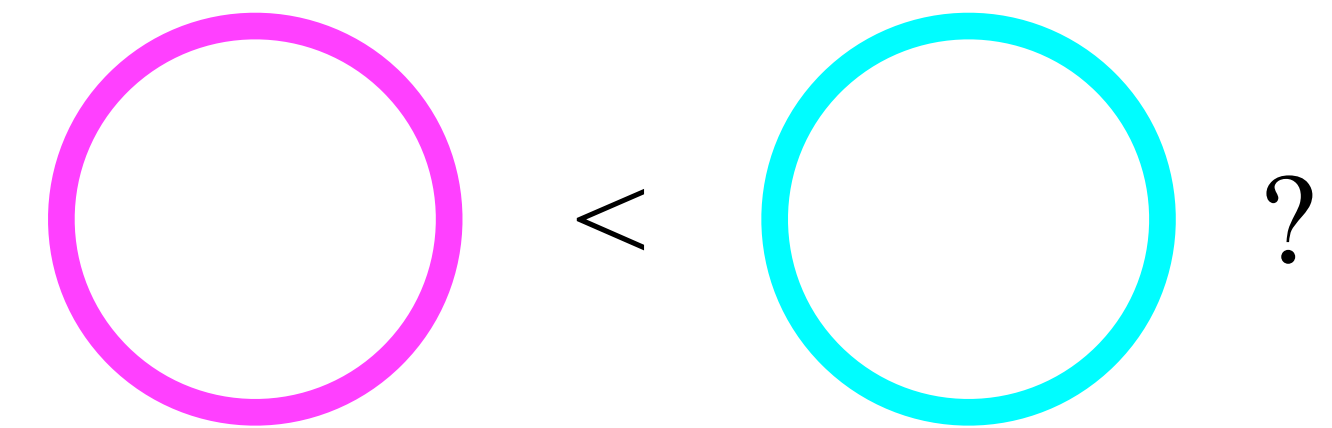
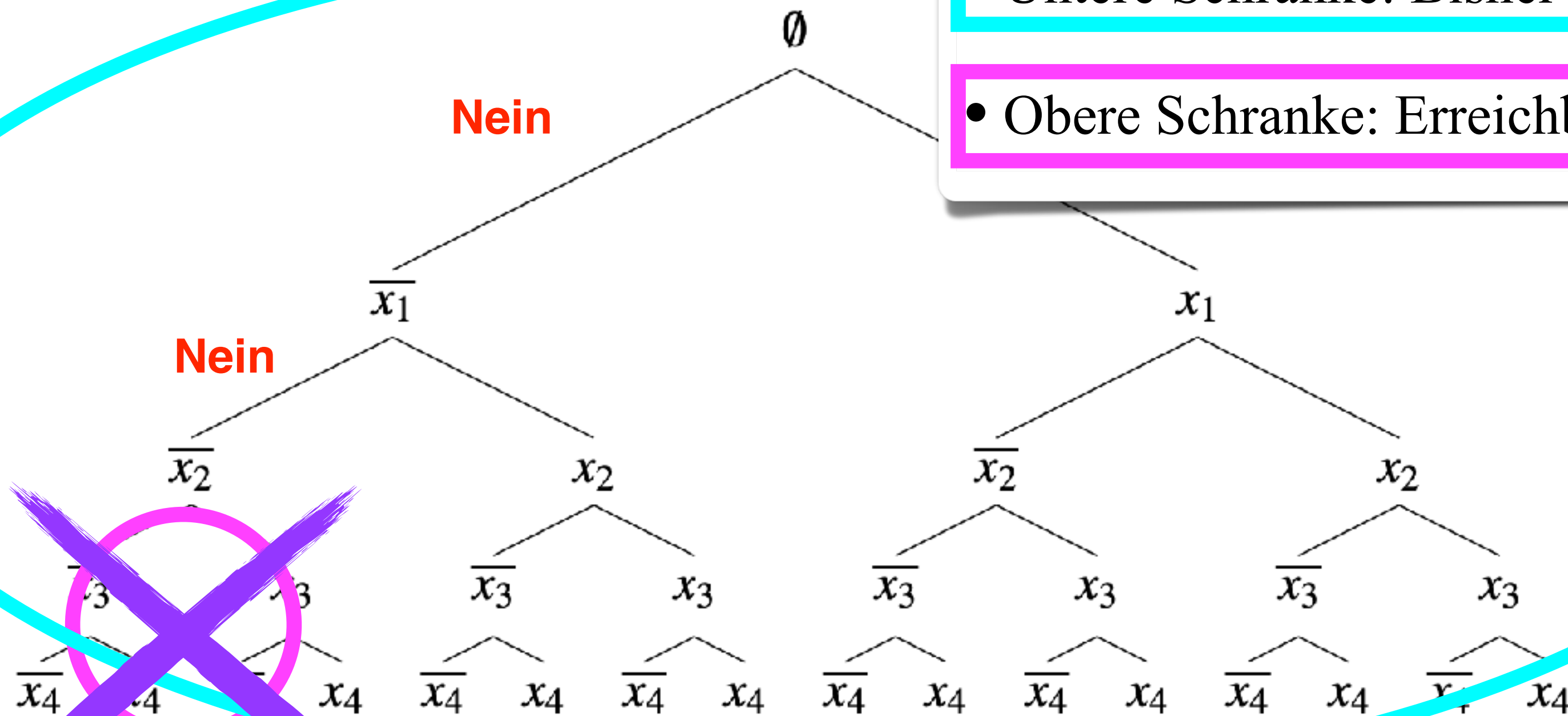
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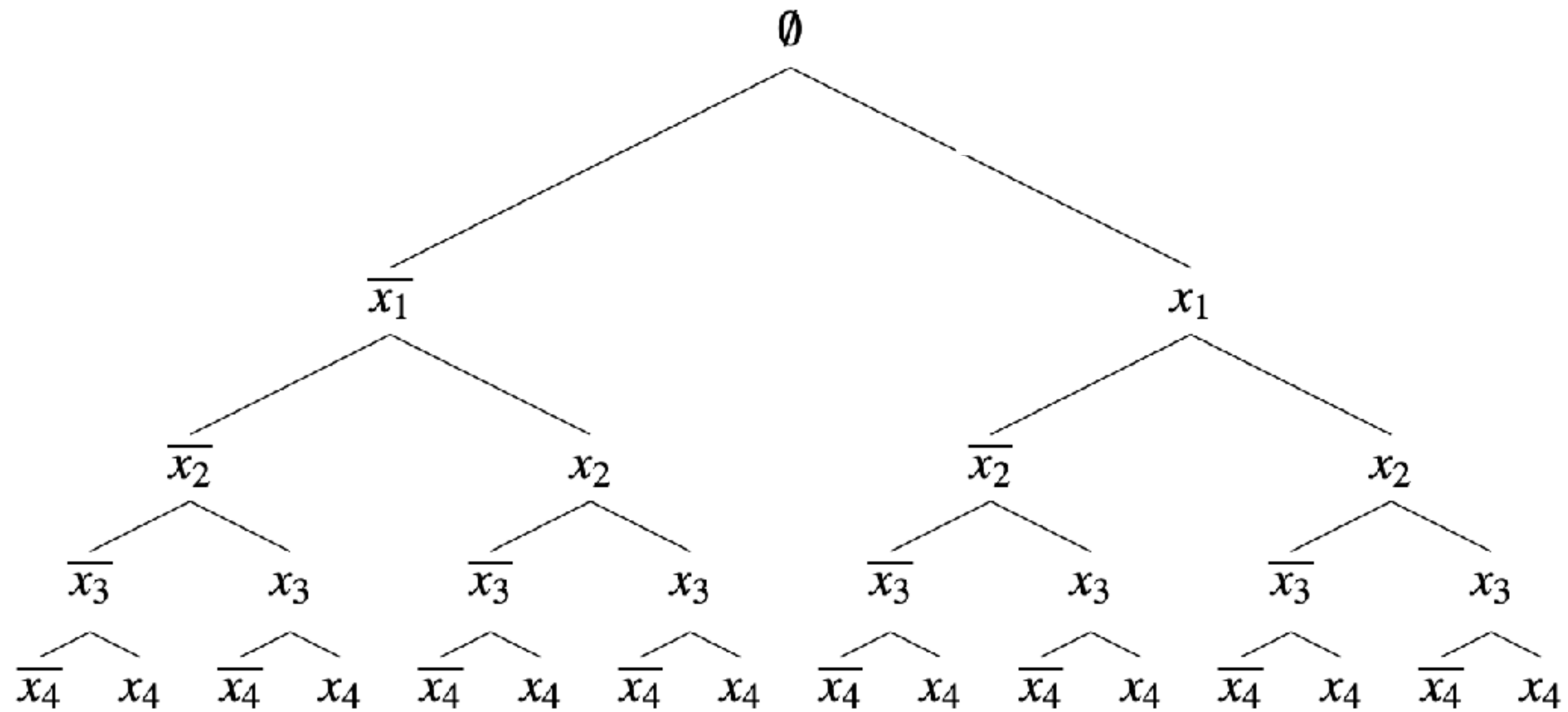
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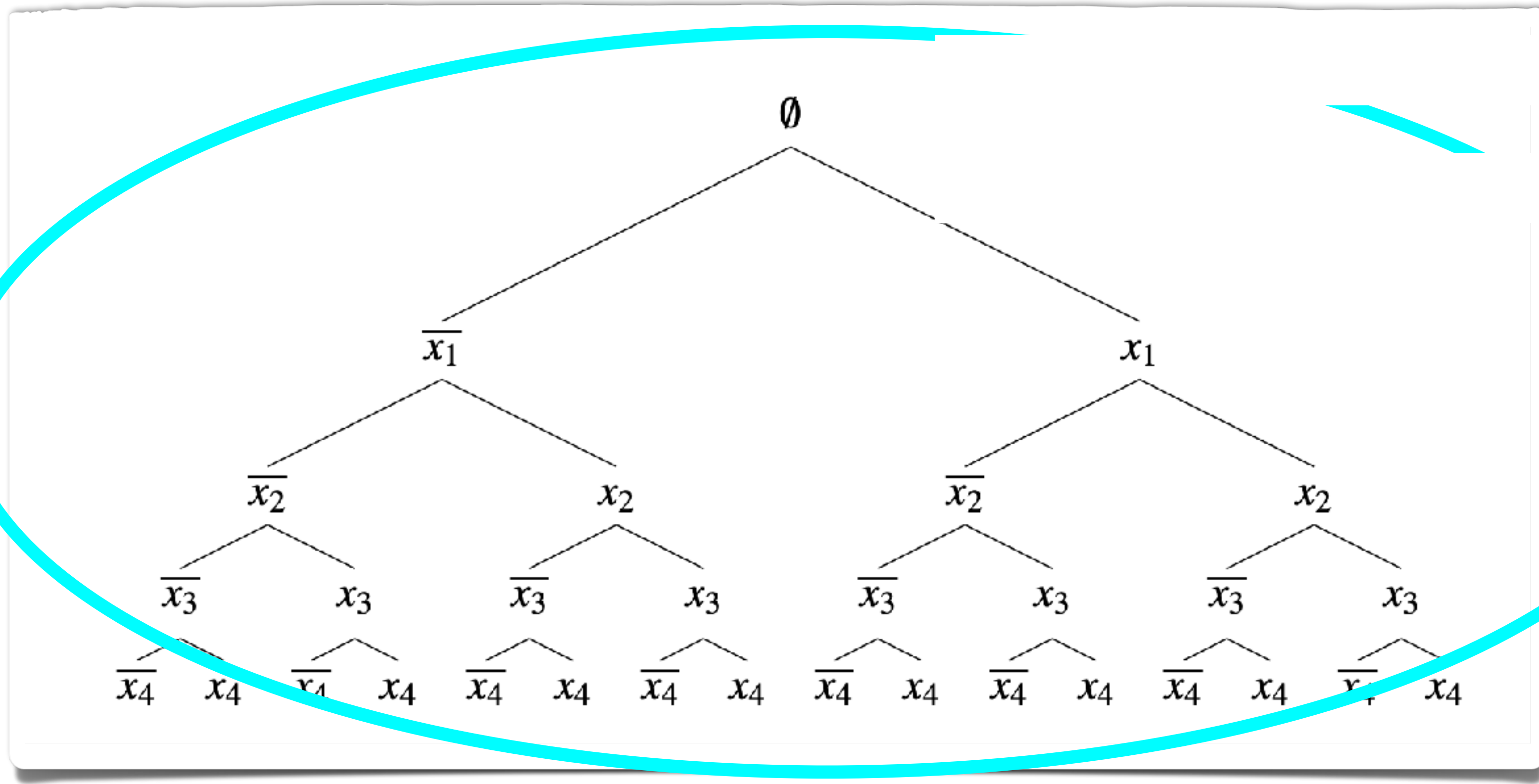
Was sind geeignete Schranken?

2. Geeignete Schranken - Knapsack

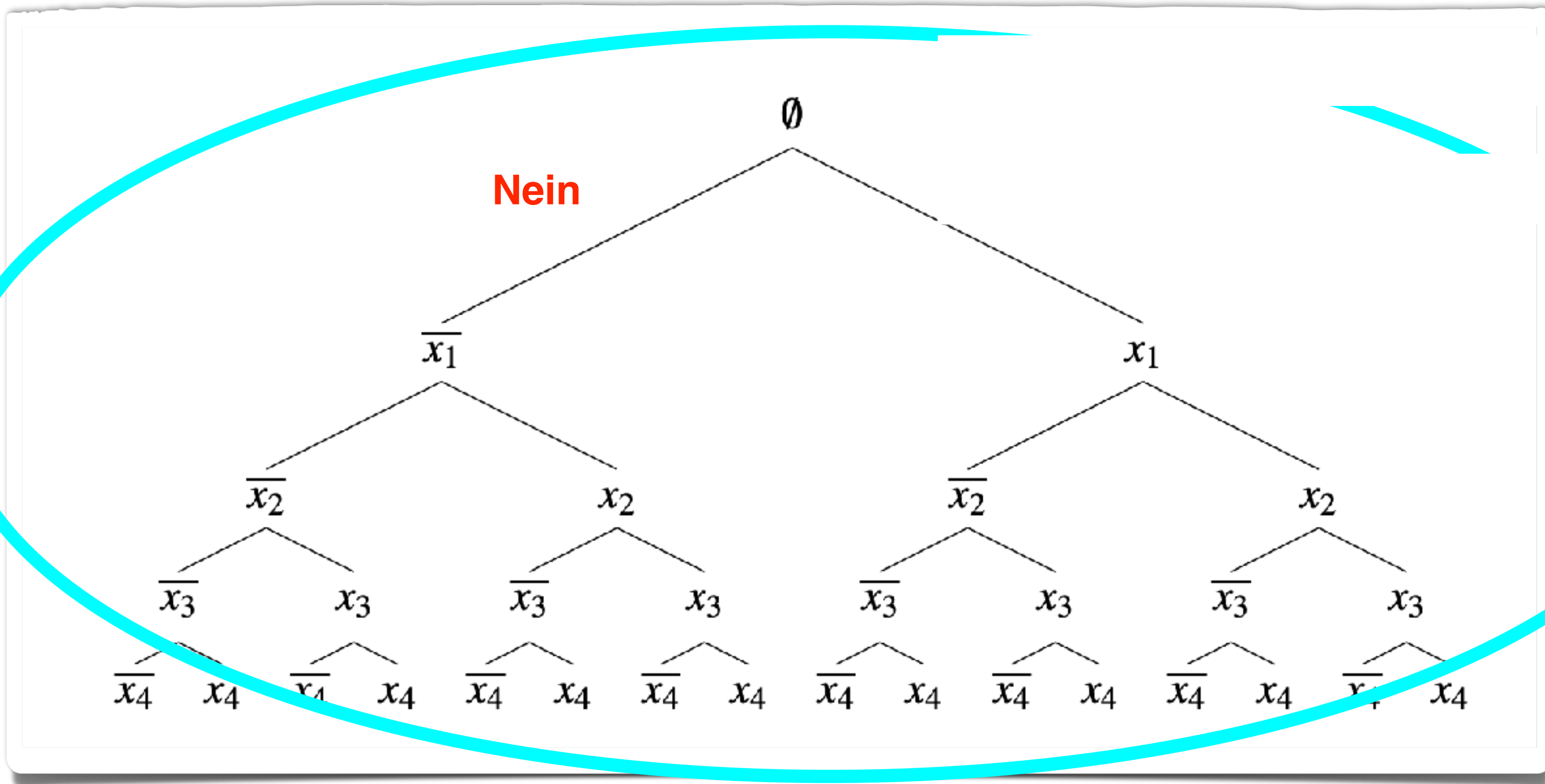
2. Geeignete Schranken - Knapsack



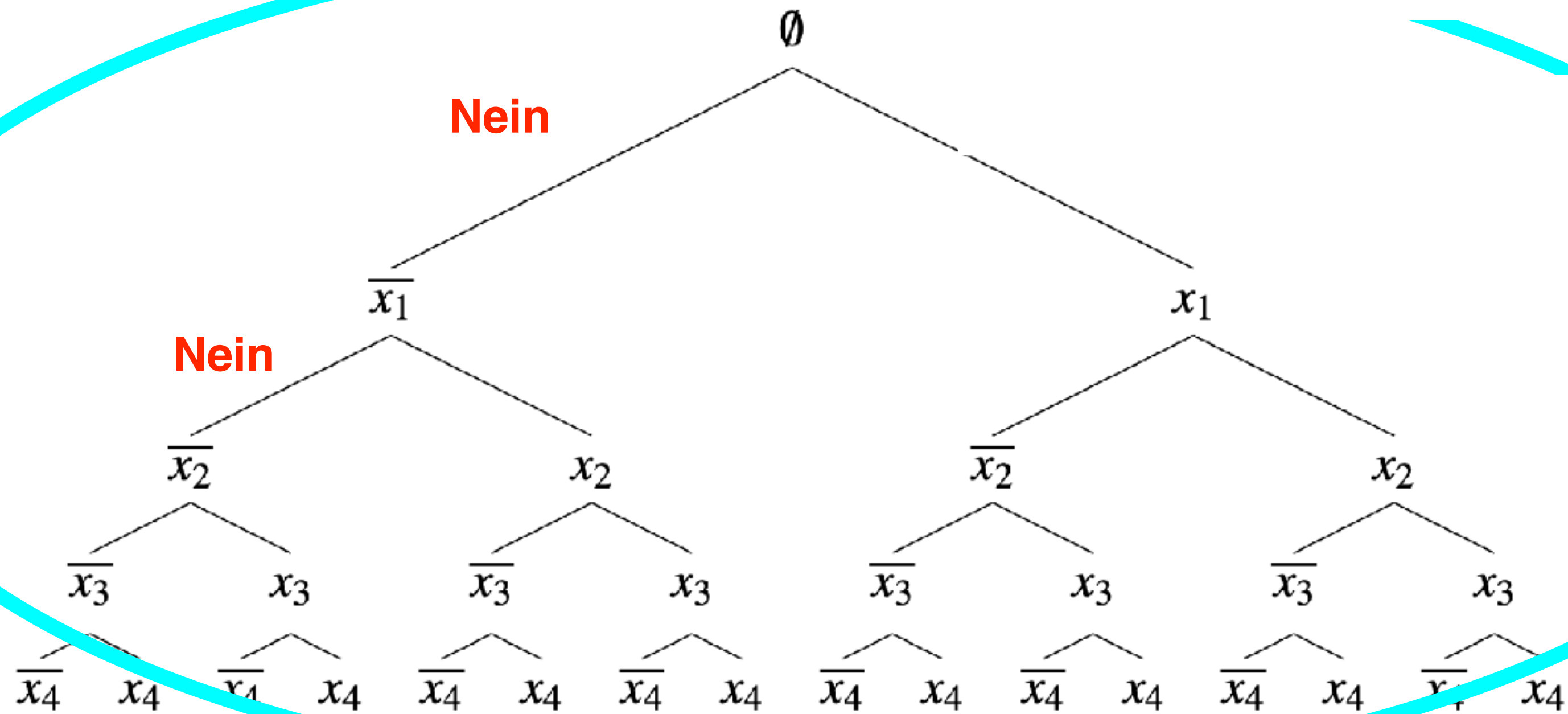
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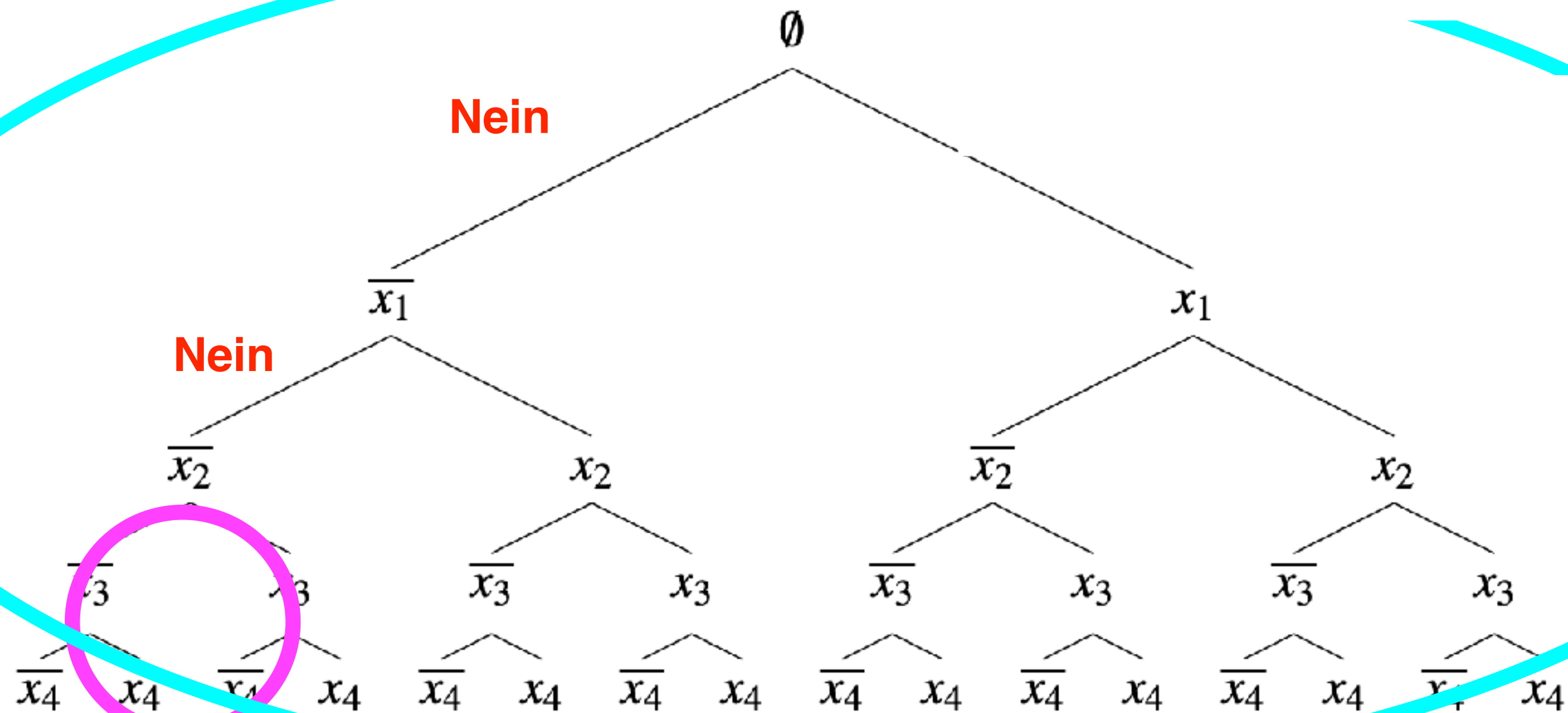
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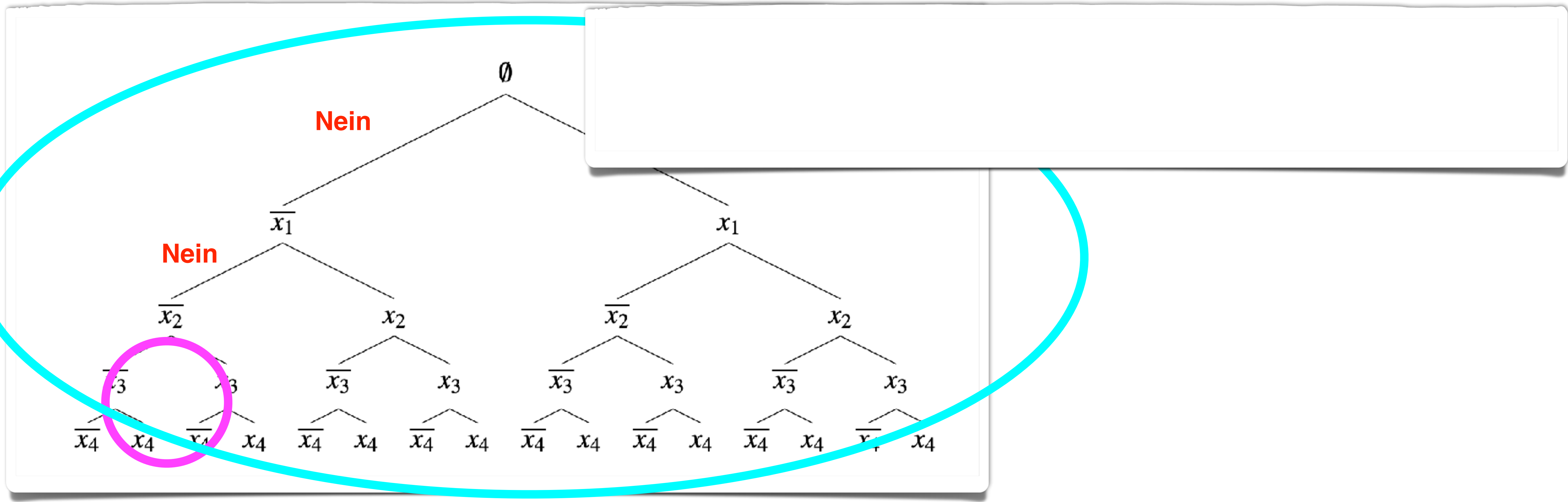
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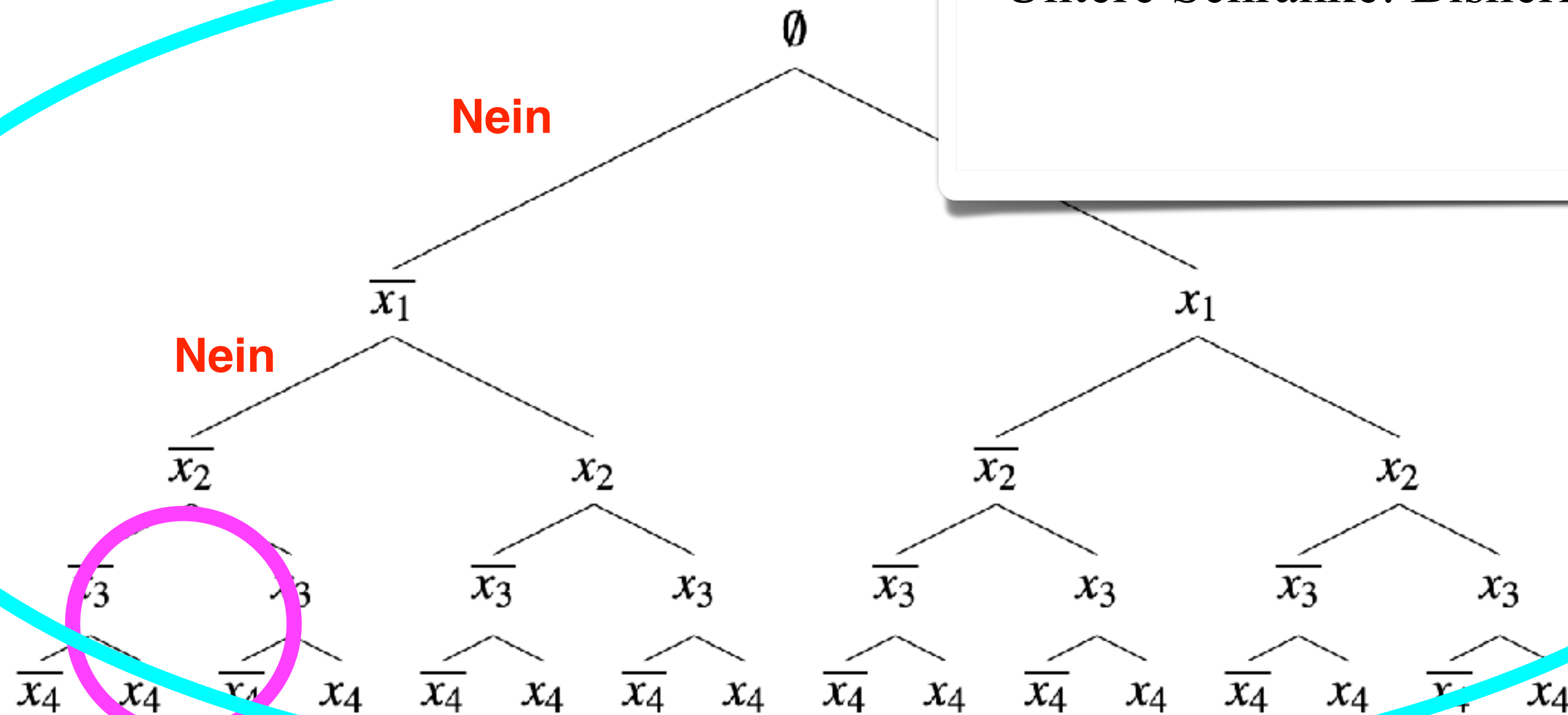


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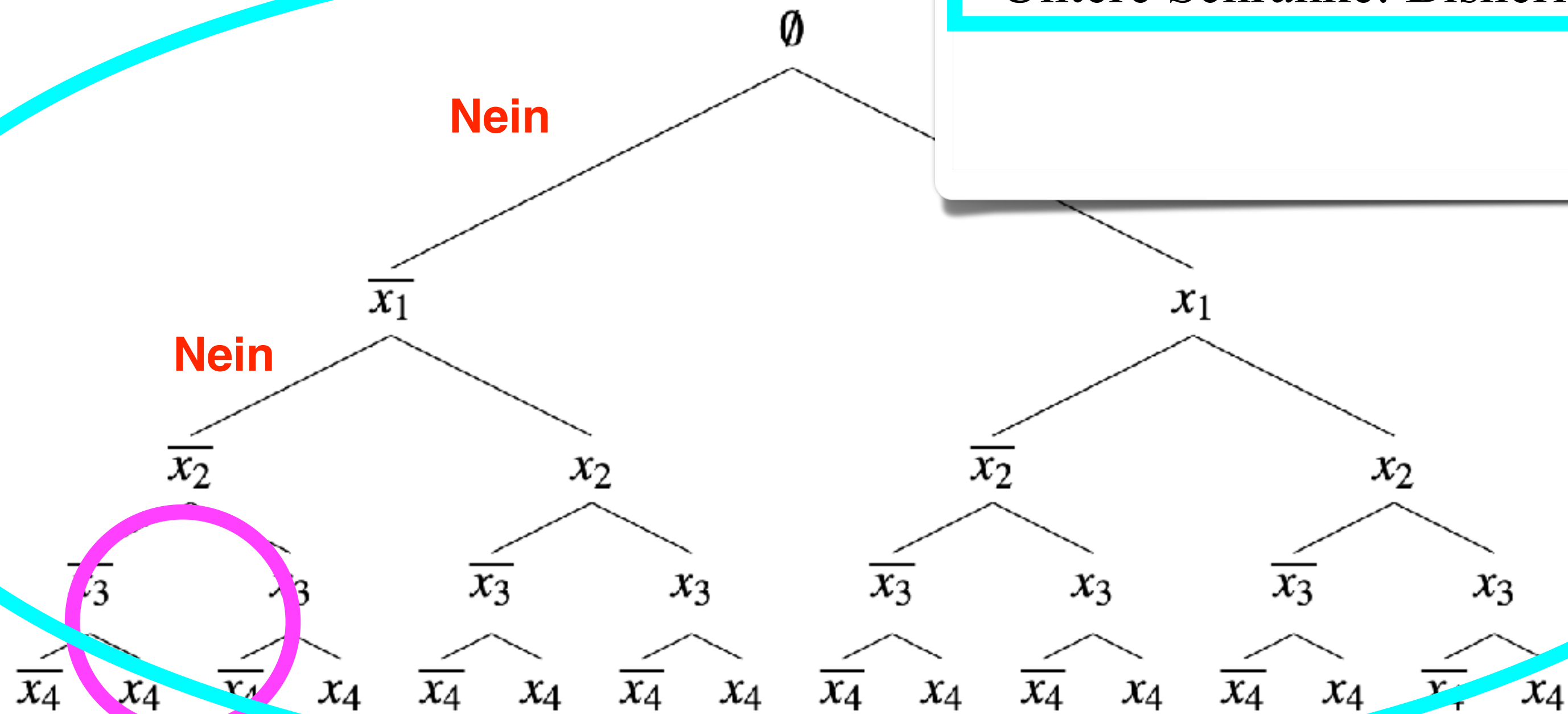
2. Geeignete Schranken - Knapsack

- Untere Schranke: Bisheriger Bestwert für Greedy (ganzzahlig)



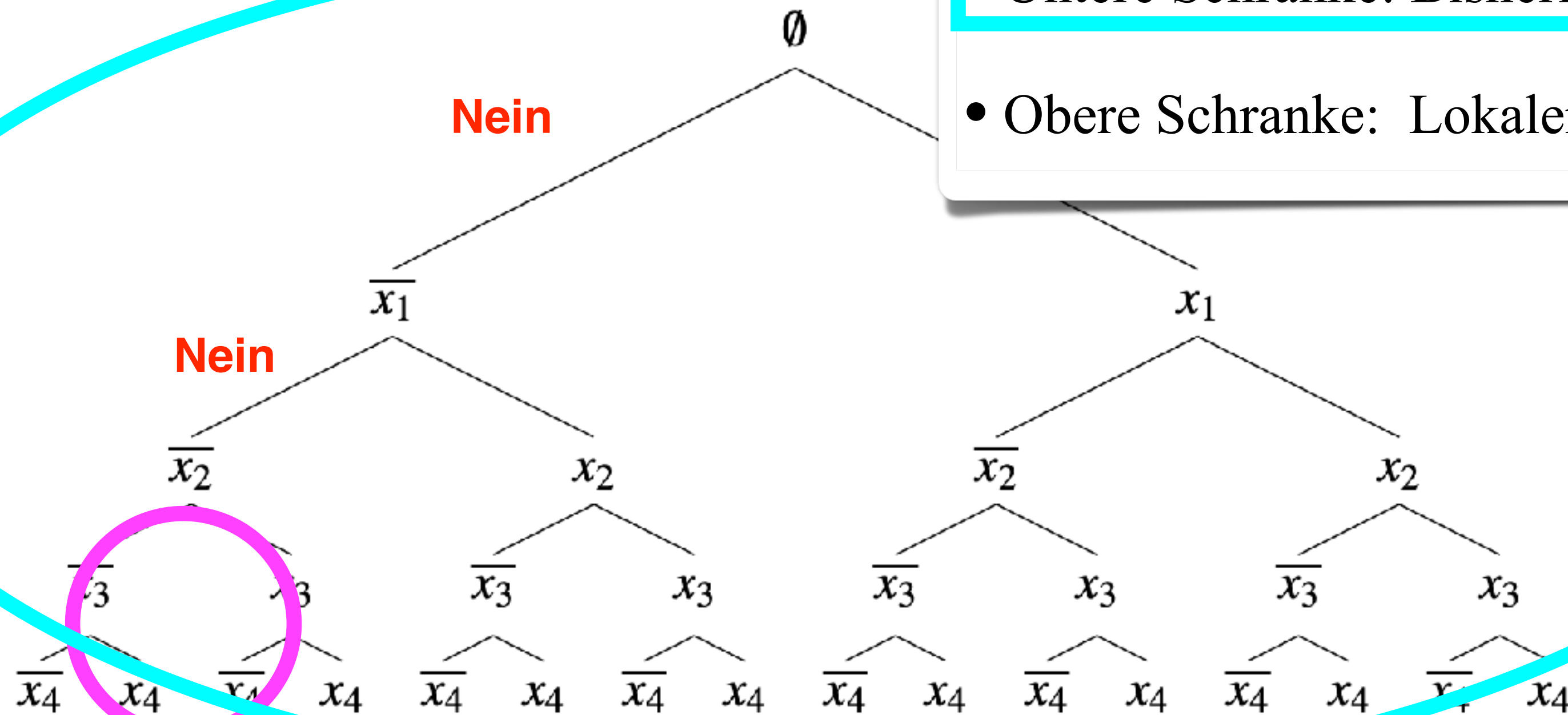
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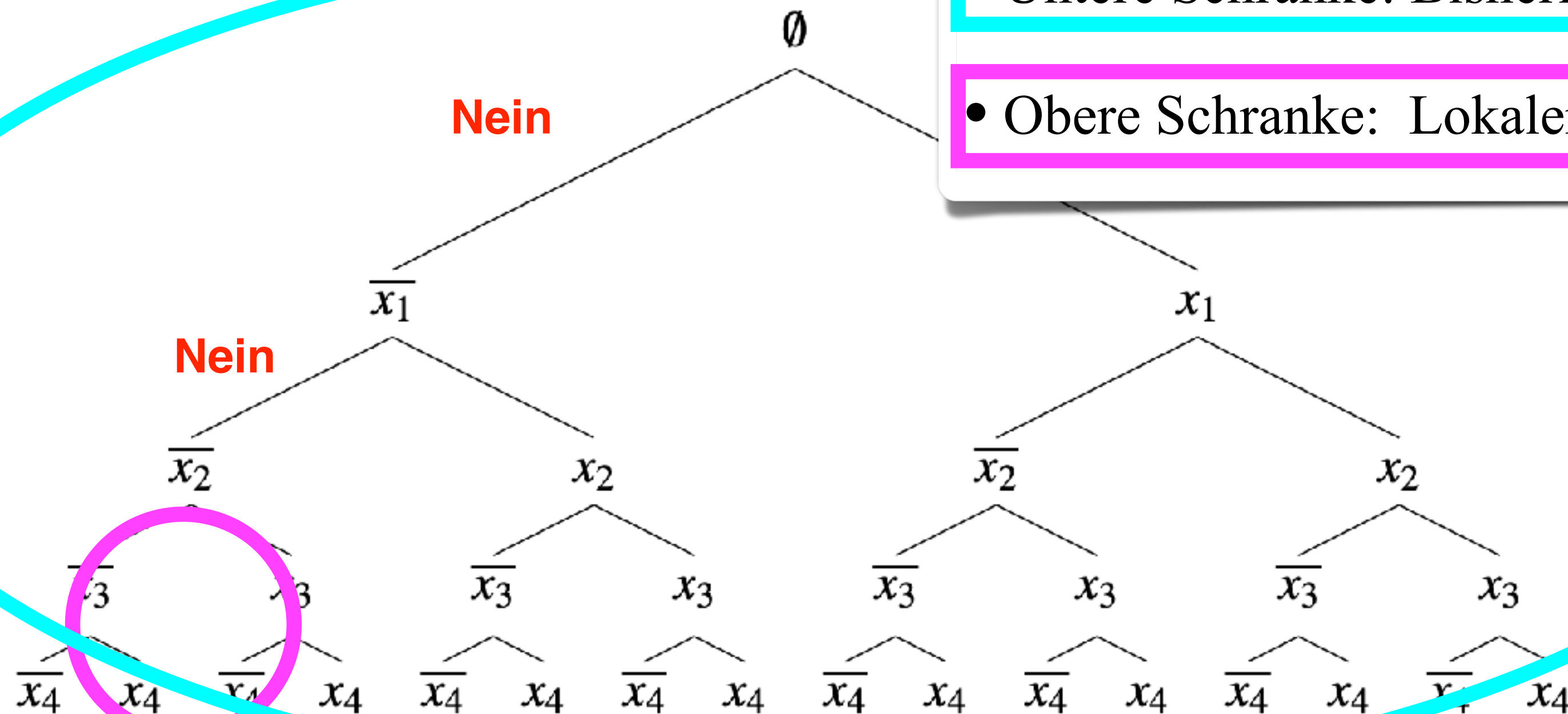
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- Obere Schranke: Lokaler erreichbarer Bestwert für Greedy (fraktional)



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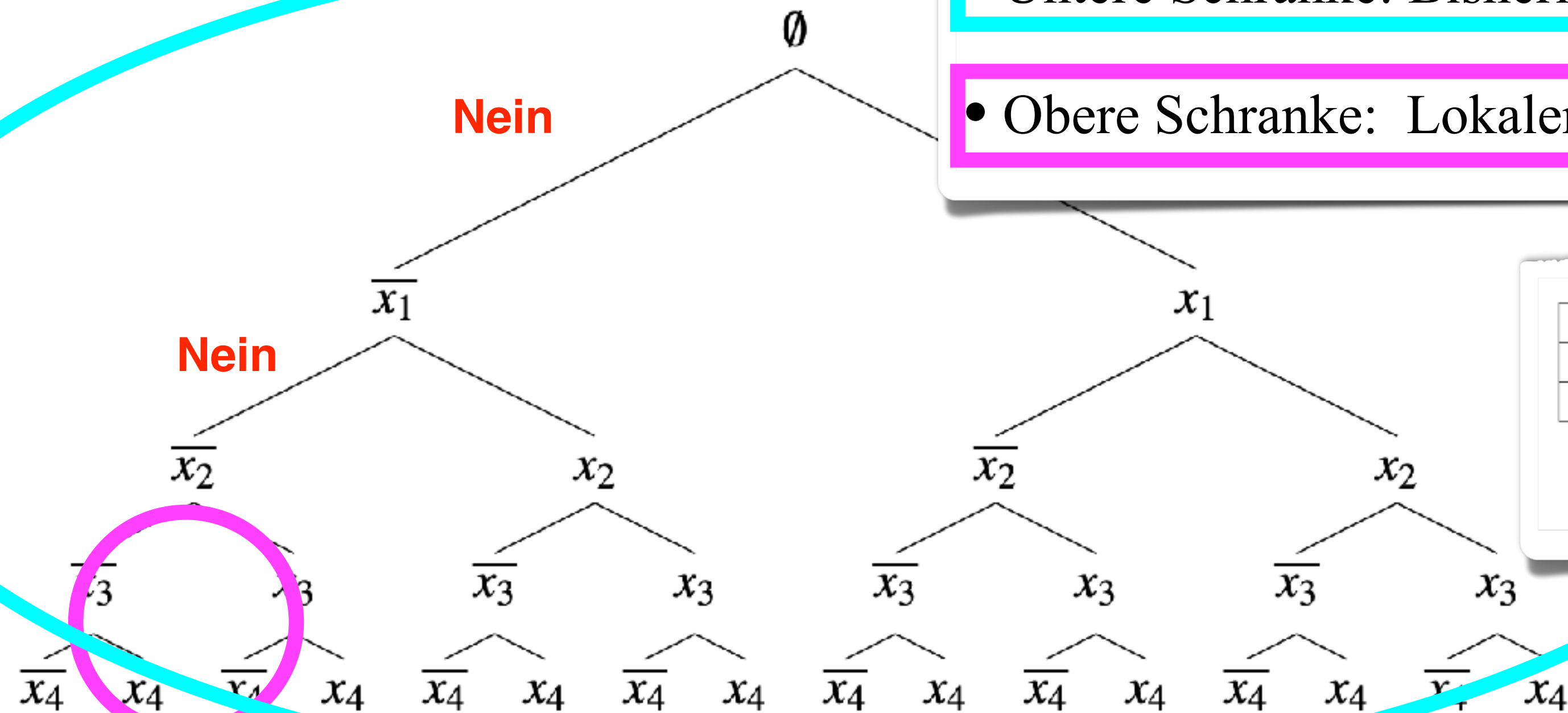
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i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

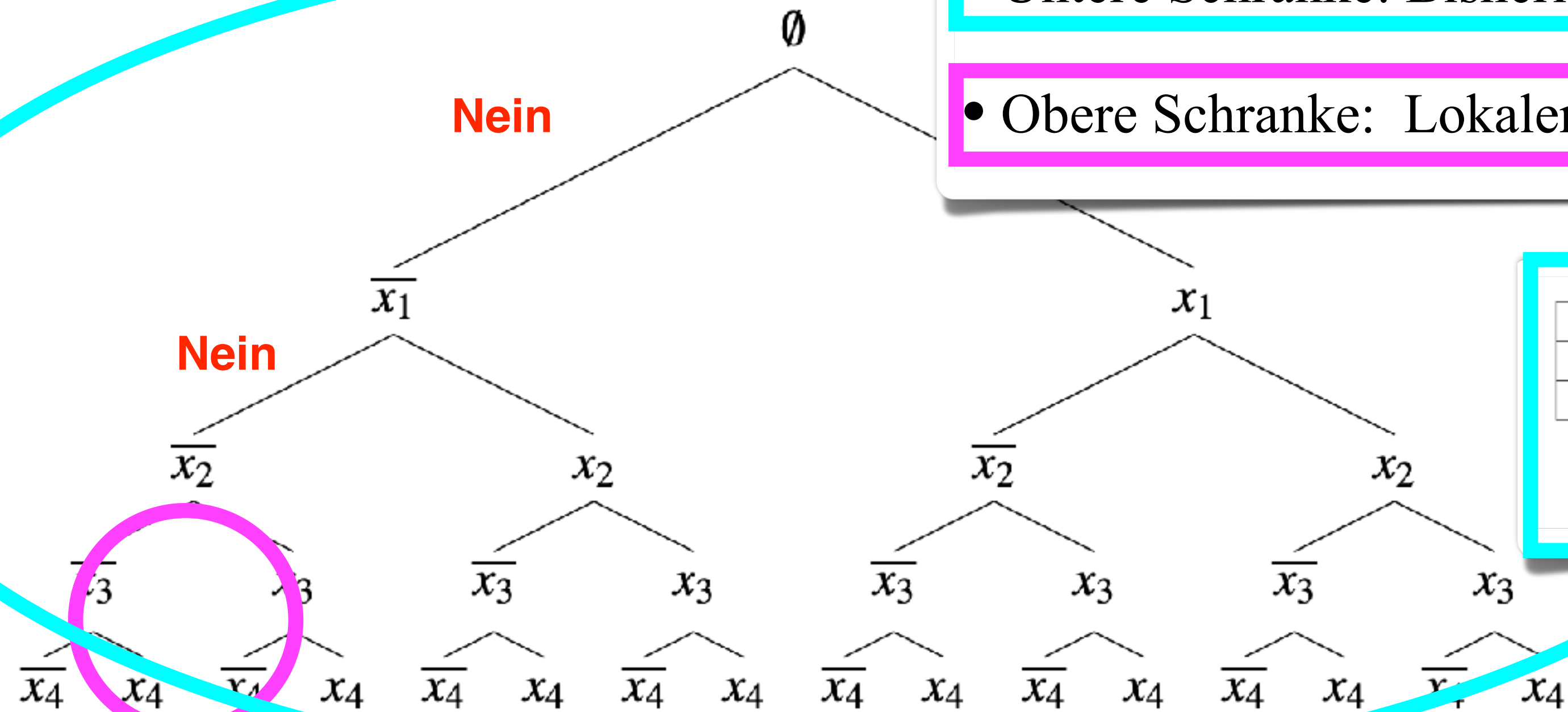
$\sum_{i=1}^n p_i x_i = 44$

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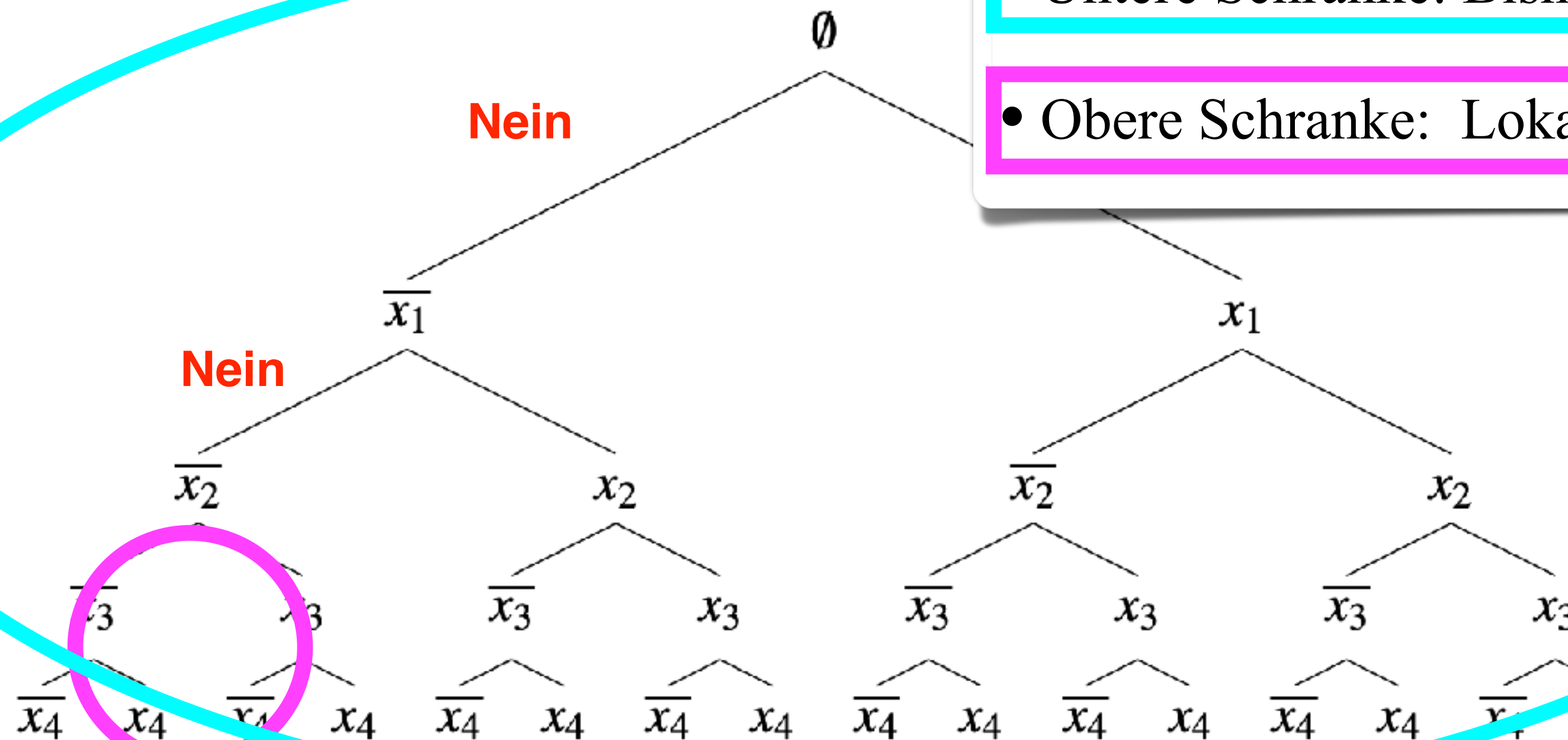
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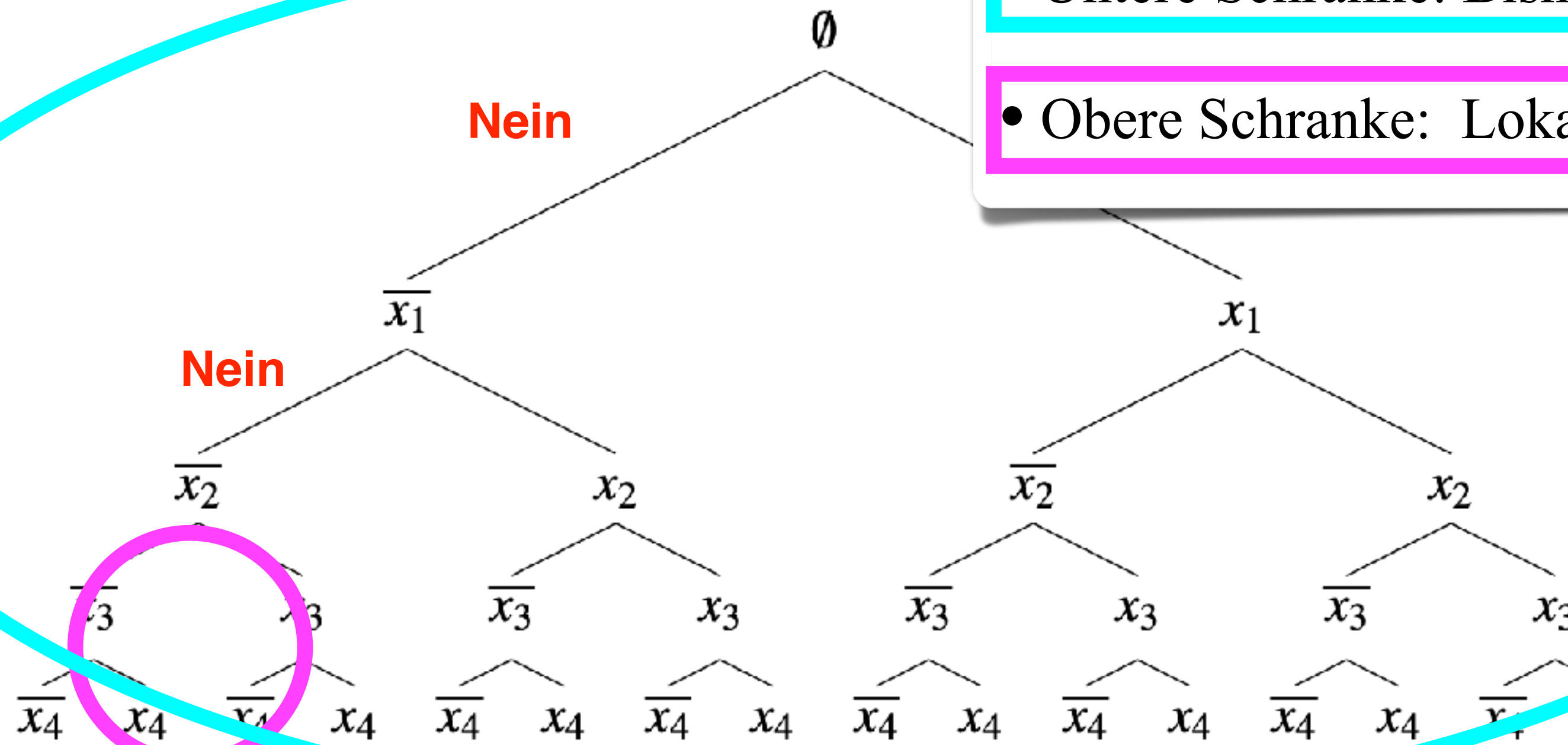
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$\sum_{i=1}^n p_i x_i = 46$ $x_{15} = 0,6$

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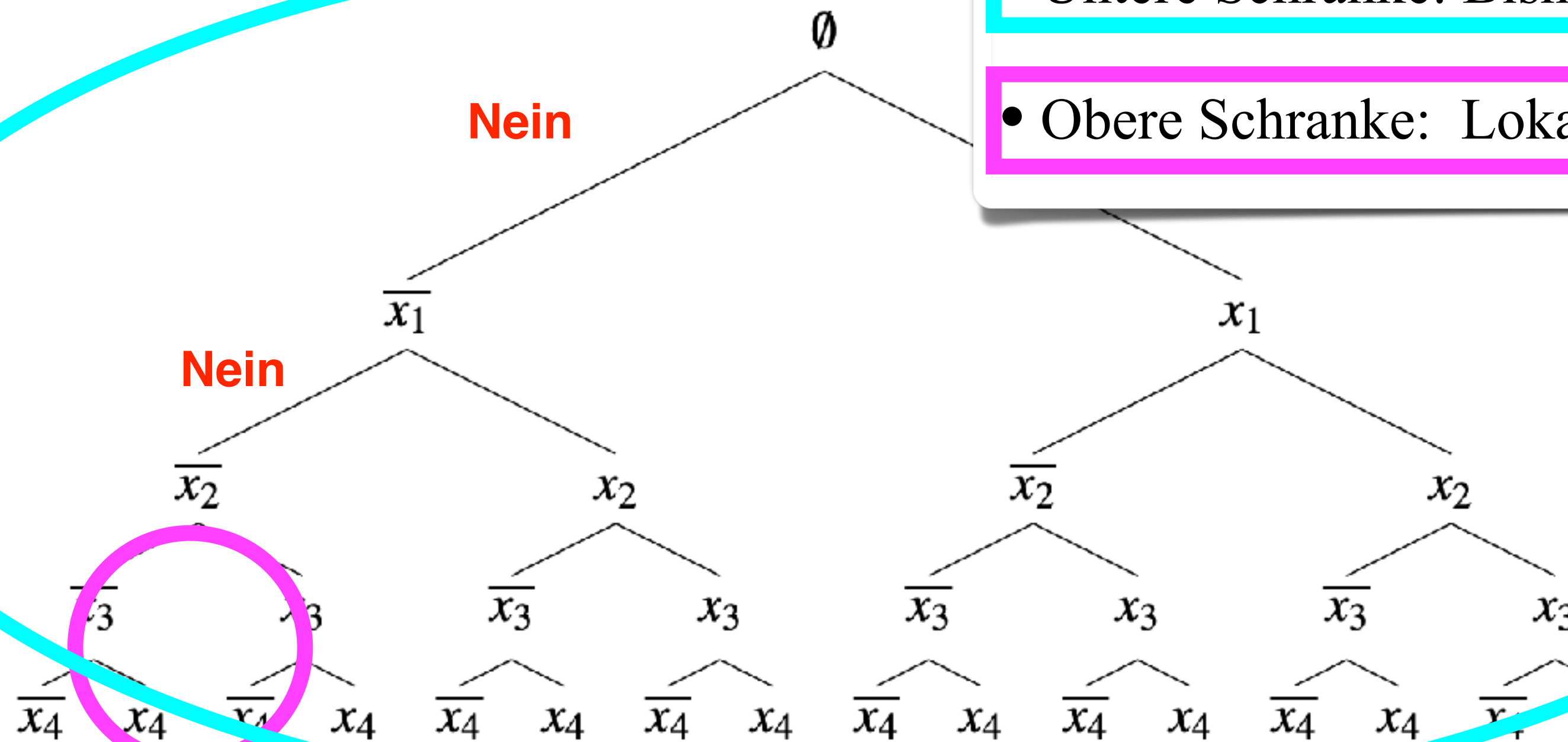
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Relaxierung: Einfacher durch größere Lösungsmenge

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Relaxierung: Schnell und optimal zu lösen durch größere Lösungsmenge

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- Untere Schranke: Bisheriger Bestwert für Greedy (ganzzahlig)
- Obere Schranke: Lokaler erreichbarer Bestwert für Greedy (fraktional)

<i>i</i>	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
<i>z_i</i>	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
<i>p_i</i>	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$\sum_{i=1}^n p_i x_i = 44$

<i>i</i>	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
<i>z_i</i>	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
<i>p_i</i>	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

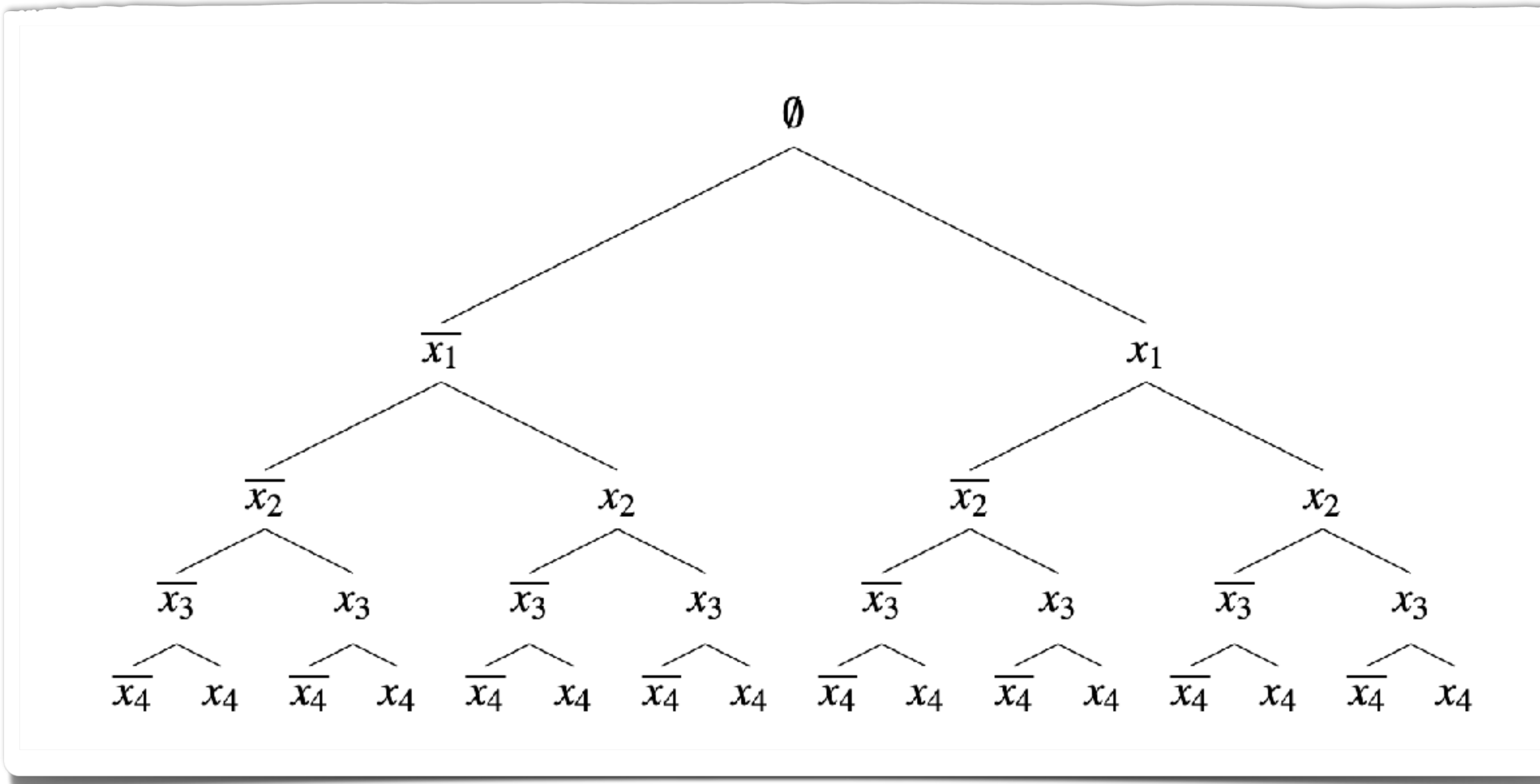
$\sum_{i=1}^n p_i x_i = 46$ $x_{15} = 0,6$

Heuristik: Schnell zu lösen, aber nicht notwendig optimal

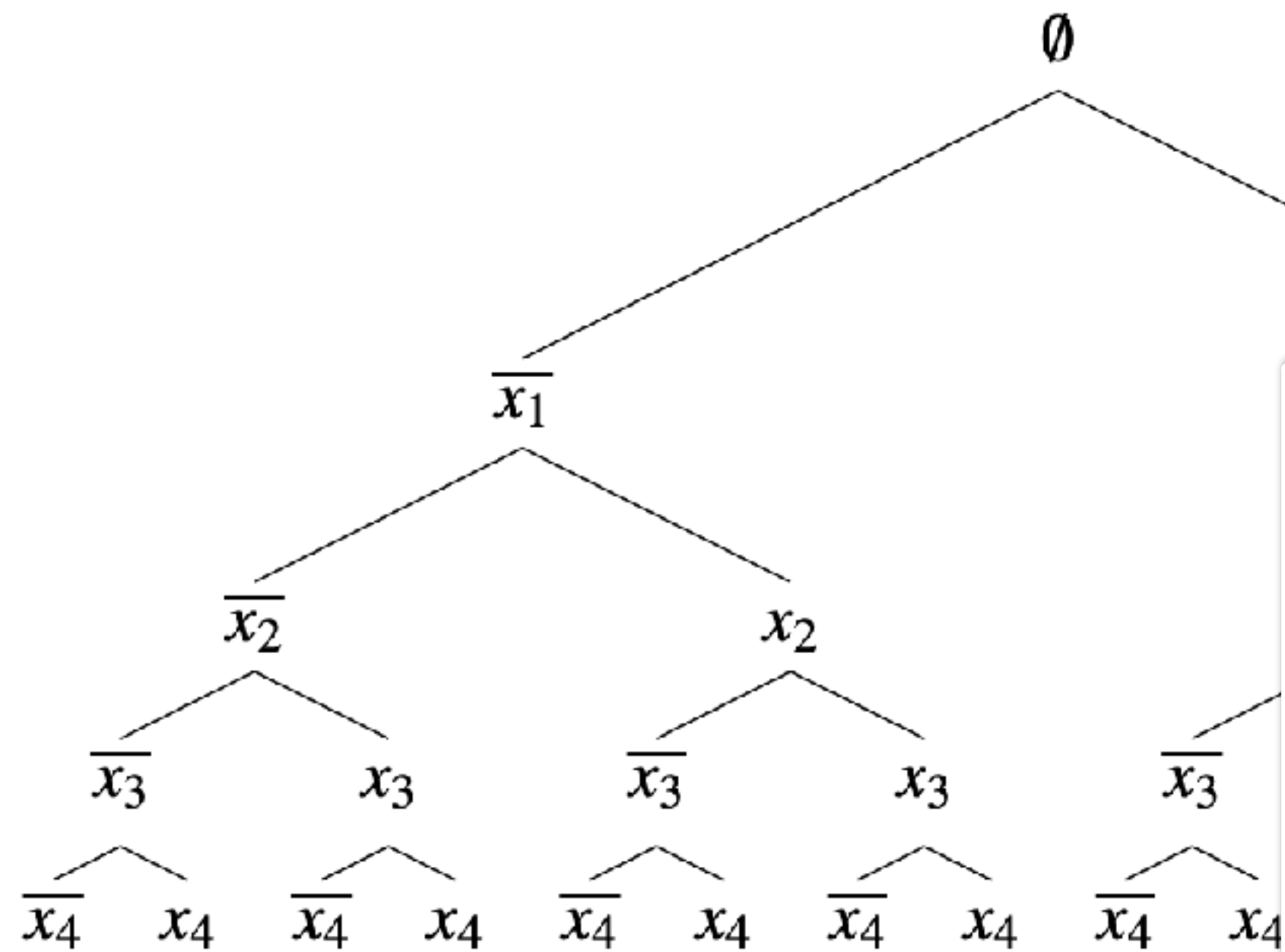
Relaxierung: Schnell und optimal zu lösen durch größere Lösungsmenge

3. Berechnung von Schranken II

3. Berechnung von Schranken II

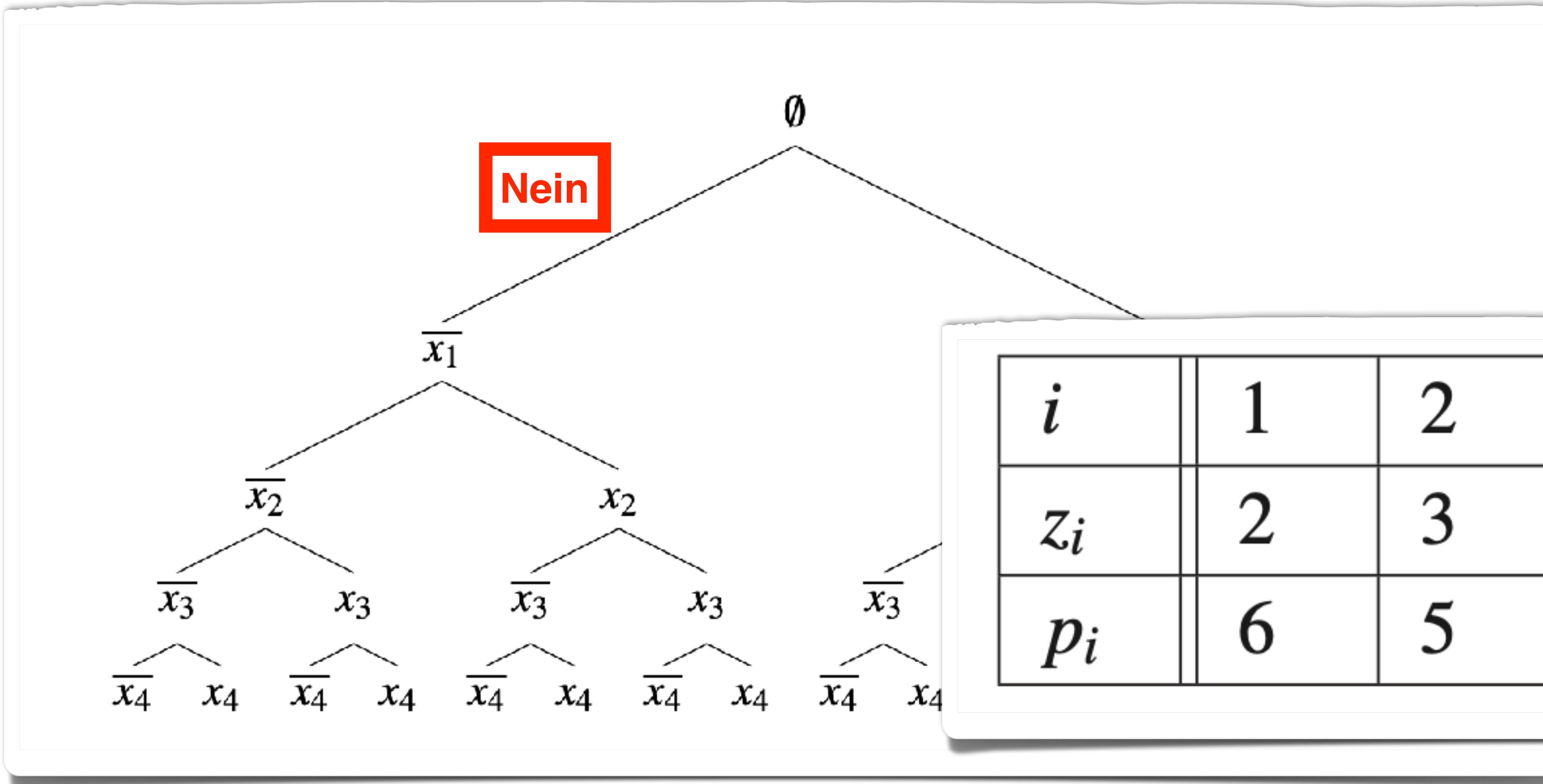


3. Berechnung von Schranken II



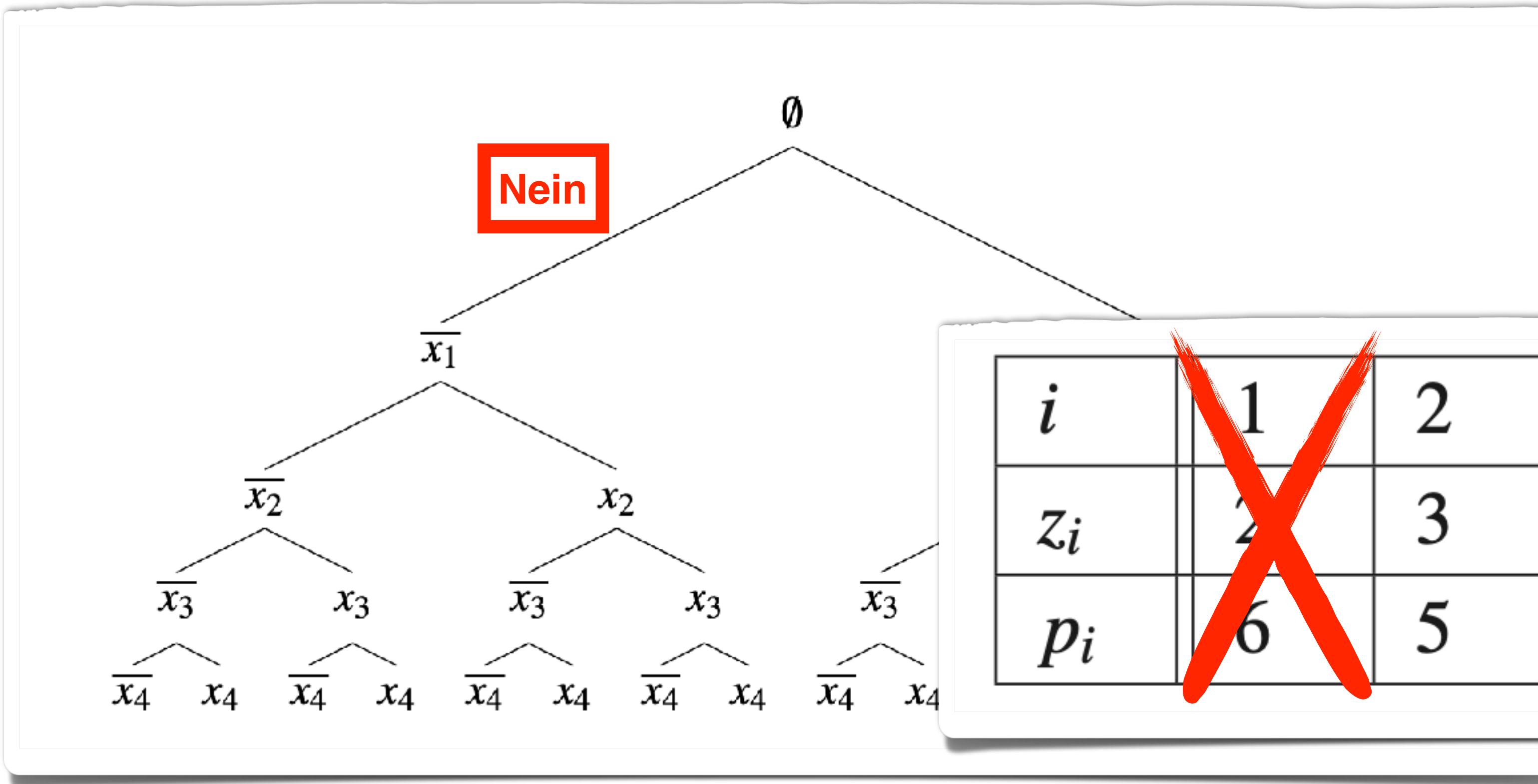
<i>i</i>	1	2	3	3	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

3. Berechnung von Schranken II



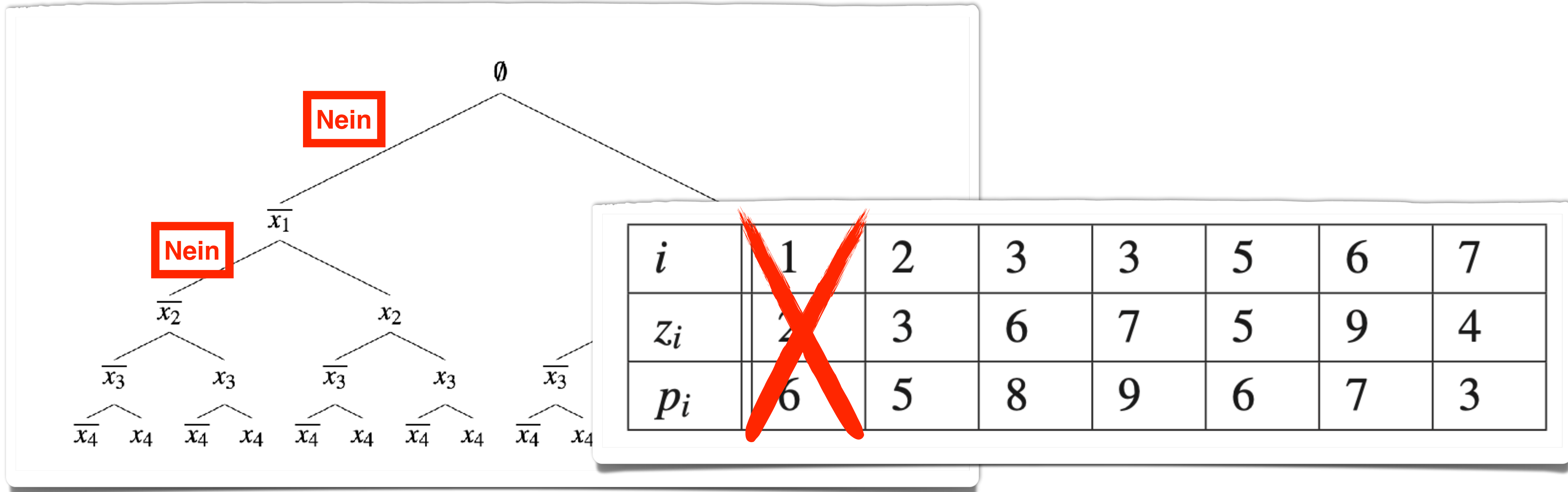
<i>i</i>	1	2	3	3	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

3. Berechnung von Schranken II

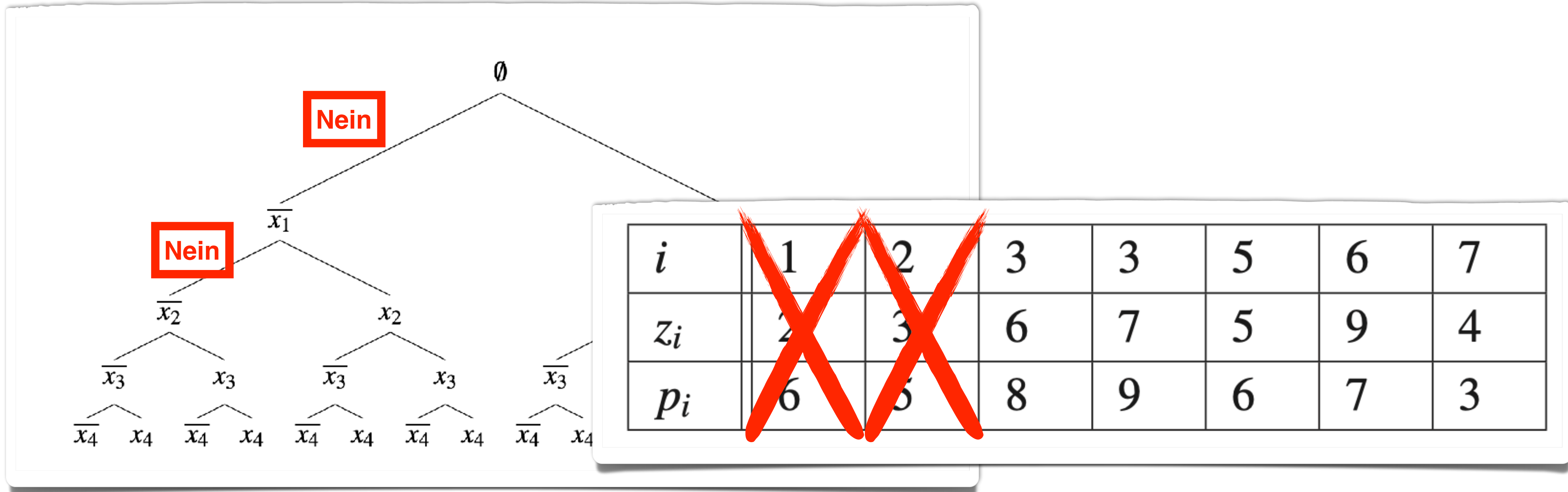


i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

3. Berechnung von Schranken II

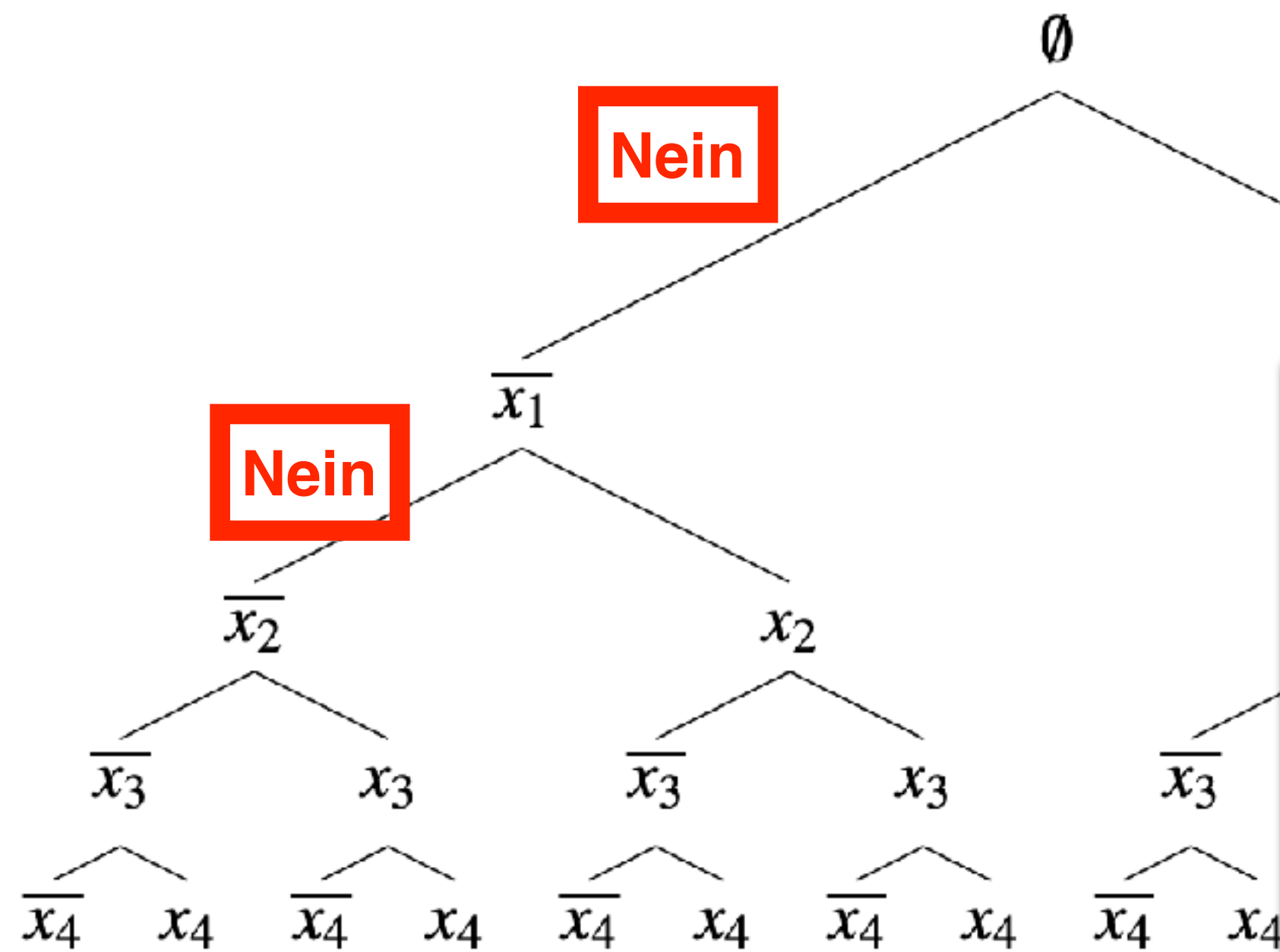


3. Berechnung von Schranken II



3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)
 \bar{S} : negativ fixiert ($x_i = 0$)

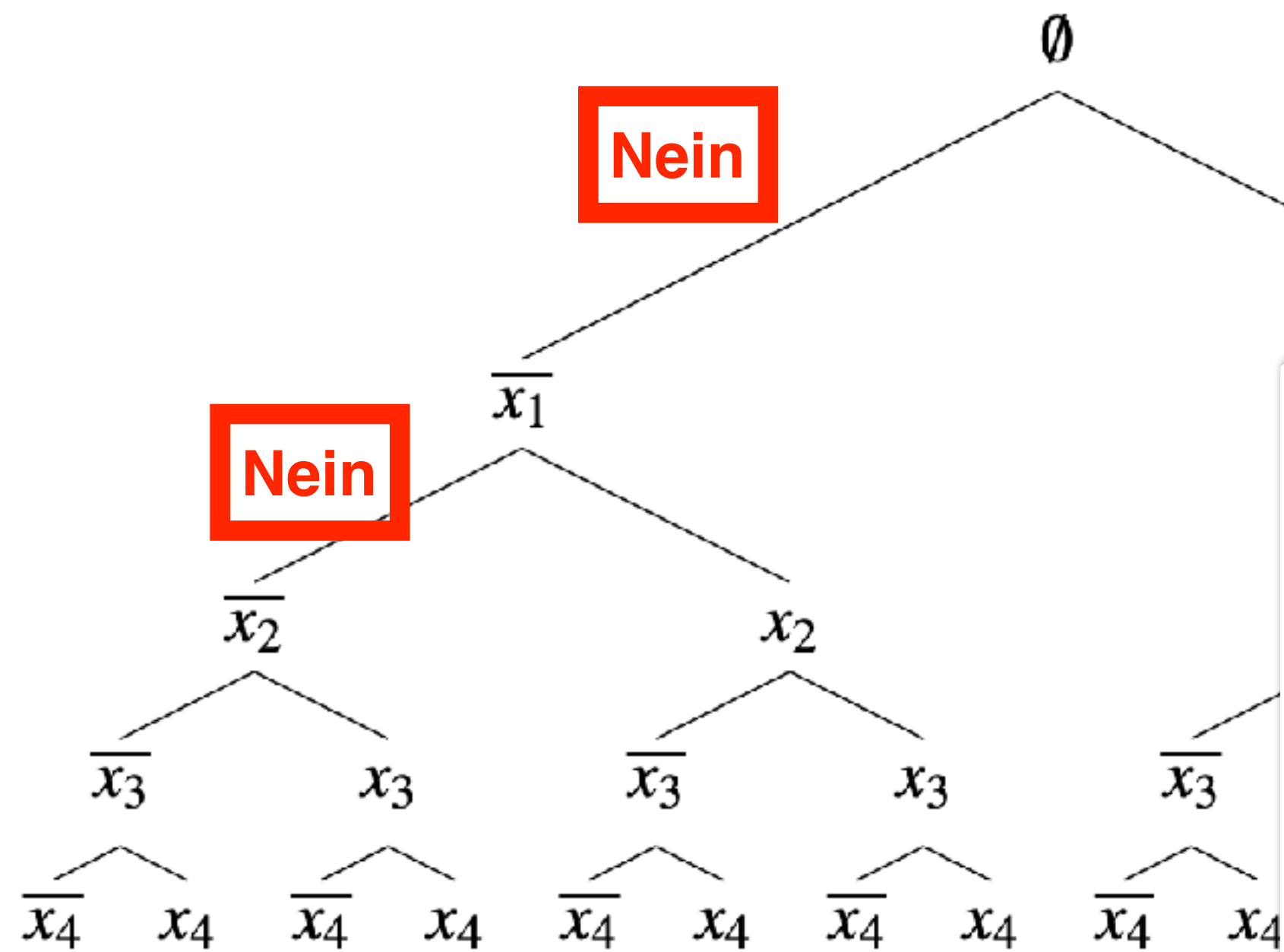


i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

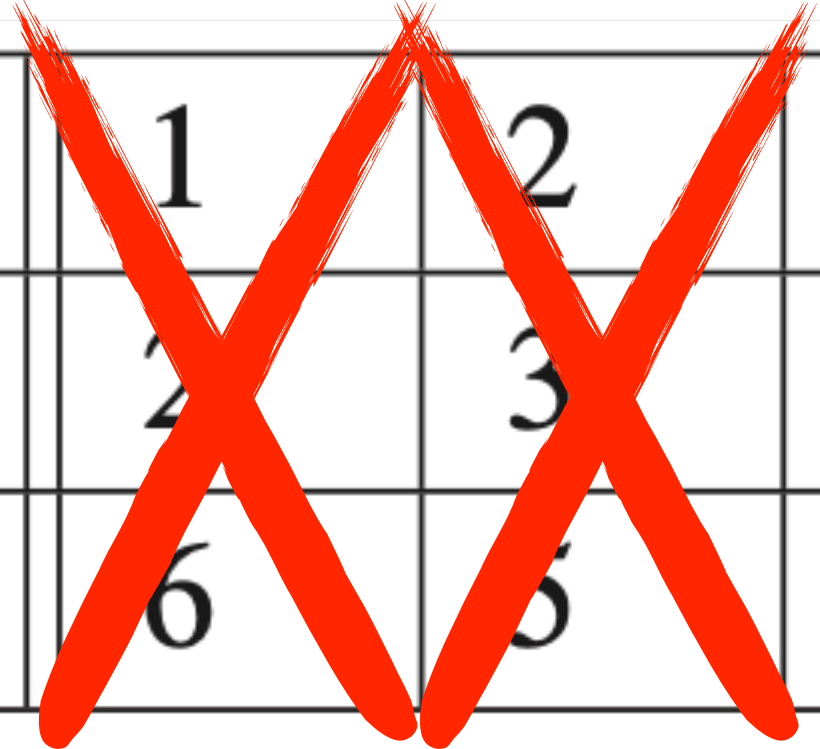
3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)

S: negativ fixiert ($x_i = 0$)



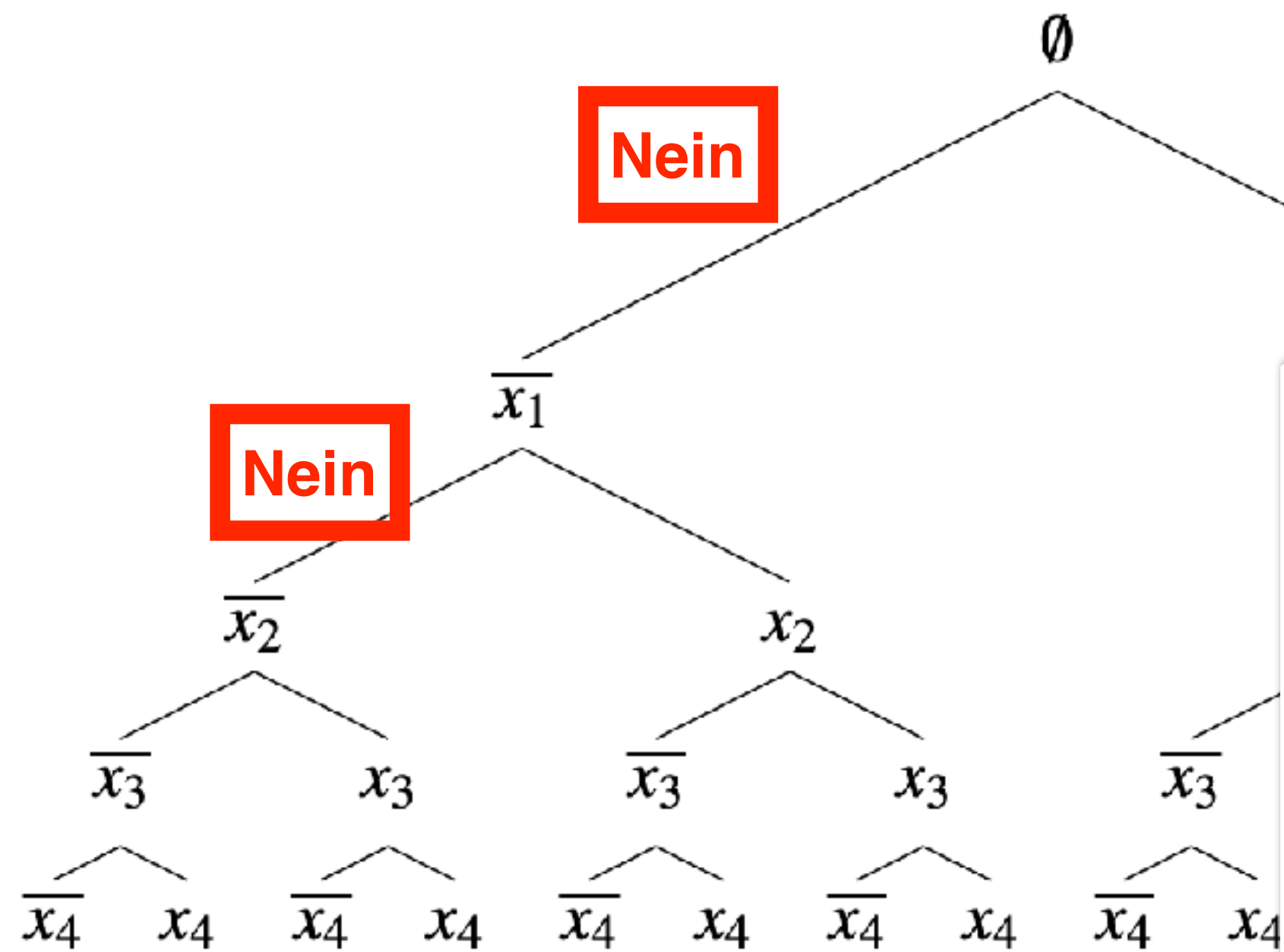
i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3



3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)

S: negativ fixiert ($x_i = 0$)

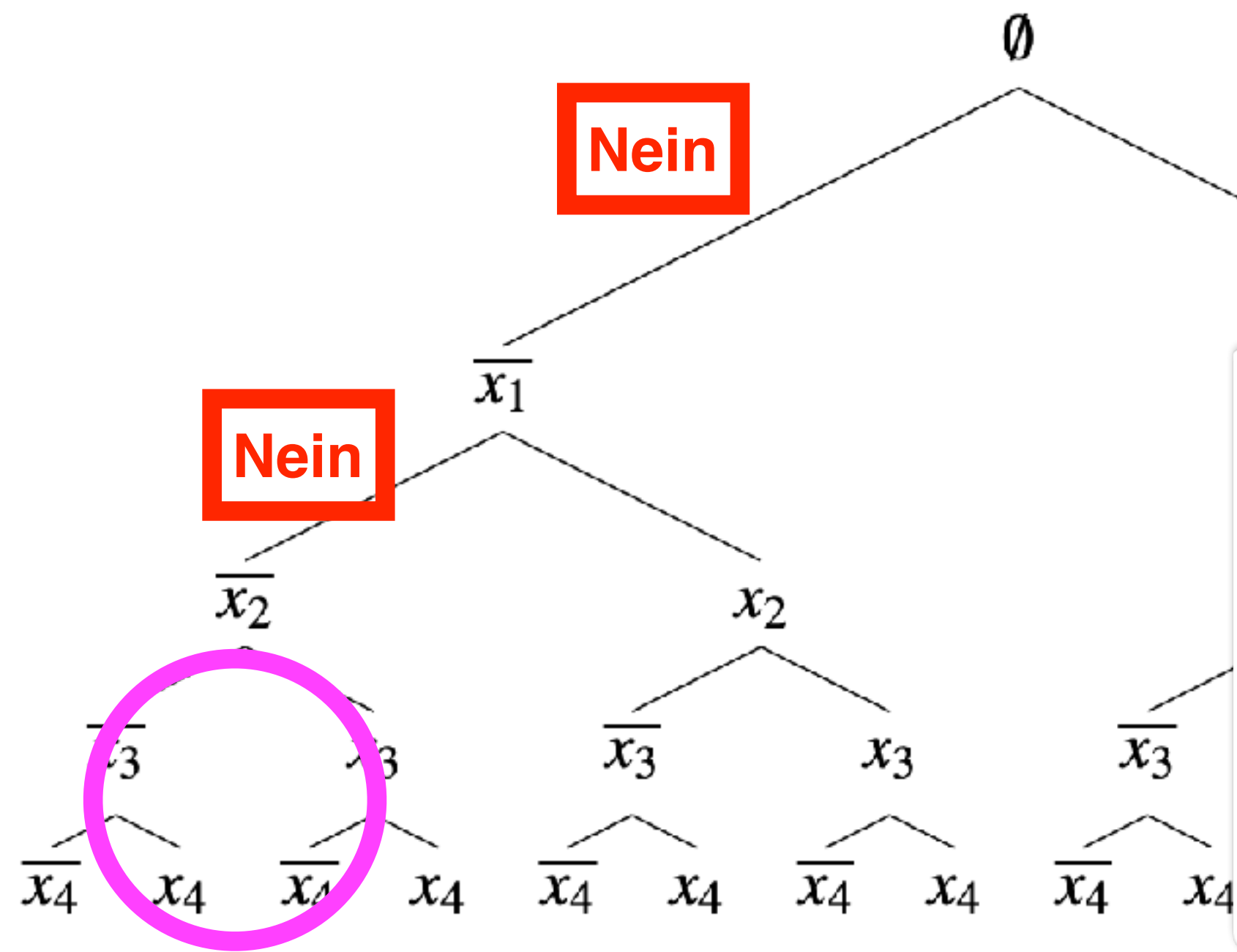


i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

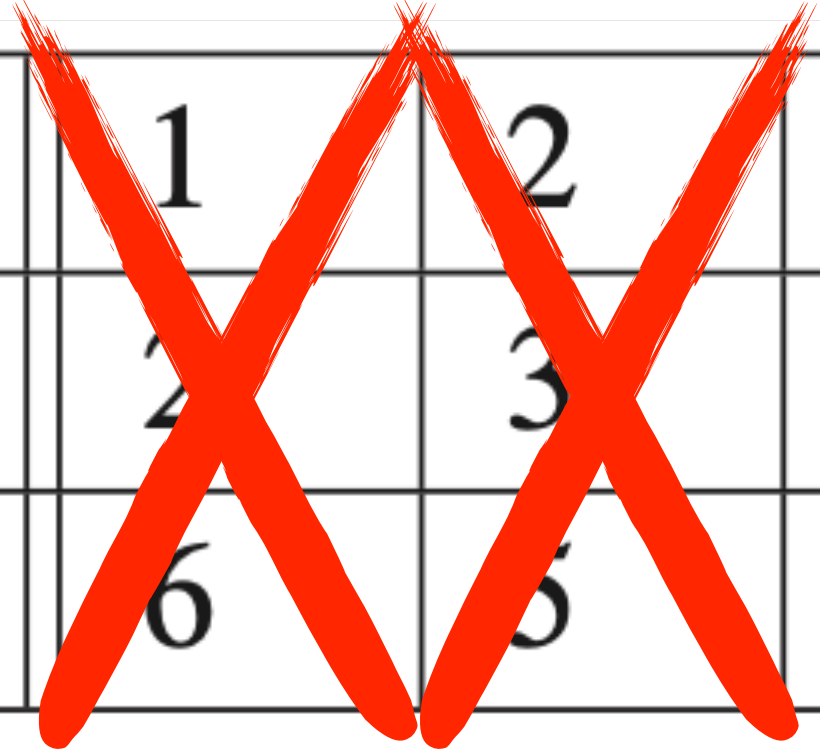
3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)

S: negativ fixiert ($x_i = 0$)



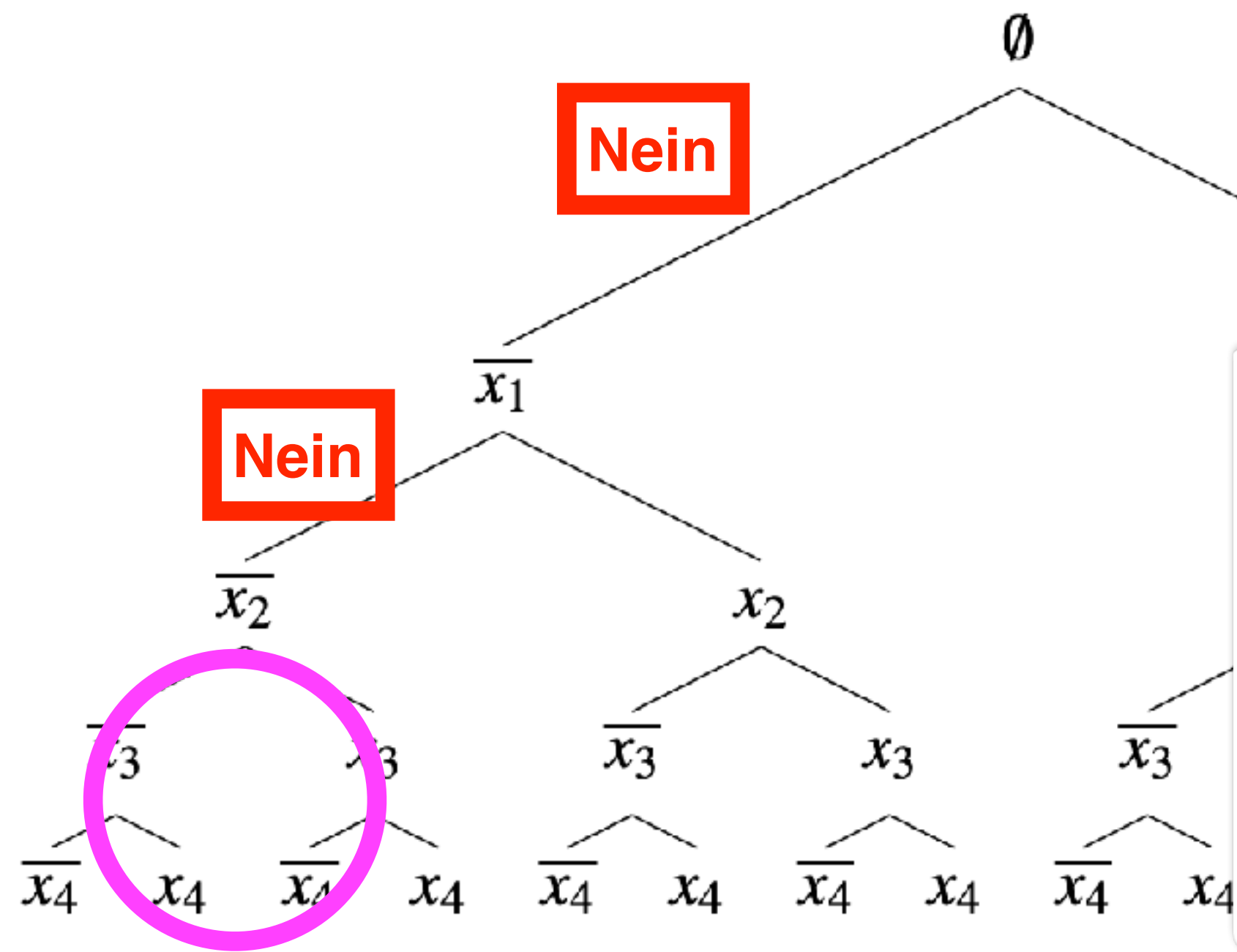
i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3



3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)

S: negativ fixiert ($x_i = 0$)



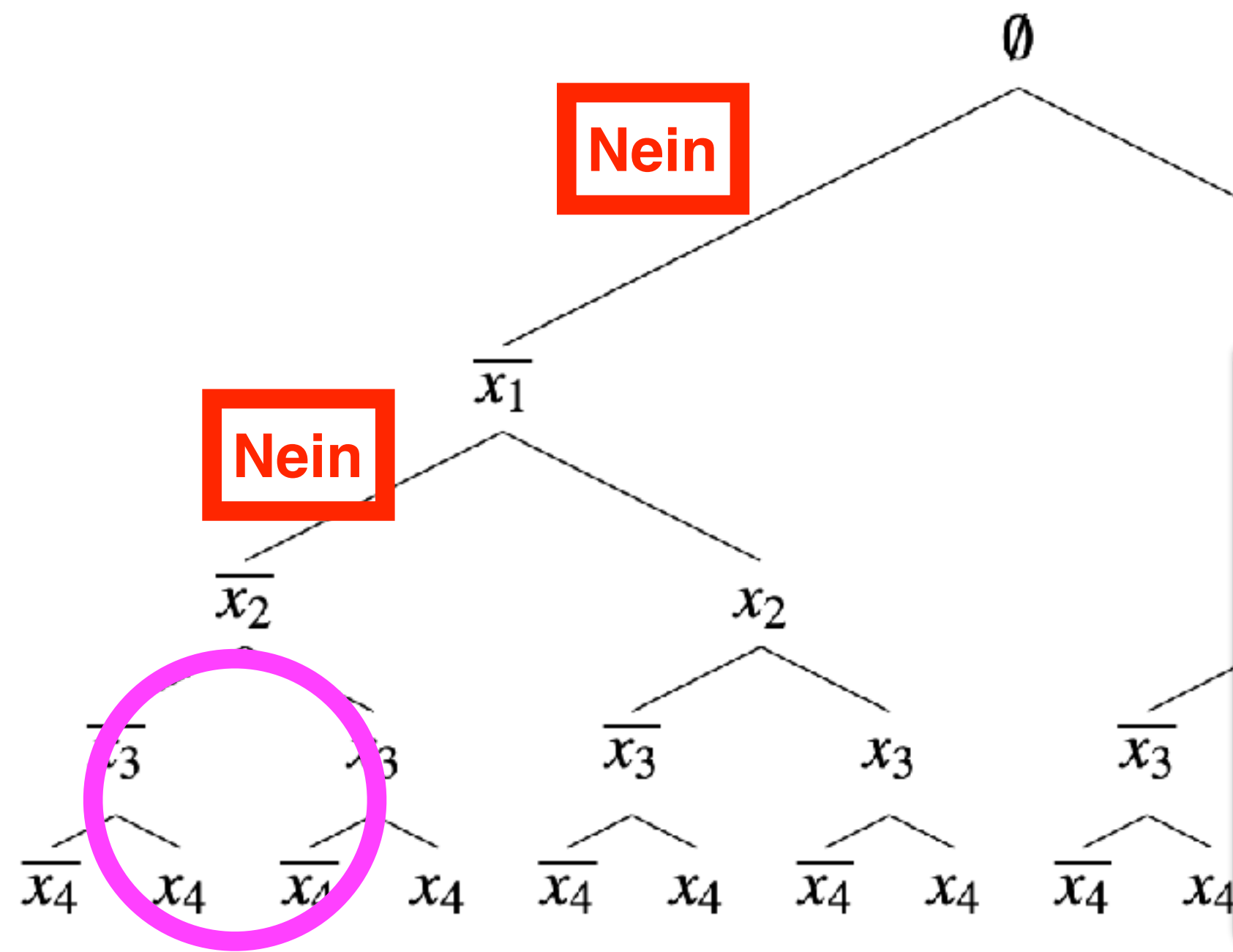
<i>i</i>	1	2	3	3	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)

S: negativ fixiert ($x_i = 0$)

Bisherige Entscheidungen fixieren Teilmenge!



i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

3.3 Durchführung

Beispiel: Knapsack

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i
z_i
p_i

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1
z_i	2
p_i	6

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2
z_i	2	3
p_i	6	5

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3
z_i	2	3	6
p_i	6	5	8

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4
z_i	2	3	6	7
p_i	6	5	8	9

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5
z_i	2	3	6	7	5
p_i	6	5	8	9	6

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5	6
z_i	2	3	6	7	5	9
p_i	6	5	8	9	6	7

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Untere Schranke: Greedy (ganzzahlig)

Untere Schranke: Greedy (ganzzahlig)

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Untere Schranke: Greedy (ganzzahlig)

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

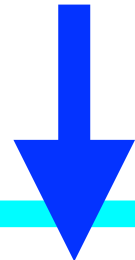
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



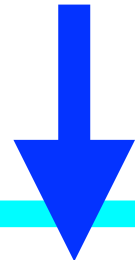
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



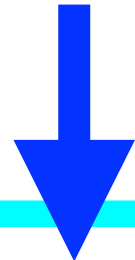
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

$$i=1 : \sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3


$$Z = 9$$

$$i=1 : \sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$




i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$




i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3


$$Z = 9$$

$$i=2 : \sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

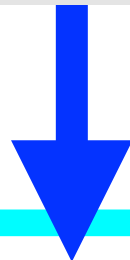
$$Z = 9$$

$$i=2 : \sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



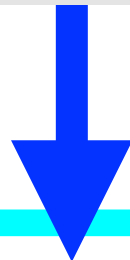
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



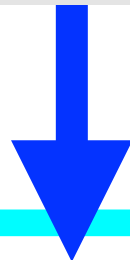
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



<i>i</i>	1	2	3	4	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

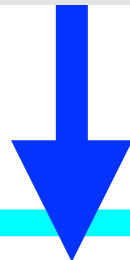
$$Z = 9$$

$$i=3 : \sum_{i \in S} z_i = 11 > 9, X$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



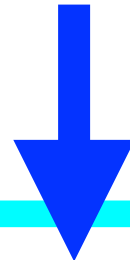
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



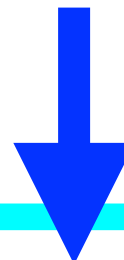
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



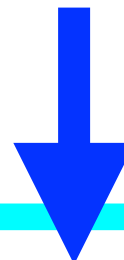
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

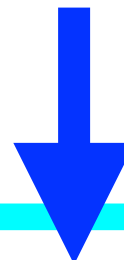
$$Z = 9$$

$$i=4 : \sum_{i \in S} z_i = 12 > 9, X$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$




i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$




i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$




i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3


$$Z = 9$$

$$i=5 : \sum_{i \in S} z_i = 10 > 9, X$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



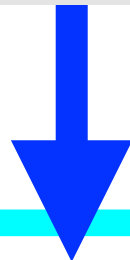
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



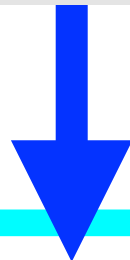
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



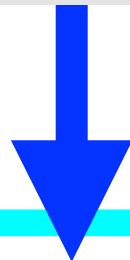
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



<i>i</i>	1	2	3	4	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

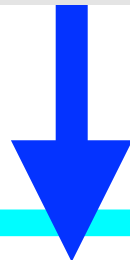
$$Z = 9$$

$$i=6 : \sum_{i \in S} z_i = 14 > 9, X$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

<i>i</i>	1	2	3	4	5	6	7
<i>z_i</i>	2	3	6	7	5	9	4
<i>p_i</i>	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

$$i=7 : \sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

$$i=7 : \sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$$Z = 9$$

$$i=7 : \sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$$

Untere Schranke: Greedy (ganzzahlig)

$$S = \emptyset$$

$$\bar{S} = \emptyset$$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
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$$i=1 : \sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$$

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$$\text{Damit: } \sum_{i \in S} x_i z_i = 9, \sum_{i \in S} x_i p_i = 11 + \frac{2}{3} * 8 = 16, \bar{3}$$

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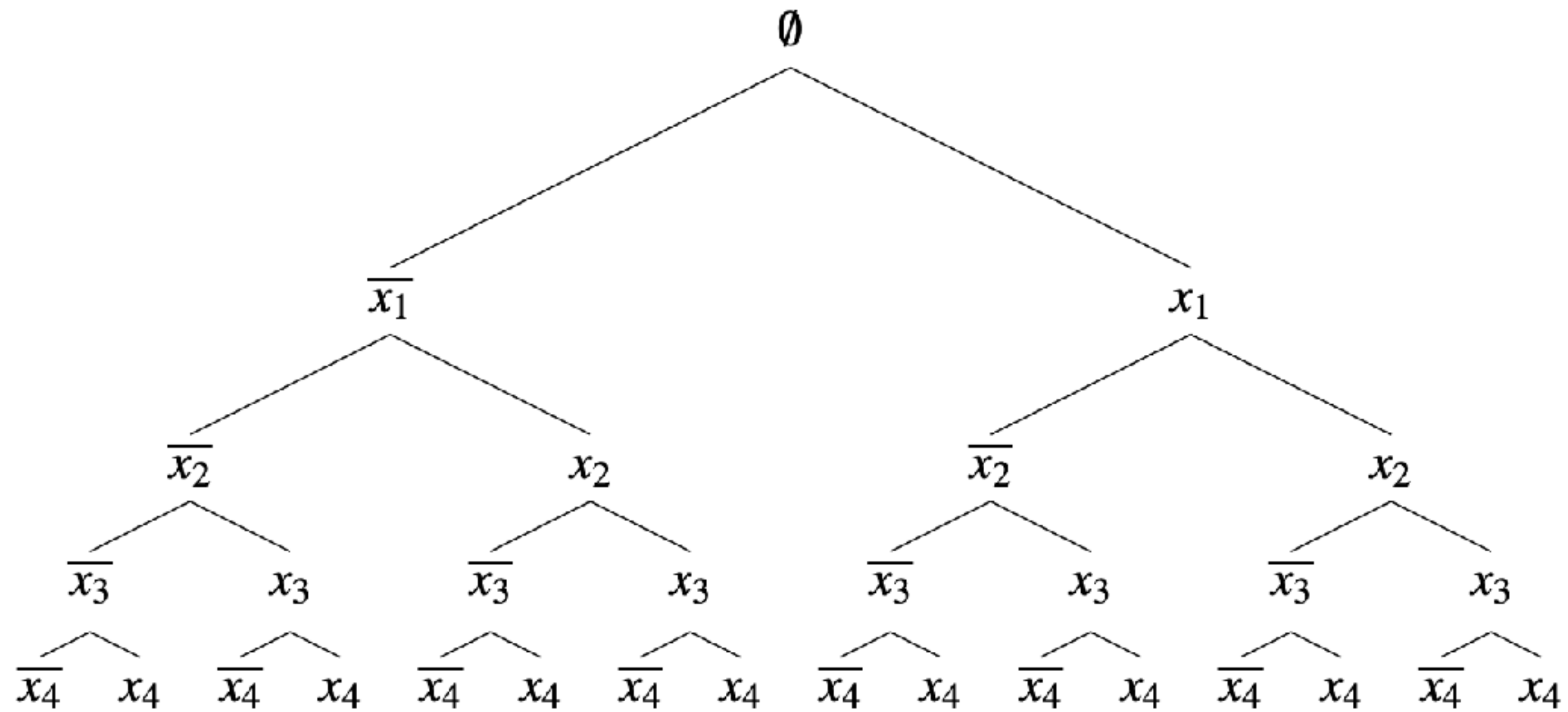
$$\text{Bleibt: } Z - \sum_{i \in S} z_i = 4, \text{ also } x_3 = \frac{2}{3} \left(= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3} \right)$$

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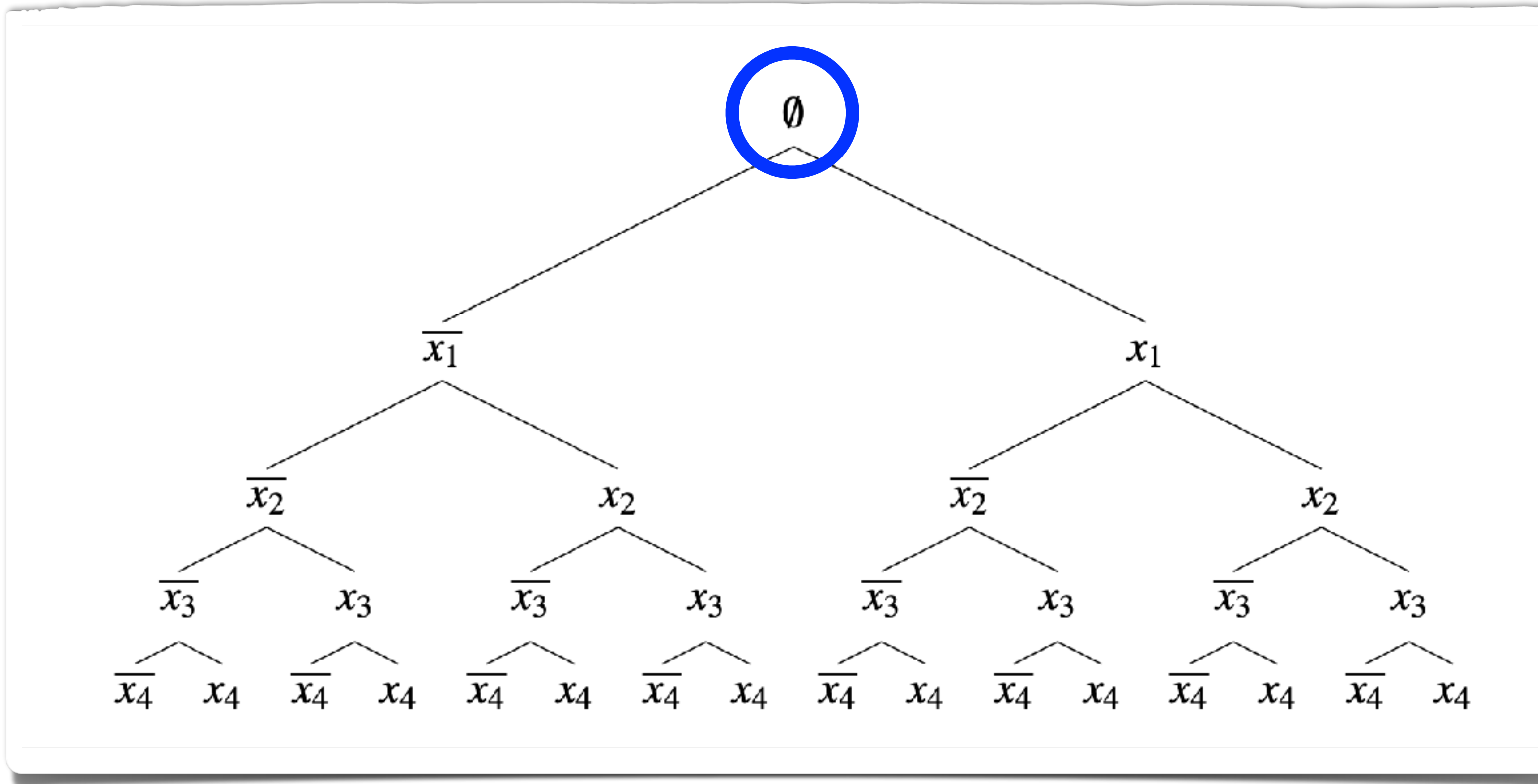
$$UB = 16$$

Gesamtbeispiel

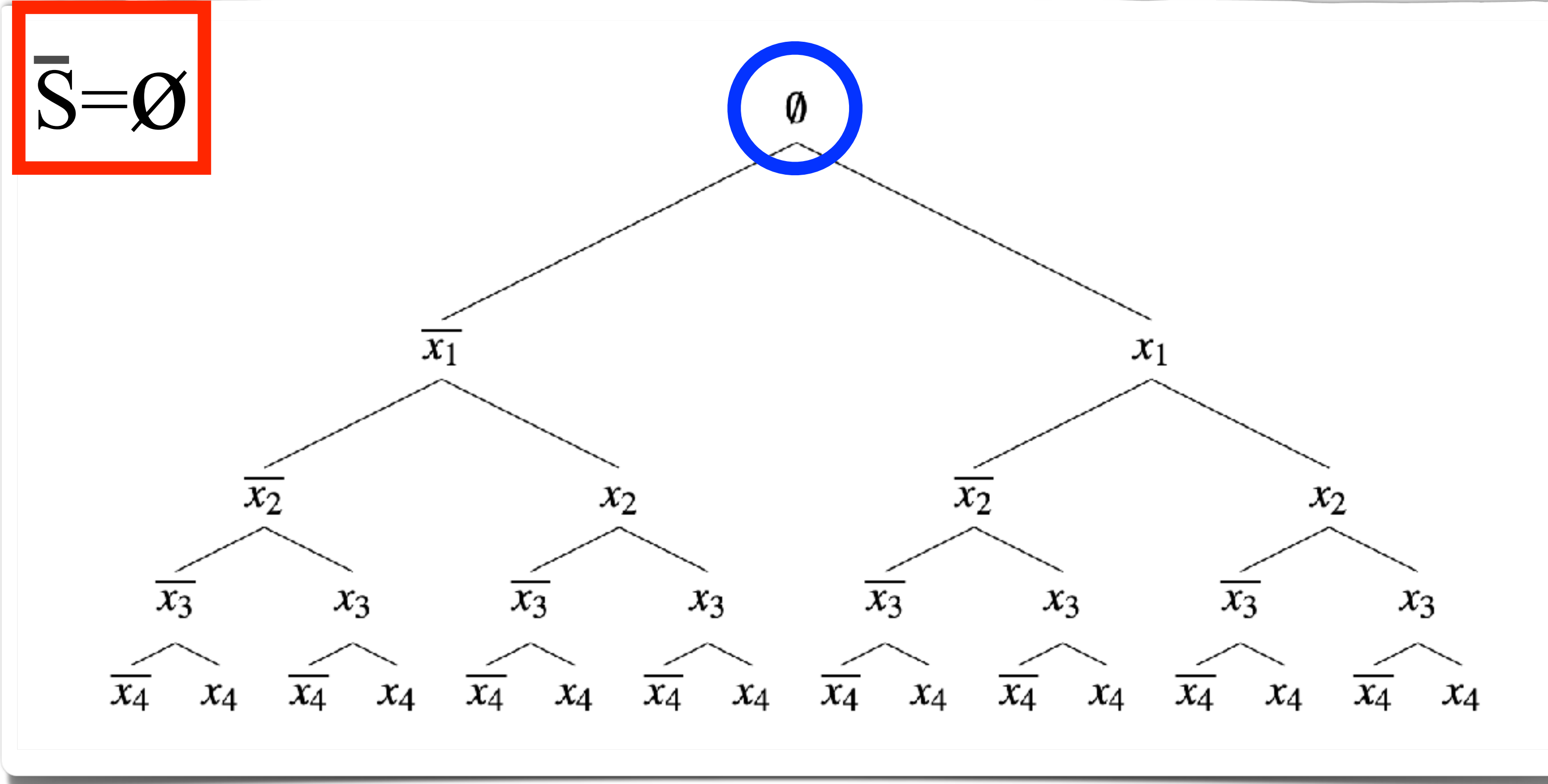
Gesamtbeispiel



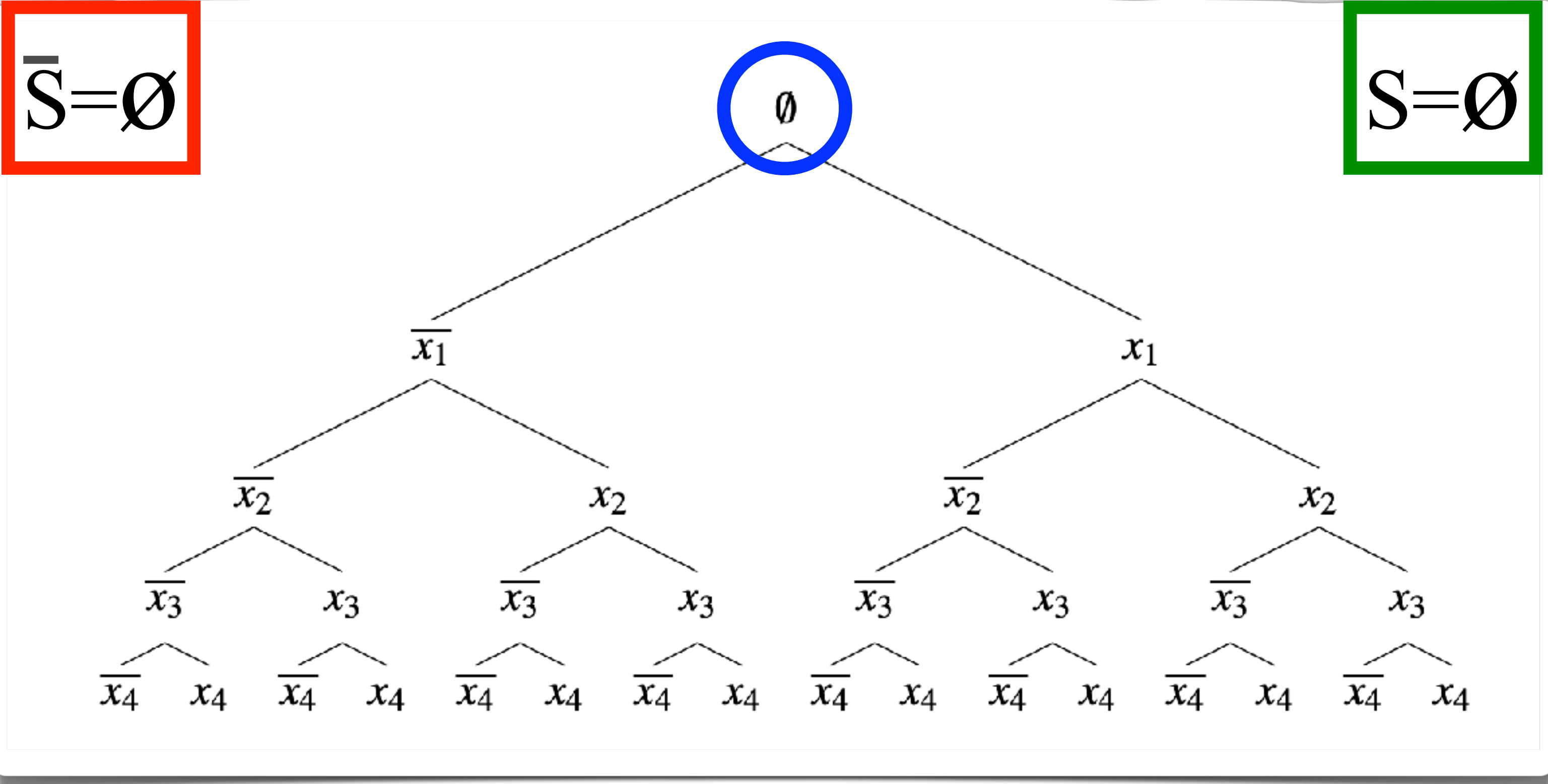
Gesamtbeispiel



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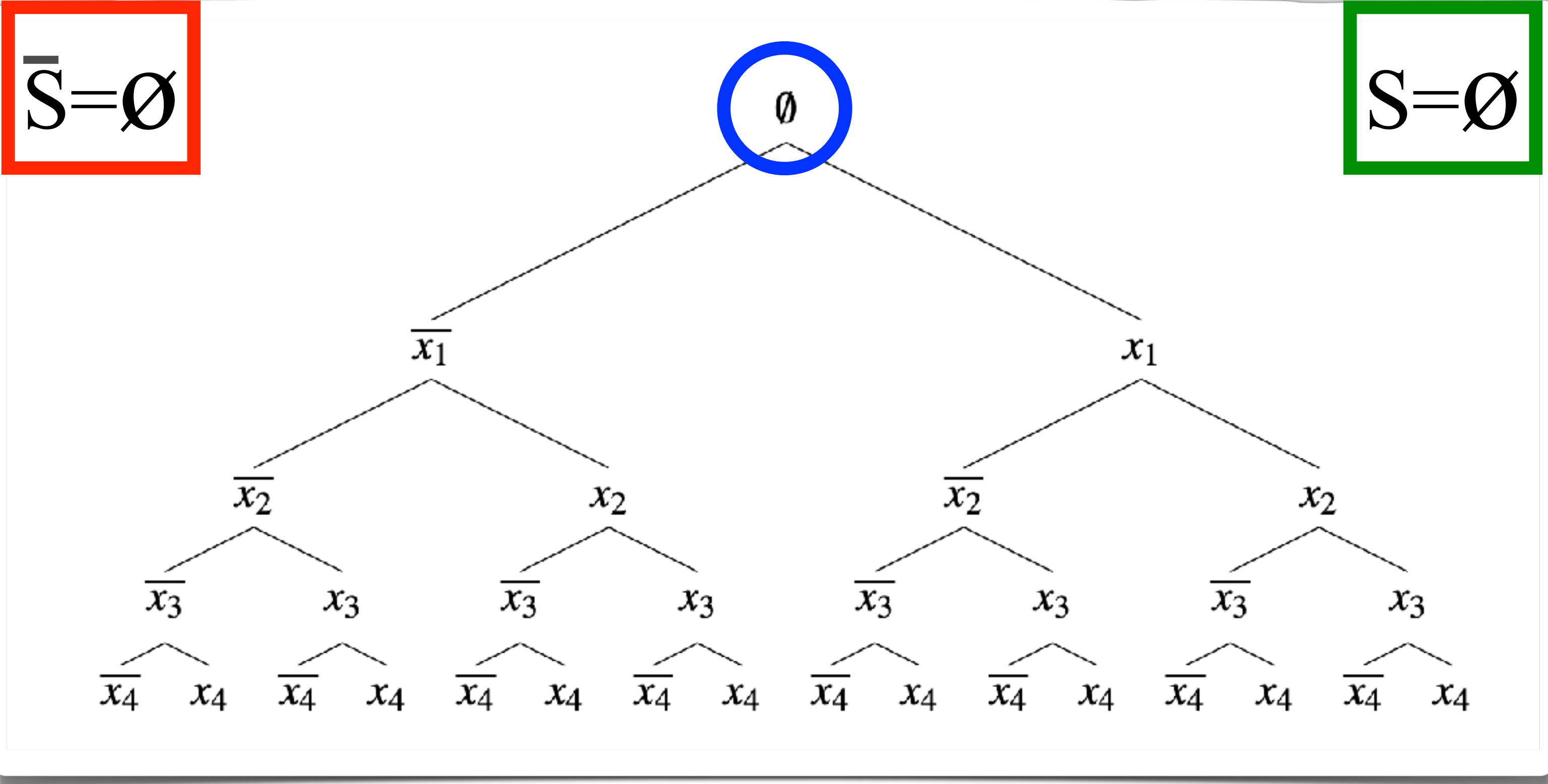
Gesamtbeispiel



Gesamtbeispiel

$$\bar{S} = \emptyset$$

$$S = \emptyset$$

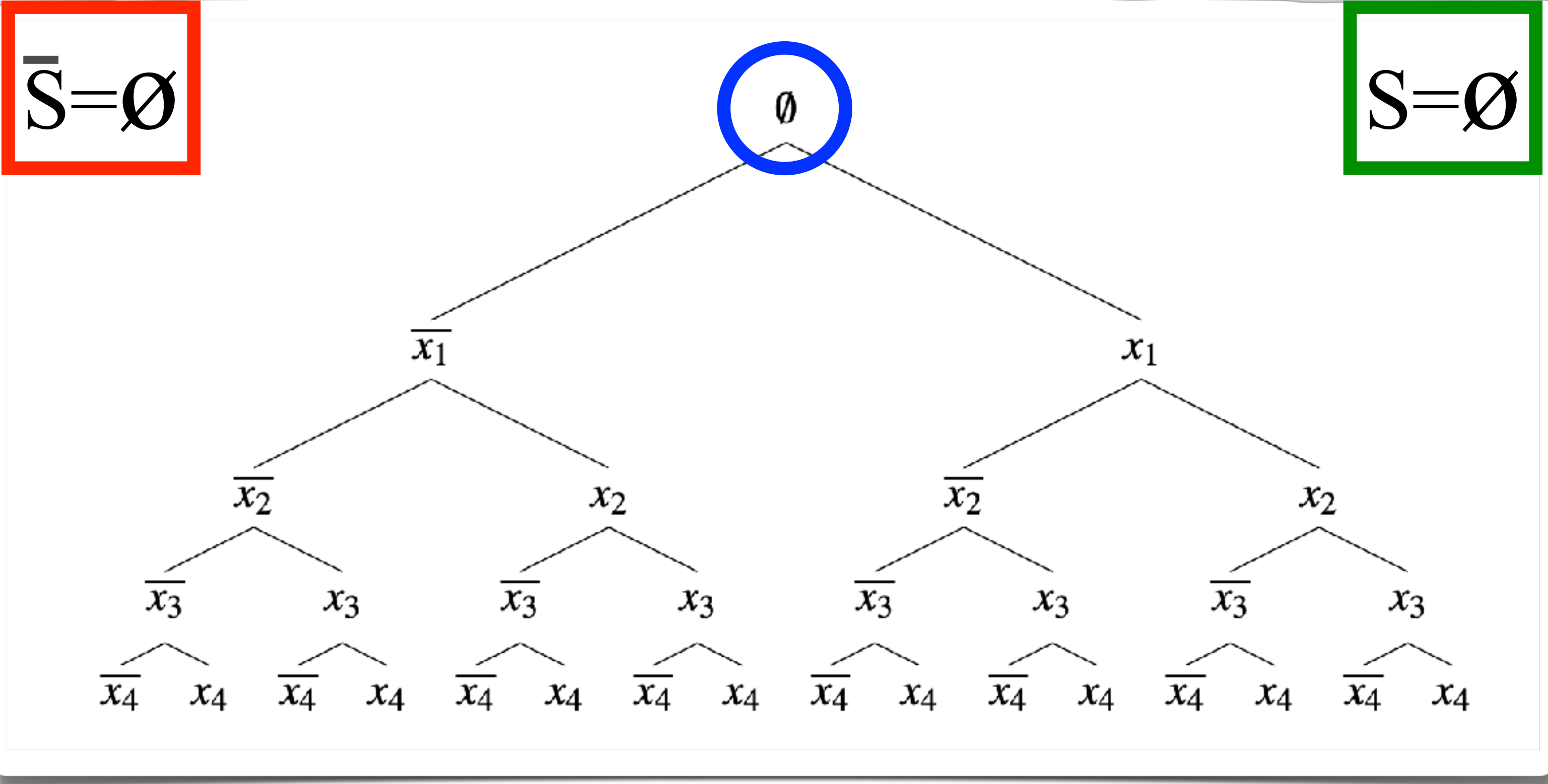


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z_i	2	3	6	7	5	9	4
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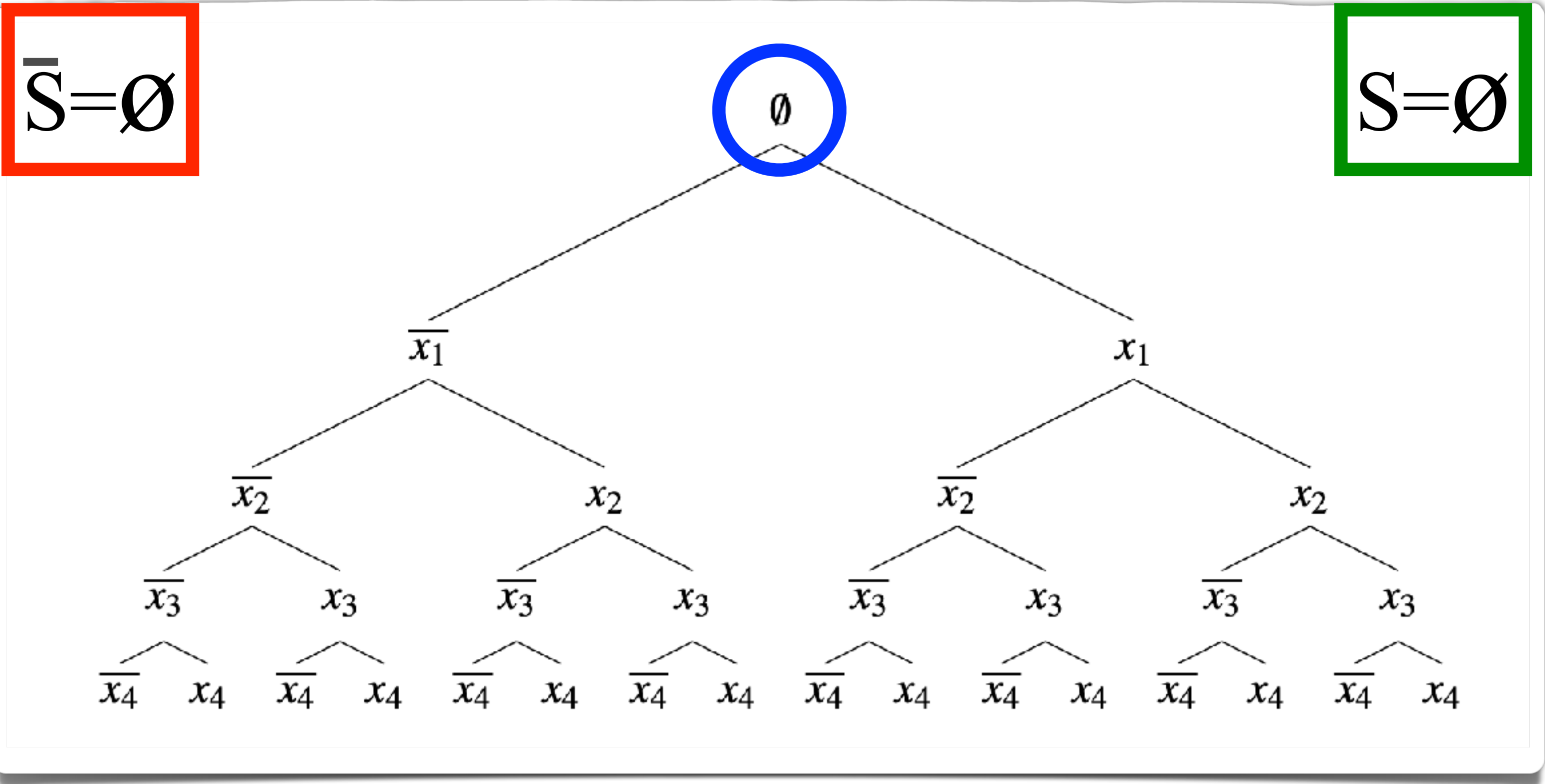


i	1	2	3	4	5	6	7
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Gesamtbeispiel

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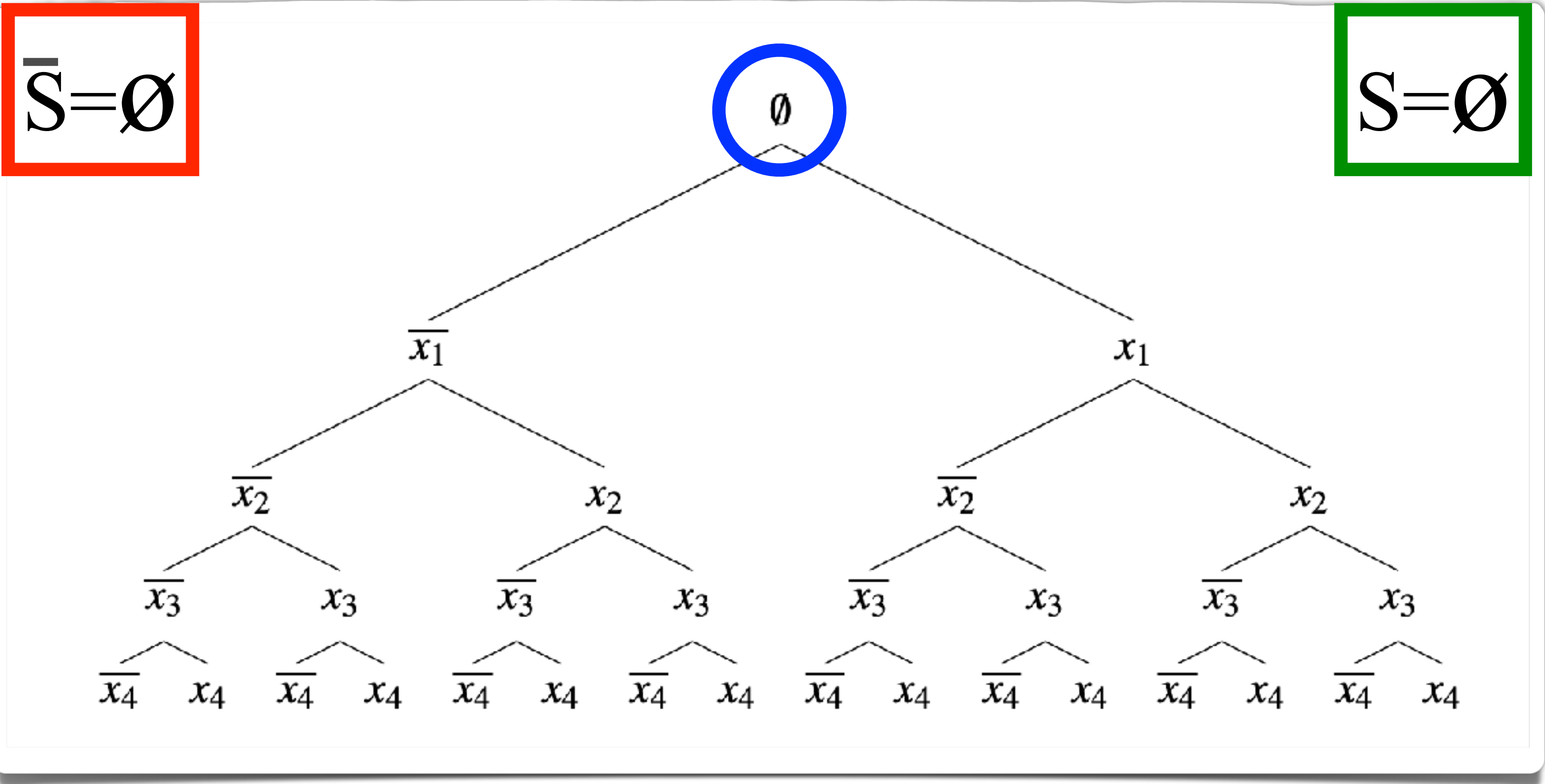


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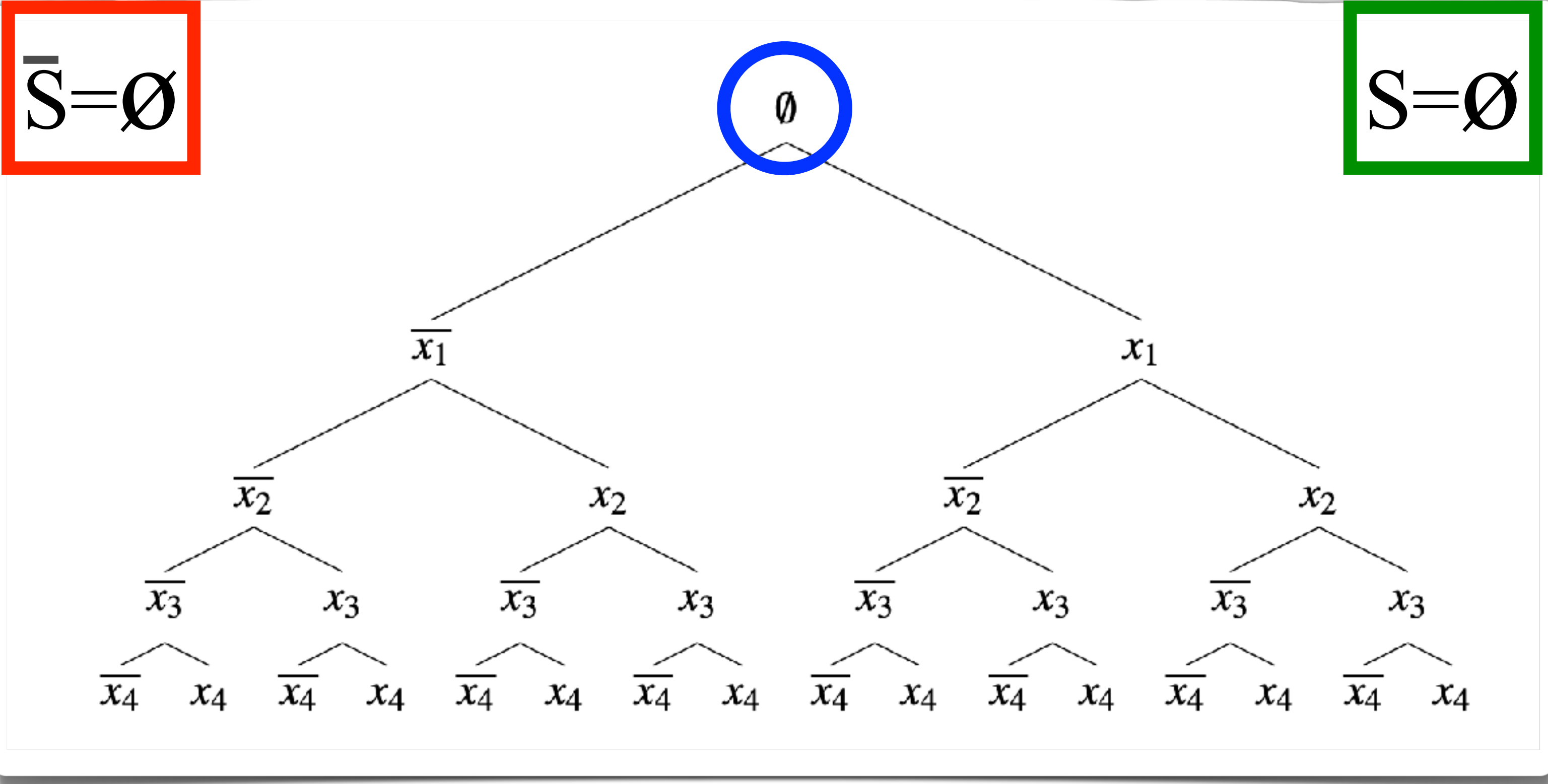


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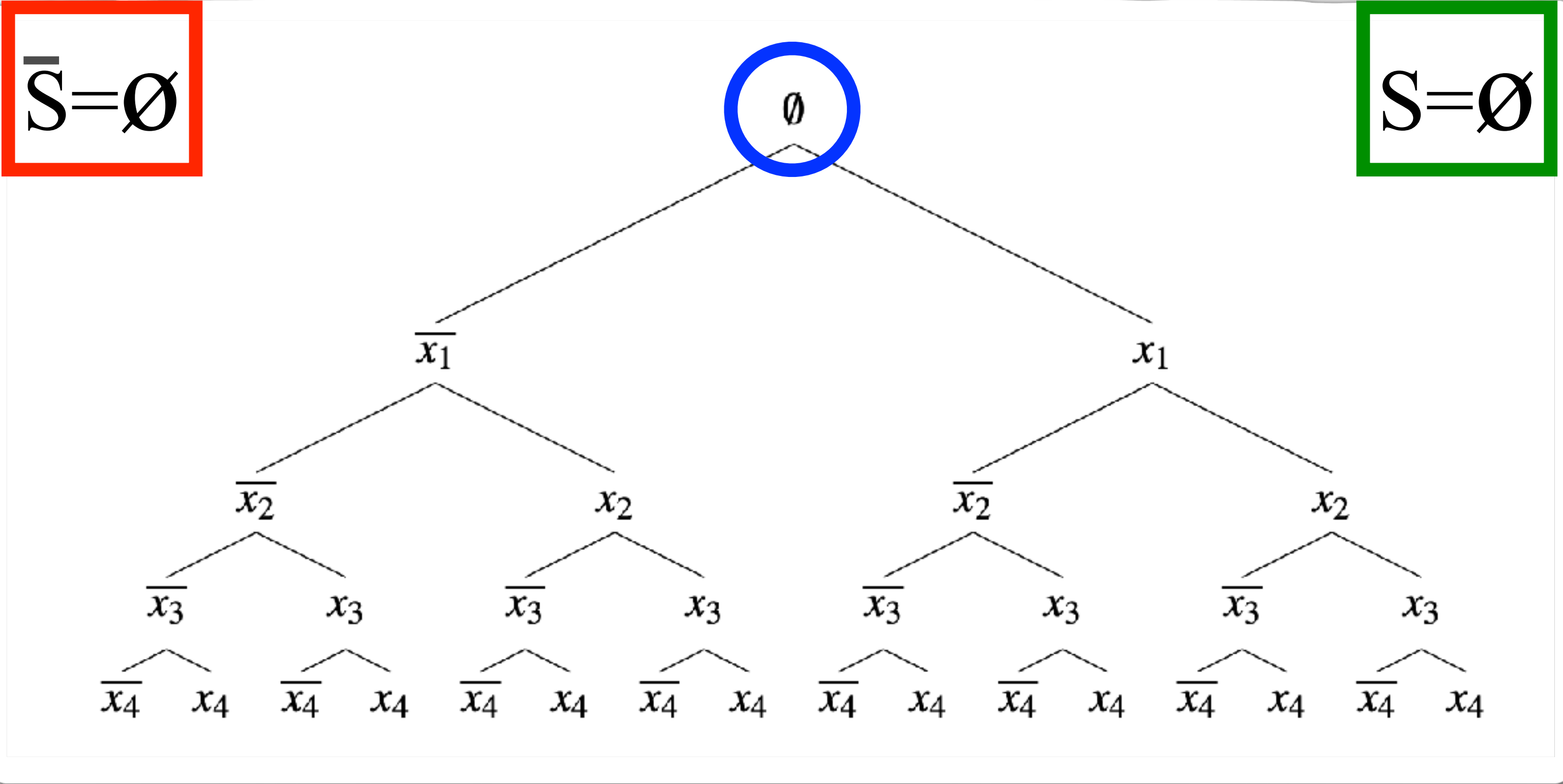
$$LB = 14$$

Gesamtbeispiel

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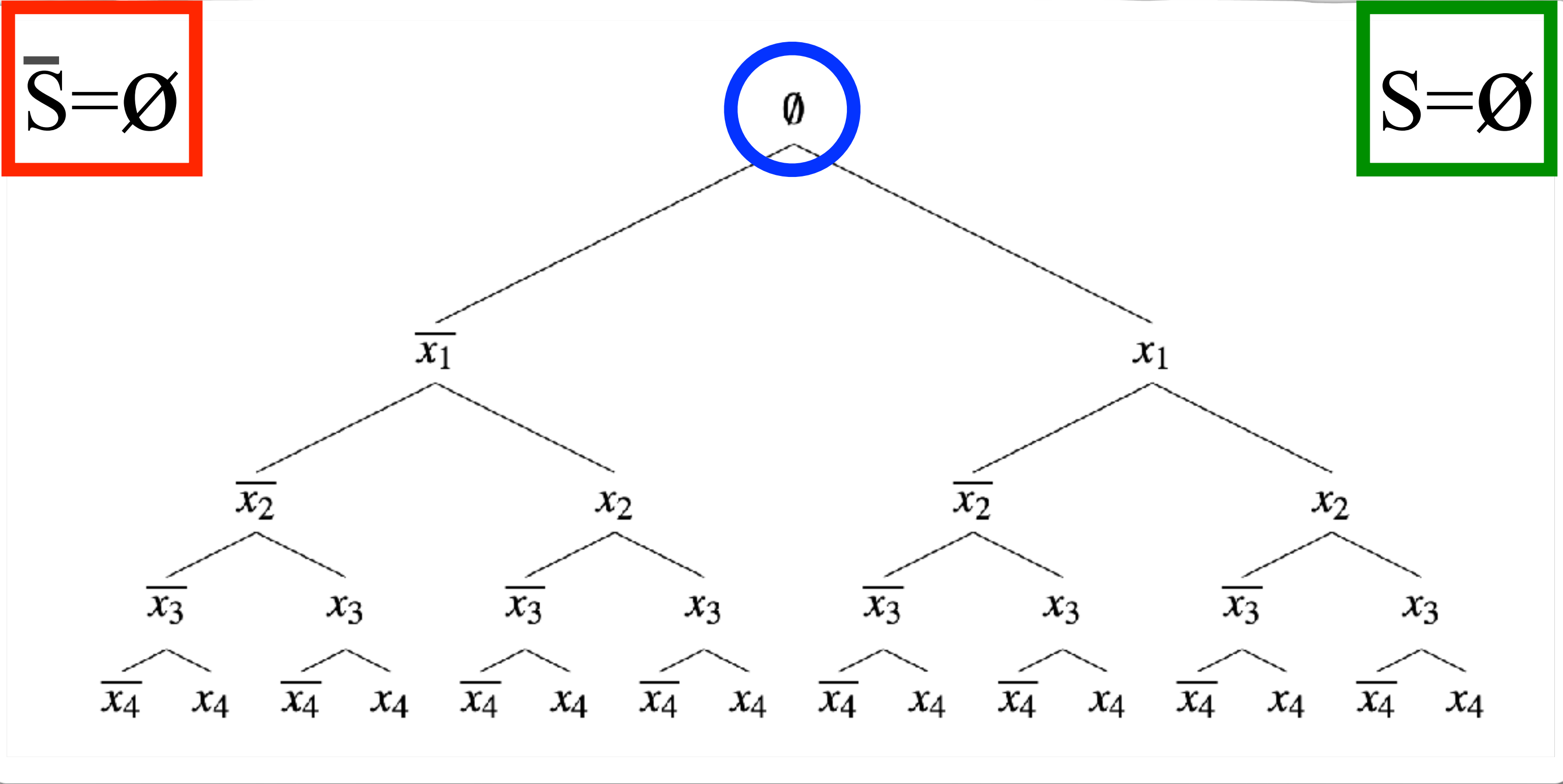
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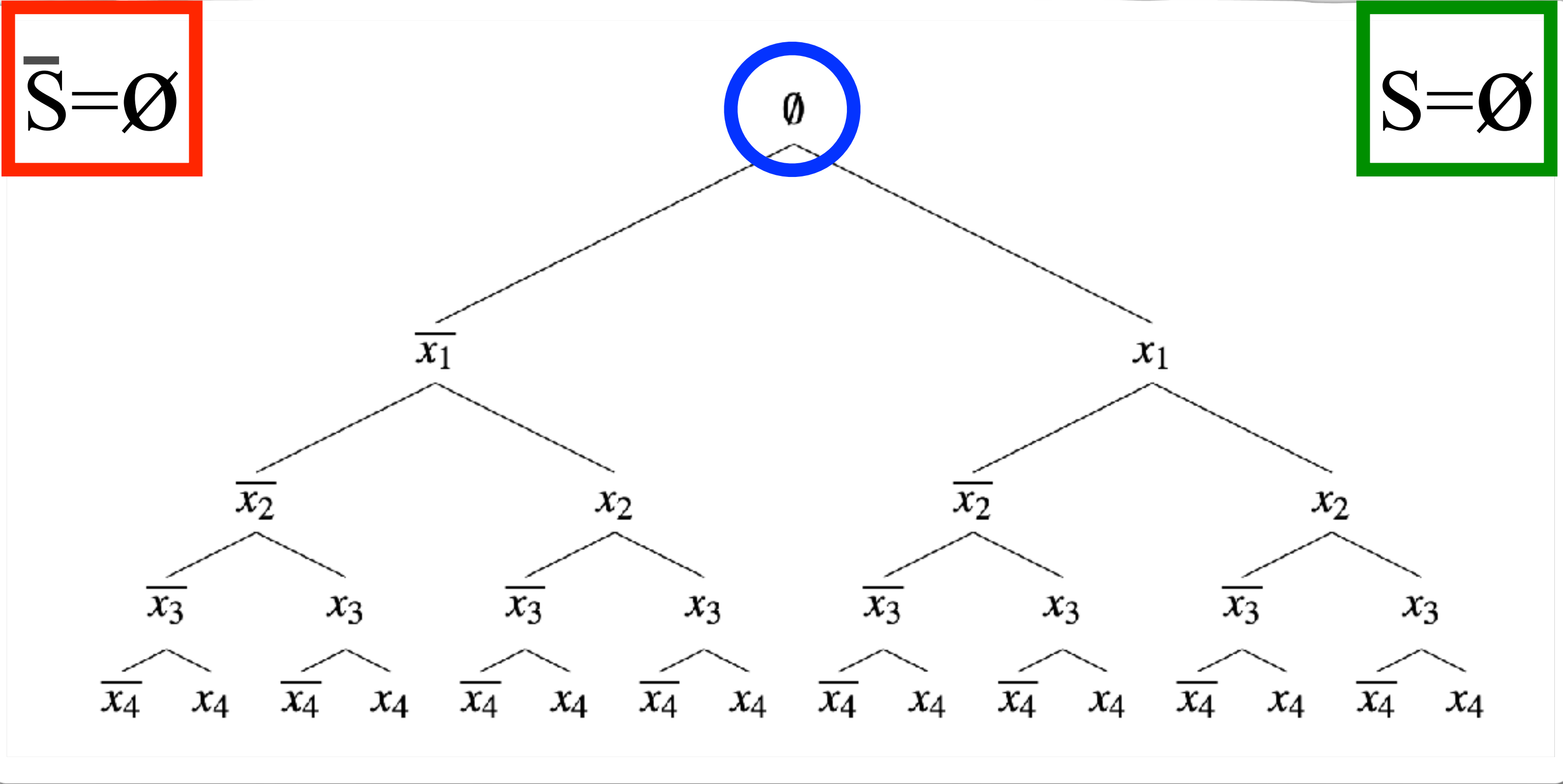
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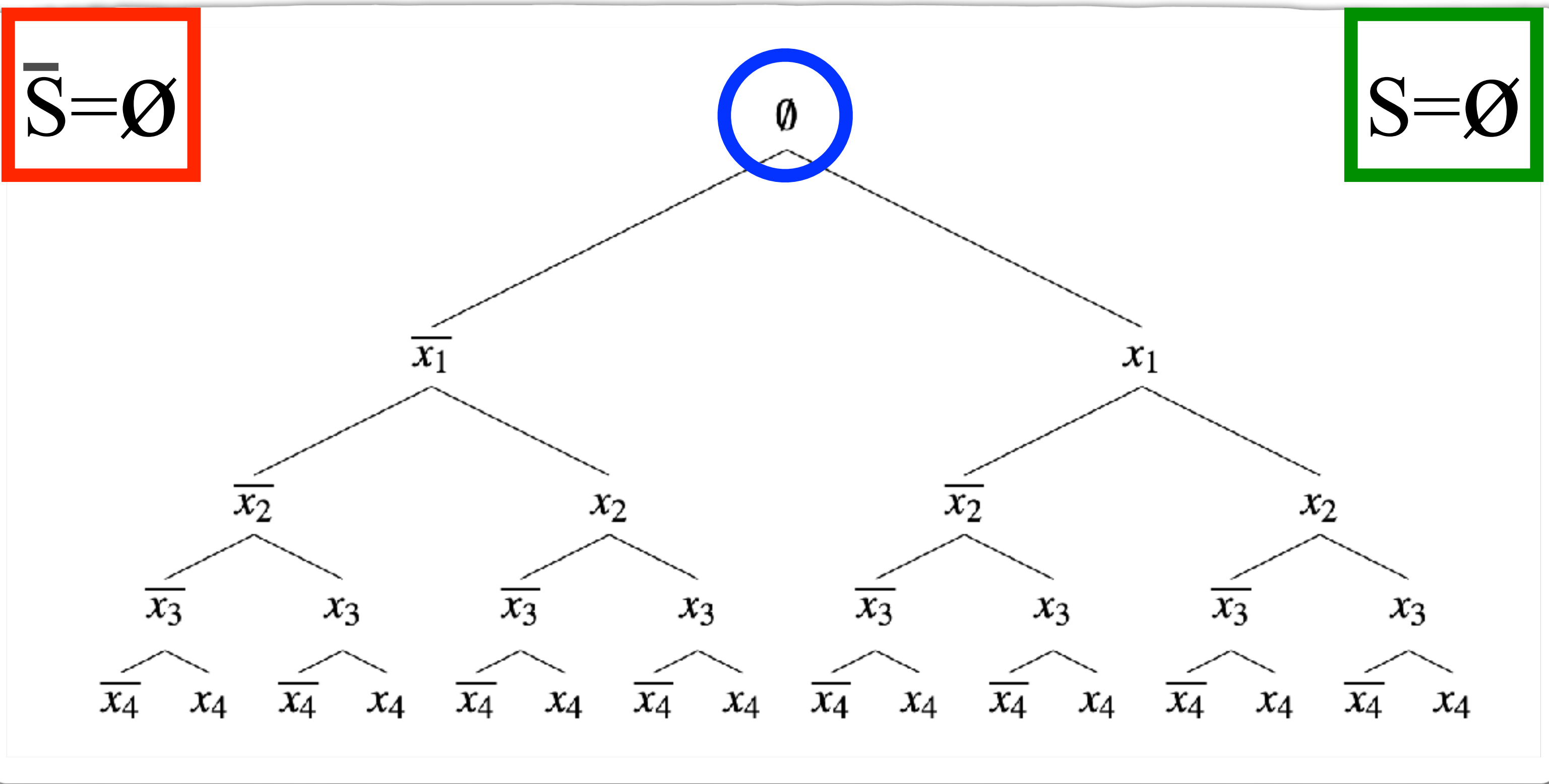
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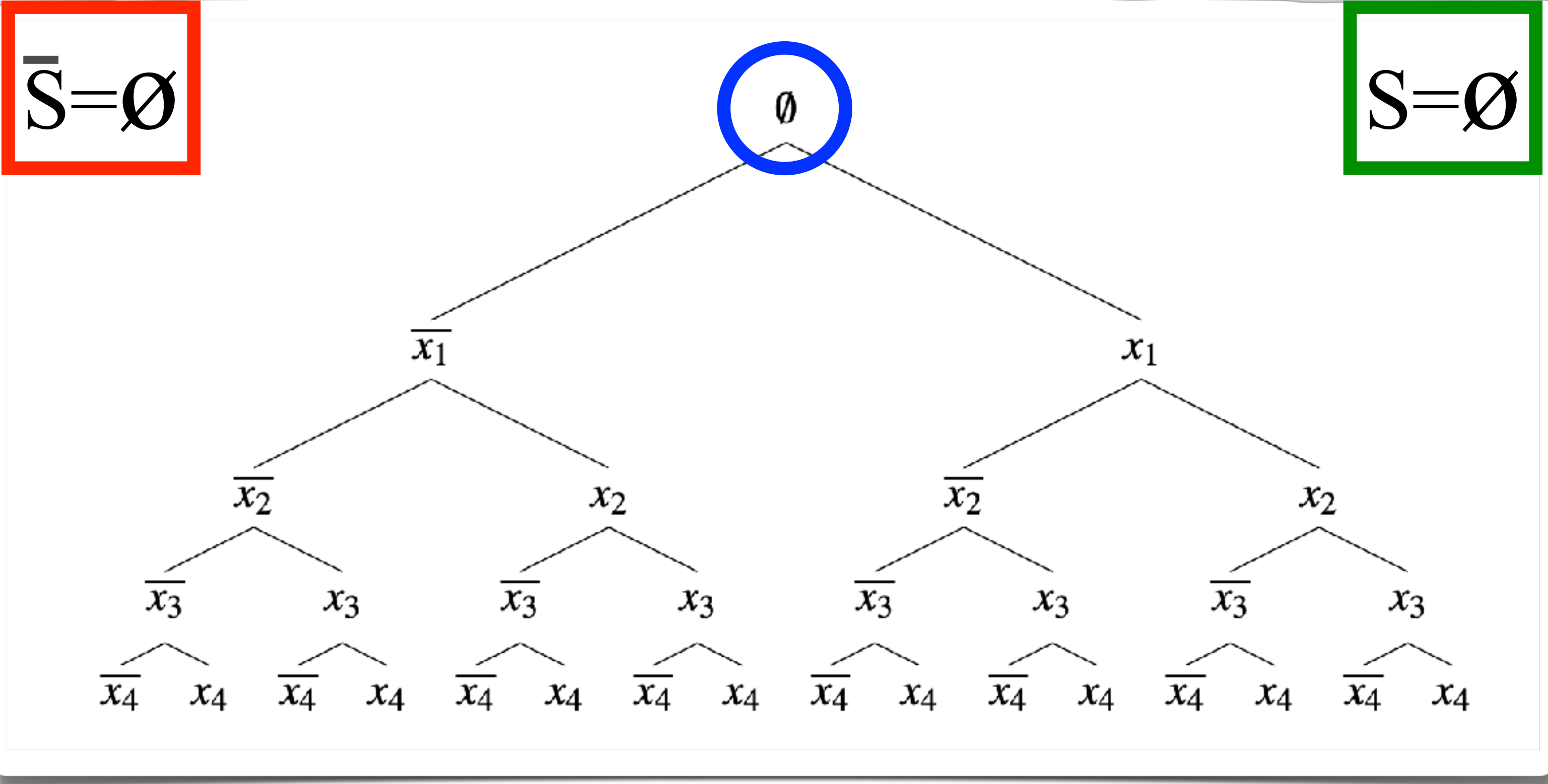
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UB=16

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

LB=14

Vielen Dank!

s.fekete@tu-bs.de