Algorithms Group **Summer 2020**
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Online Algorithms
Exercise 2
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Hand in your solutions as PDF file until June 3, 2020, 11:30 AM via e-mail to `v.sack@tu-bs.de`, with CC to `keldenich@ibr.cs.tu-bs.de`. If you cannot turn your solution into a PDF file (for example by writing it in LaTeX or Word), you can also submit photographs or scans. In that case, be careful to keep the file size acceptable (about 3 MB per page) by using appropriate compression and resolution; however, make sure that your solutions are still readable.

Exercise 1 (Online Bin Covering):
In this exercise, we consider the problem of Bin Covering in an online scenario. Analogous to the situation for online bin packing, we are given a sequence of items of unknown weights $a_1, \dots, a_n \in [0, 1]$ and want to assign these items to bins in an online fashion; however, the bins do not have limited capacity. In the Bin Covering problem, we want to *maximize* the number of *covered* bins, i.e., the number of bins that receive items of total weight at least 1.

a) Find an online algorithm for Bin Covering with an absolute competitive ratio of 2 and prove its competitive ratio. Prove that no deterministic online algorithm can have an absolute competitive ratio $c < 2$.

b) Prove that no deterministic online algorithm for Bin Covering can have an asymptotic competitive ratio $c < 3/2$.

(10+10 pts.)

Exercise 2 (Directed Graph Exploration):
In this exercise, we consider the problem of exploring an unknown directed graph. Each vertex is labeled by a natural number $i \in \{1, \dots, n\}$. We do not know the set of arcs in advance; whenever we visit a vertex i for the first time, we get to know its outgoing arcs and the vertices they point to.
For example, if vertex 3 has an outgoing arc to vertices $1, 2, 4, 5, 6$, we get to know this only once we visit 3 for the first time. This holds even if we visited vertices $1, 2, 4, 5, 6$ before; in other words, we do not learn anything about the incoming arcs of v when visiting v.
We start at vertex 1. Our goal is to visit each vertex of the graph at least once and return to 1 with minimum possible cost. Traversing any arc costs 1.

a) Prove that no deterministic online algorithm has a competitive ratio strictly less than $\frac{n+1}{2} - \frac{1}{n}$.

b) Devise a deterministic online algorithm with competitive ratio $\frac{n+1}{2} - \frac{1}{n}$ and prove this upper bound.

(10+10 pts.)