Online Algorithms
Tutorial 3 — Recap & Exam

Lecture Recap

What topics did we cover?

- Ski Rental
- Paging
- File Migration
- 2D packing

Ski Rental: What did we prove?

Lecture Recap — Ski Rental

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Lecture Recap — Ski Rental

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Ratio $D = {}^{b}\!/_{r}$

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Lecture Recap — Paging

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Model: cache size k, n total pages, fault costs 1

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File migration — $\textsc{Move To Min}$

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Can we get better than H_k? No (no details).

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File migration: Coin Flipping!

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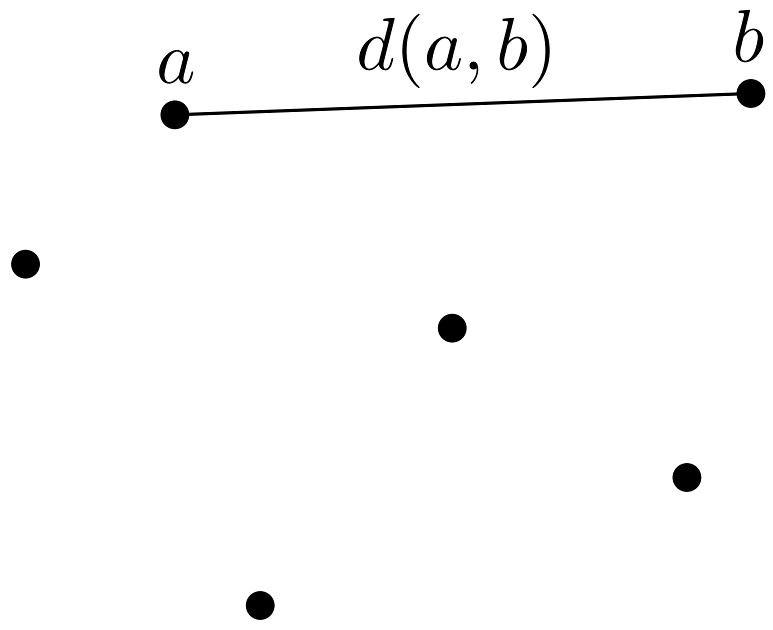

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Moving: $D \cdot d(a, b)$

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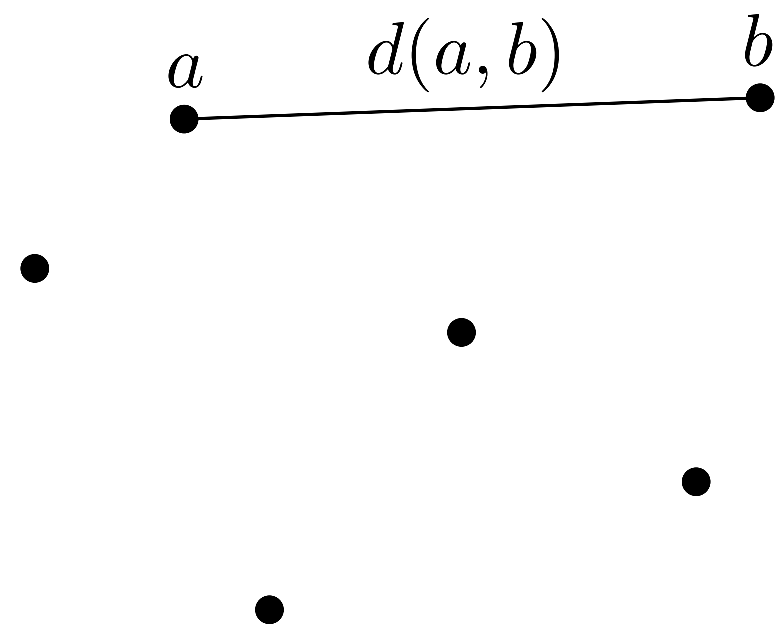

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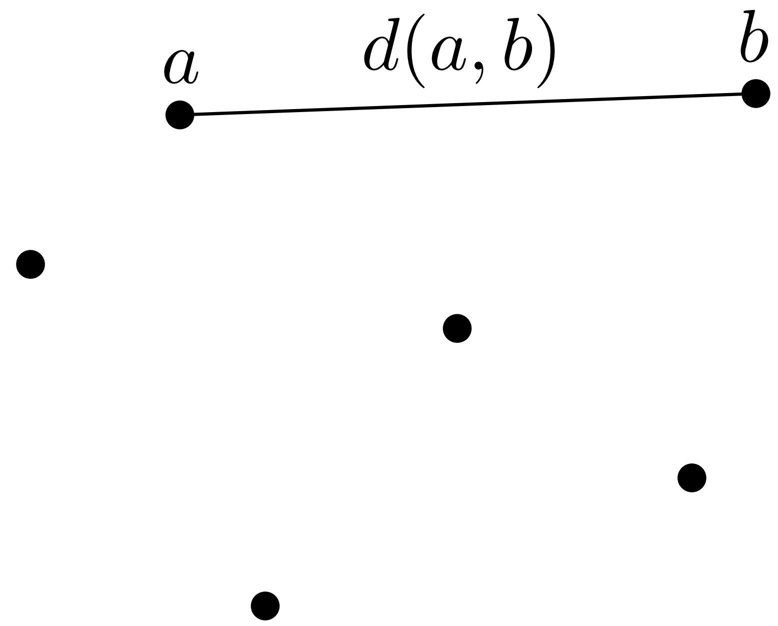

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We can move anywhere we want after a request!

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Can we be better against adaptive online?

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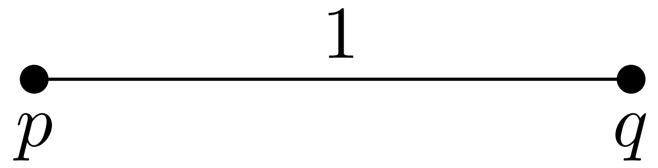

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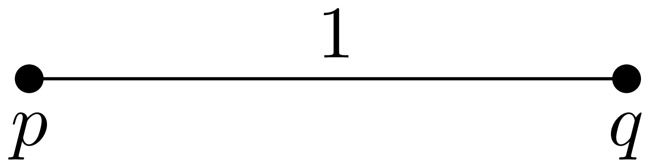

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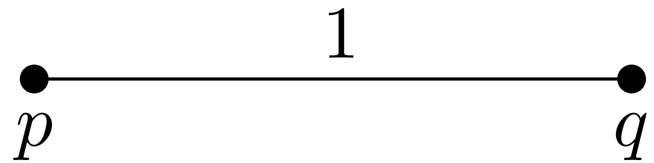

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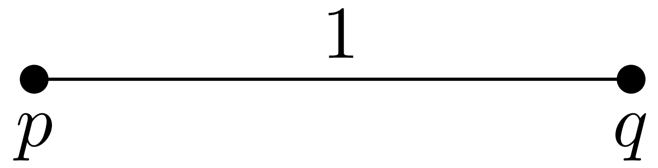

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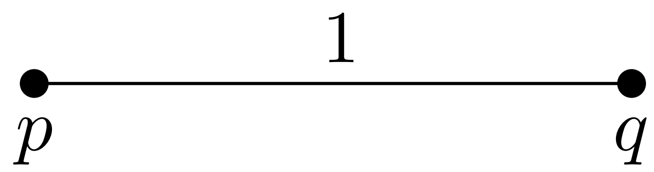

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All adversarial strategies add up to A

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Yes, $4.086\ldots$ (no details); what is a lower bound?

2D packing

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How does the analysis work? What did we prove?

- $H_{on} \leq 4H_{\mathsf{OPT}} + 3$
- Free shelves
- Asymptotic competitive ratio?
- Absolute competitive ratio?
- Where else did we have asymptotic competitive ratio?

Questions?

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Tips for exam preparation:

- Do not prepare alone
- Try to come up with questions
- Explain stuff to each other
- Make sure you know the basic definitions
- Make sure you know the topics
- Make sure you know the algorithms
- Make sure you know the competitive ratios
- Do not learn a *stream*