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Online Algorithms  
Exercise 0  
May 2, 2022

You do not have to hand in this exercise sheet. The solutions will be presented in the small tutorial on Monday, the 9th of May 2022.

**Exercise 1 (Memory):**

In this exercise, we consider a single-player version of the card game MEMORY. There is a deck of  $n$  pairs of cards lying face down on the table in front of the player. The player has to remove the cards by uncovering matching pairs of cards in as few moves as possible.

In each move, the player selects a first card and turns it around to see its face. After that, he selects and uncovers a second card. If the two cards match, they are removed. Otherwise, the cards are turned face down and remain on the table.

Knowing all cards' positions, the optimal offline algorithm removes the cards in  $n$  moves.

- (a) Design an online algorithm for MEMORY that requires at most  $2n - 1$  moves.
- (b) Prove that there is no online algorithm that requires at most  $2n - 2$  moves.
- (c) Design a  $\frac{3}{2}$ -competitive online algorithm for MEMORY.
- (d) In the two-player game, a player gets another free move after uncovering a matching pair of cards. In the single player variant, this corresponds to reducing the costs of a move to 0 if that move successfully removes a matching pair of cards. Give a  $c$ -competitive online algorithm for some constant  $c > 1$  for this version of MEMORY, or prove that no such algorithm exists.

(0 pts.)