



Technische
Universität
Braunschweig

Online Algorithms - Tutorial 01

Summer term 2022, 02. May 2022

Part I -Organization

Organization

Small tutorial

- Held by Peter Kramer (kramer@ibr.cs.tu-bs.de)
- Monday, every other week, starting from 09. May 2022
- Same time & place as the big tutorial

Exercises

- Will appear after the small tutorial to be handed in one week later
- Exercise sheet 0 is not graded
- Hand them in either
 - Via box in front of IZ337
 - Email to both Peter and me (mperk@ibr.cs.tu-bs.de)
- At the end of semester: quizzes and preparation for the exam

Material and Videos

Course Website

- <https://www.ibr.cs.tu-bs.de/courses/ss22/oa/index.html>
- Material: <https://www.ibr.cs.tu-bs.de/courses/ss22/oa/material/>
- **Register for the mailing list:** <https://mail.ibr.cs.tu-bs.de/mailman/listinfo/oa>
- No script, but there are books
- Referenced on the website, full-text access via eduroam

Videos of lectures and tutorials

- We are videotaping lectures & tutorials
- Accessible shortly after the event on the material site

Passing the module

Exercise sheets

- You hand in your solutions to the exercises
- They will be graded individually
- To pass the module, you need 50% of the points

Exam

- Oral or written exam at the end
- You do not need 50% of the exercise points to take the exam
- Grade only depends on the exam

Part II - Introduction

Introduction

Online Algorithm

Informally: Algorithm that works with incomplete knowledge

Formal definition: See board.

Often criticized: focus on the worst case

- We only look at the worst case — is that sensible?
- Do we play against an adversary in the real world? In security contexts, we might!
- May restrict our view to unlikely inputs
- Average case analysis — but what is a likely input?

Ist der Spritpreis derzeit zu hoch?

Teure Energie

Spritpreise sinken nur leicht

Warum Tanken in Deutschland besonders teuer ist



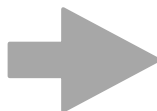
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»Krisenprofit« von mehr als 100 Millionen Euro pro Tag

So verdienen Mineralölkonzerne Milliarden am Spritpreisanstieg

ADAC

Diesel und Benzin im März laut ADAC so teuer wie nie



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Part III - The BahnCard Problem

The BahnCard Problem

- Generalization of Ski Rental
- Buying only reduces cost by a factor
- Only lasts for a finite time (e.g., a year)
- Different costs for different travels

Formal definition: See board.

Cheap and expensive intervals: See board.



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The BahnCard Problem - Offline Algorithm

Naive Offline Algorithm

- For every request (t_i, c_i) , check whether we have a valid BahnCard
 - If yes, simply buy the reduced ticket
 - Otherwise, buy a BahnCard if $[t_i, t_i + T)$ is expensive

Question: Is this algorithm optimal?

Answer: No! Proof: See board.

Correct algorithm: Homework :)

Use the following facts:

- In each expensive interval of length at most T , the optimal offline solution owns a BahnCard at some point.
- The optimal solution never buys a BahnCard if it still owns one.

The BahnCard Problem - Lower Bound

Theorem 1.1: No deterministic online algorithm can achieve a competitive ratio better than $2 - \beta$.

Question: If you were an evil adversary, what would you do?

Proof: See board.

The BahnCard Problem - Online Algorithm

Question: How do we apply the idea for Ski Rental to the BahnCard Problem?

Ski Rental buys when it notices OPT would have bought...

Algorithm SUM

- For a request (t_i, c_i) , buy BahnCard iff
 - we do not own one,
 - the cost of all regular requests in $(t_i - T, t_i]$ is at least the critical cost c^*

Theorem 1.2: SUM is $(2 - \beta)$ -competitive.

Proof: Homework :)