Technische Universität Braunschweig

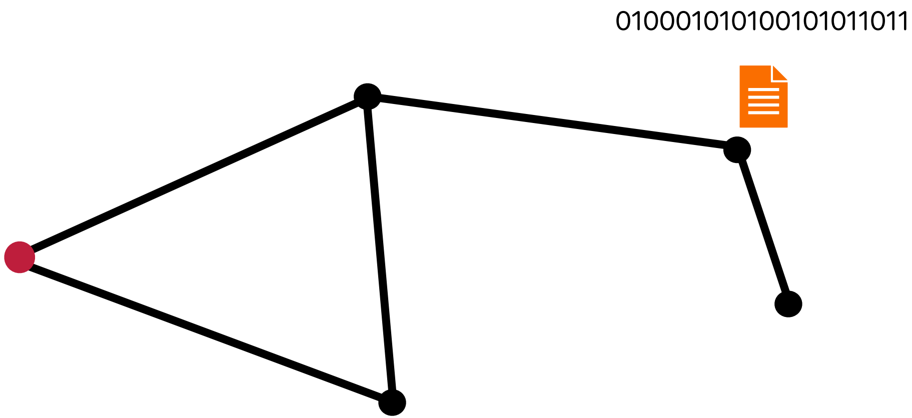
Online Algorithms - Tutorial 03

Summer term 2022, 30. May 2022

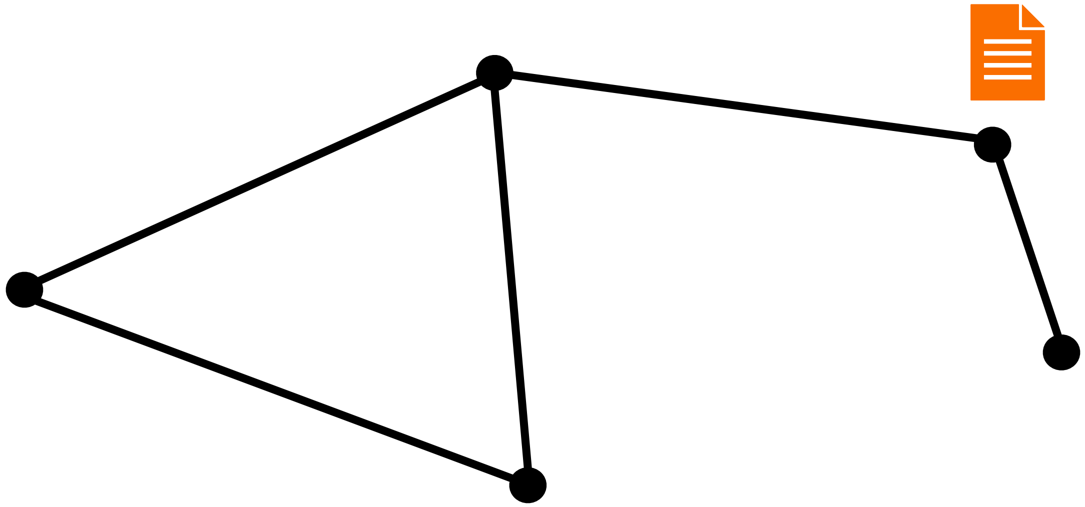
Distributed data management

Distributed data management

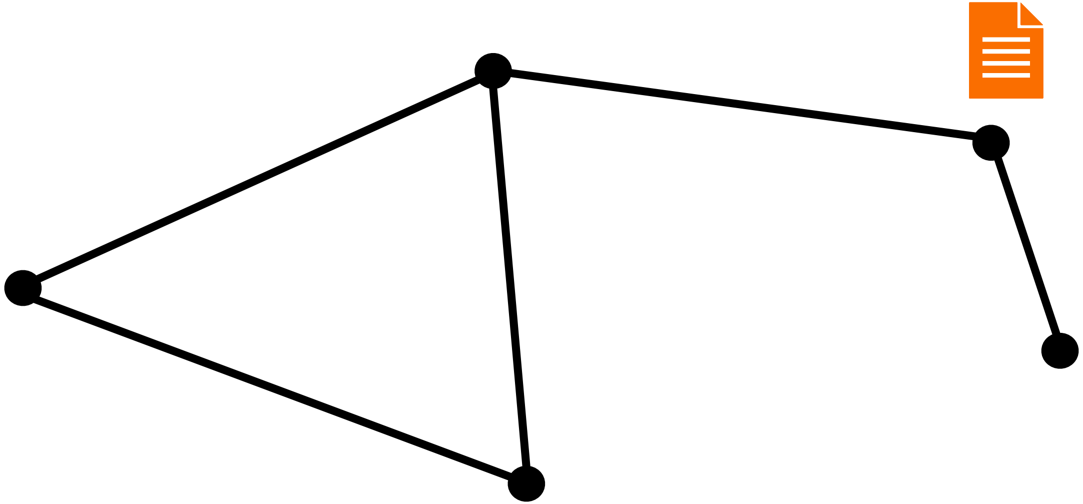
- Consider a network of processors with local memory
- Global shared memory is modeled by distributing the physical pages among the local memories



File replication



File migration



File migration

- Undirected graph G with edge lengths (metric distance function)
- A read/write request occurs at some node v
  - Cost 0 if v holds the page
  - Cost $\delta(v, w)$ if w holds the page
- **After** the request is satisfied, the page may be migrated from w to v
- We only consider centralized migration (each node always knows the location of p)

Formal definition: See board.

File migration

Randomized Algorithm COUNTER

- Global counter $C \in [0,k]$ for some value k
- Counter initialized uniformly at random to an integer in $[1,k]$
- On each request decrement C by 1
- If $C = 0$
    - Move page to the requesting node and set $C = k$

Theorem 3.1: COUNTER is c-competitive, where $c = \max\left\{2 + \frac{2d}{k}, 1 + \frac{k+1}{2d}\right\}$

Proof: See board.

Two types of events:

1. COUNTER and OPT serve the request. COUNTER may move the page
2. OPT moves the page

File migration

Theorem 3.1: COUNTER is c-competitive, where $c = \max\left\{2 + \frac{2d}{k}, 1 + \frac{k+1}{2d}\right\}$

For $k = d + \frac{1}{2}(\sqrt{20d^2 - 4d + 1} - 1)$:

$$\lim_{d\to\infty} \max\left\{2 + \frac{2d}{k}, 1 + \frac{k+1}{2d}\right\} = 1 + \Phi$$

$\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the golden ratio

Bin Packing

Bin Packing

- Among the first and best studied online problems
- Pack items of different sizes into bins
- Goal: Minimize the number of bins

- Offline variant: NP-hard
    - Reduction to **Partition** or **Subset Sum**
    - Strong NP-hardness: **3-Partition**

Formal definition: See board.

Bin Packing - Lower Bound

Theorem 3.2: No deterministic online algorithm can achieve a better competitive ration than $\frac{5}{3}$

Proof: See board.

Theorem 3.3: There is an online algorithm for Bin Packing that is $\frac{5}{3}$-competitive

- Is this definition of a competitive ration sensible?
- What if we are interested in large instances?
- What if we ignore a constant number of inputs?

➡ Asymptotic competitive ratio

Bin Packing - Next Fit

Algorithm Next Fit

- Keep only one open bin at a time
- If the σ_i fits into the bin: Pack it
- Else: Close the bin and start a new bin

Question: What is the (asymptotic) competitive ratio of Next Fit?

Theorem 3.4: Next Fit is c^∞-competitive, where $c^\infty = 2.$

Proof: See board.