Technische
Universität
Braunschweig

Online Algorithms - Tutorial 04

Summer term 2022, 20. June 2022

Bin Packing

Bin Packing - Recap

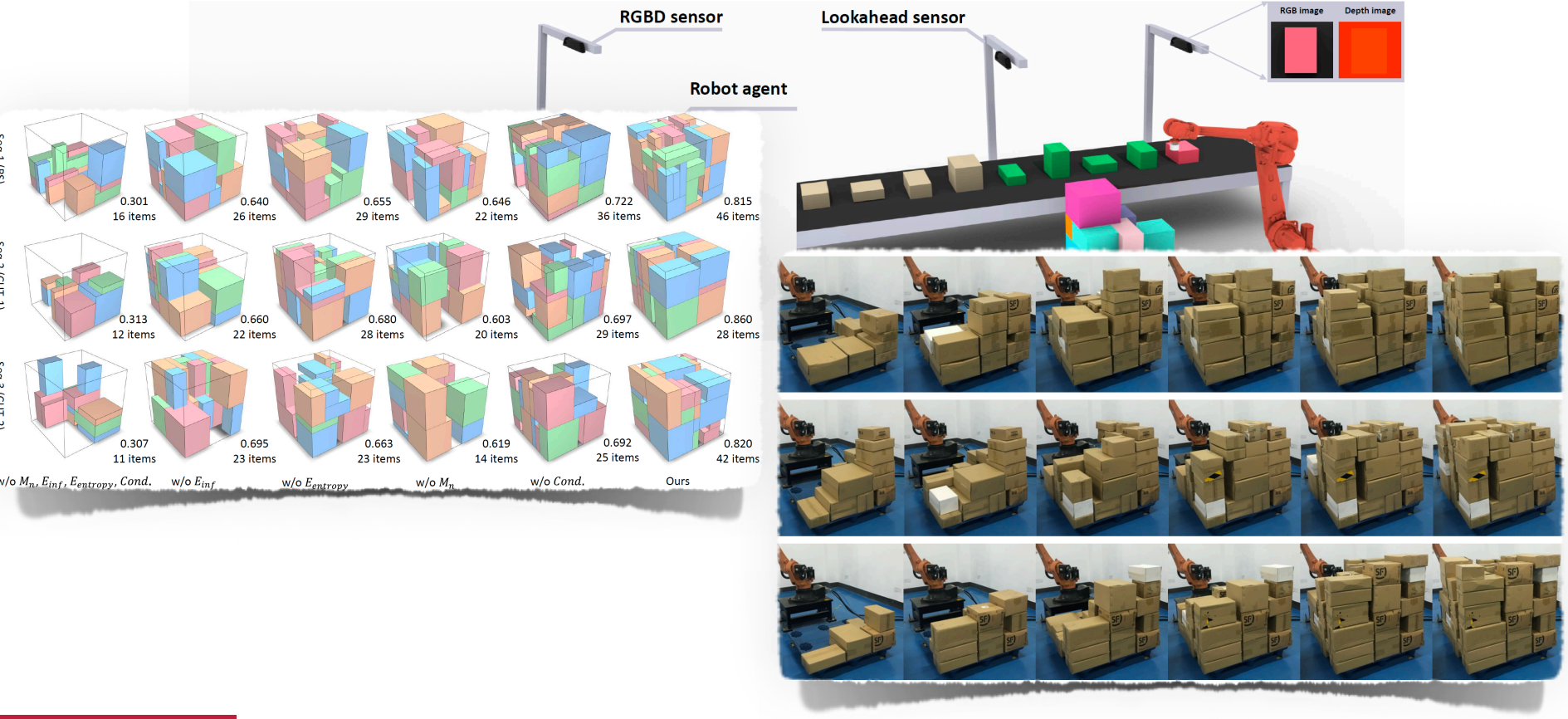
- Small example for absolute competitive ratio: 5/3
- Asymptotic competitive ratio:

$$c_A^{\infty} = \limsup_{M \to \infty} \left(\sup_{\sigma \in \Sigma^*} \{ c_A(\sigma) / c_{OPT}(\sigma) \mid c_{OPT}(\sigma) = M \} \right)$$

- Bin packing algorithms:
  - **NextFit**: packs item into bin until it does not fit anymore, then opens a new one **2-competitive**
  - **FirstFit**: keeps list of all bins, checks all bins and takes the first suitable one **1.7-competitive**
  - **BestFit**: like FirstFit, but uses the fullest suitable bin **1.7-competitive**

- Note: FirstFit and BestFit use unbounded space

Source: „Learning Practically Feasible Policies for Online 3D Bin Packing“ SCIENCE CHINA Information Sciences 2022



Harmonic Online Algorithm

- Define intervals $I_1, \ldots, I_M$

Harmonic$_M$ (Lee & Lee, 1985)

- For each incoming item
  - Determine interval k for the item, i.e. $\sigma_i \in I_k$
  - If there is room for σ_i in the open I_k bin: Pack the item
  - Otherwise: open a new I_k bin and pack the item

Harmonic Online Algorithm

Let $\bar{G}_M = \sum_{j=1}^{q} \frac{1}{h_j} + \frac{M}{h_{q+1}(M-1)}$

Theorem 4.1: Harmonic$_M$ is $\bar{G}_M$-competitive, where $q > 1$ and $h_q < M \leq h_{q+1}$

Proof: See board

- Only M active bins (thus bounded space)
- Each filled bin (except I_M bins) packs exactly k items
- Thus, for $1 \leq k < M$ only the number of items in I_k is relevant

Harmonic Online Algorithm

Theorem 4.2: No bounded space bin packing algorithm can achieve a better asymptotic competitive ratio than h_∞

- Best competitive ratio known so far: ≈ 1.581
- Best lower bound known so far: ≈ 1.5401