$$\sum_{i=1}^{n} x_i z_i \leq Z$$

$$x_i \in [0,1]$$

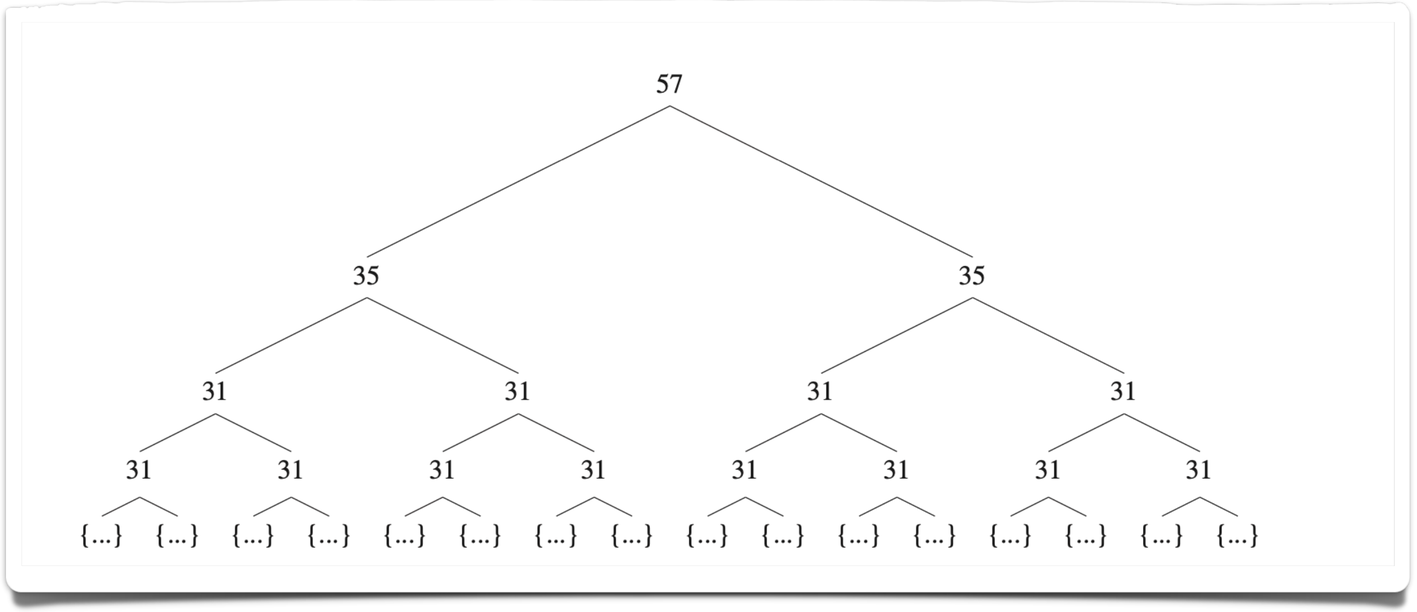
3 Branch-and-Bound

Algorithmen und Datenstrukturen 2
Sommer 2023

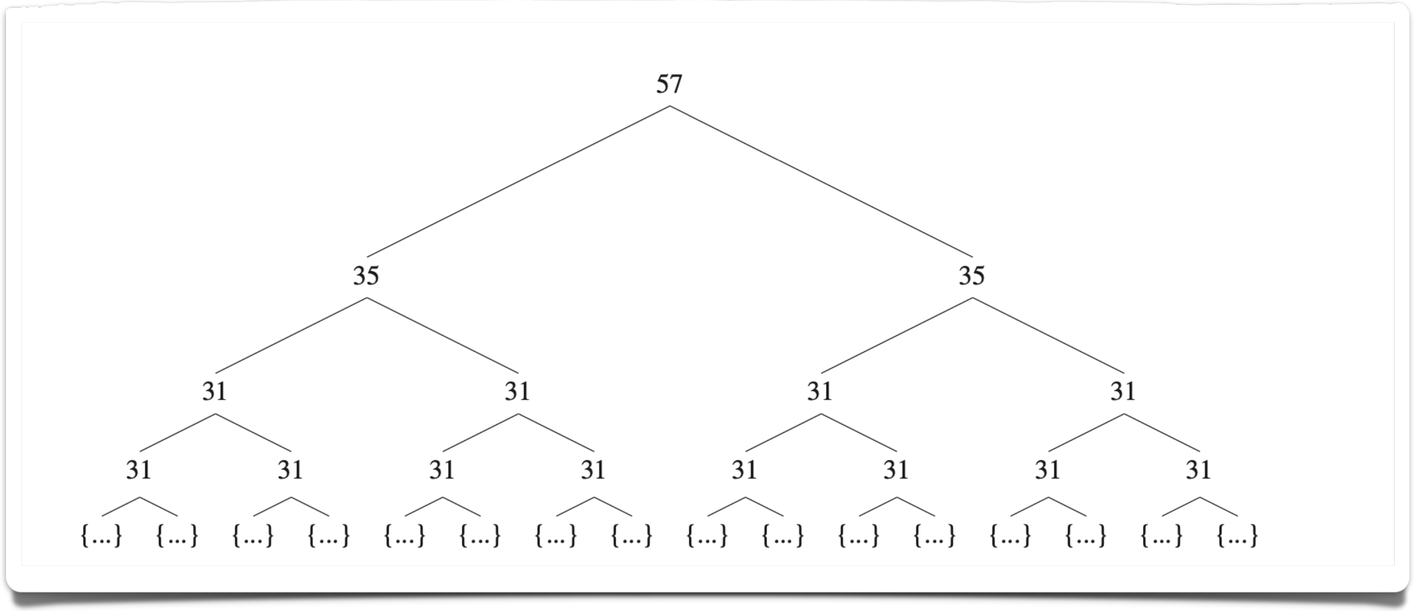
Prof. Dr. Sándor Fekete

3.1 Motivation

Enumerationsprinzip

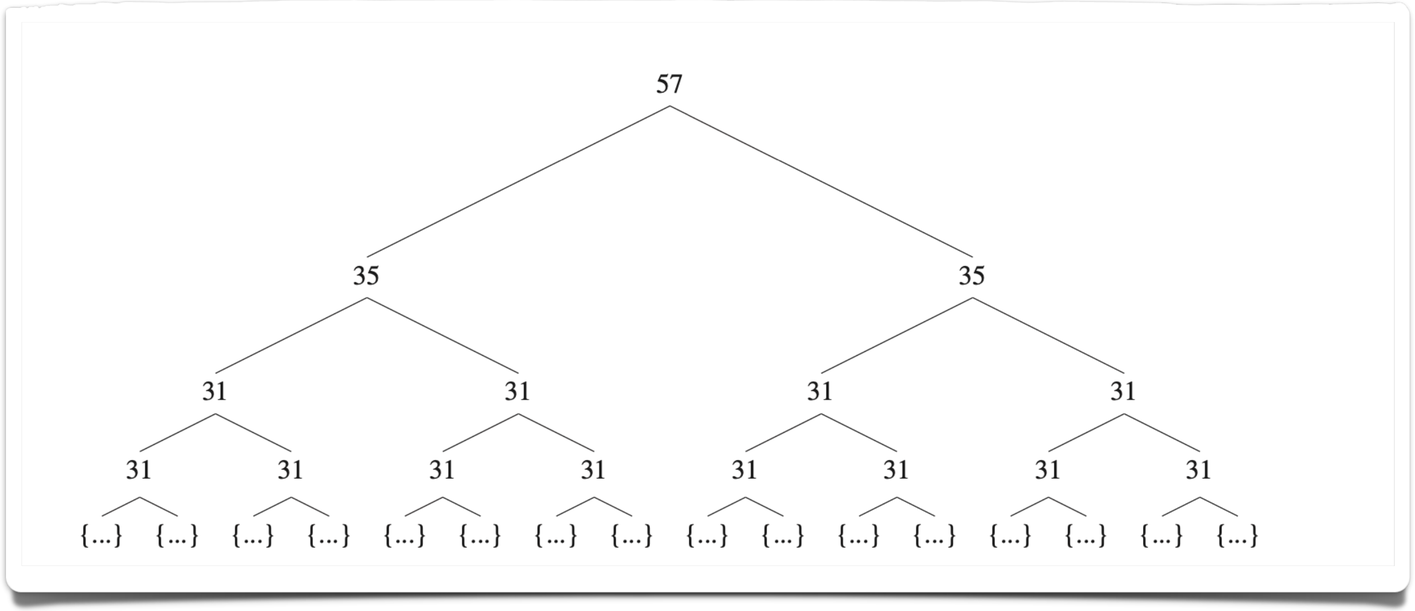


Enumerationsprinzip



- Exponentiell viele Fälle!

Enumerationsprinzip



- Exponentiell viele Fälle!
- Wie geht das systematisch?

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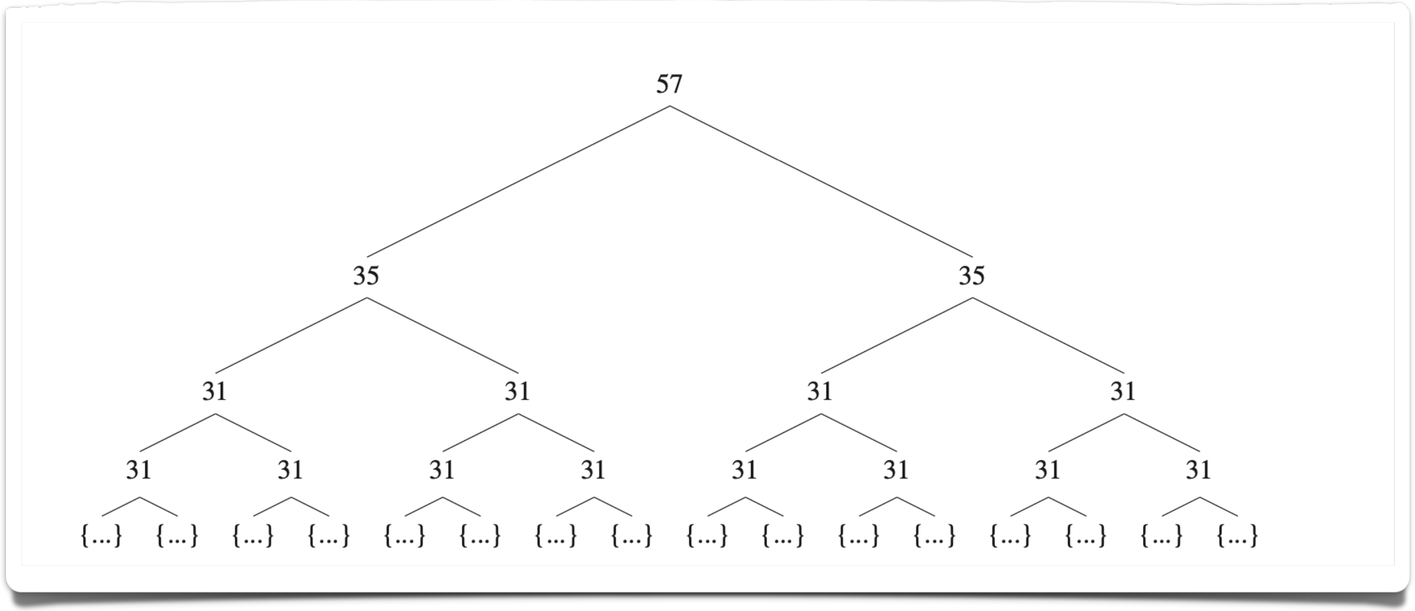

- Exponentiell viele Fälle!
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- Wo kann man Arbeit sparen?

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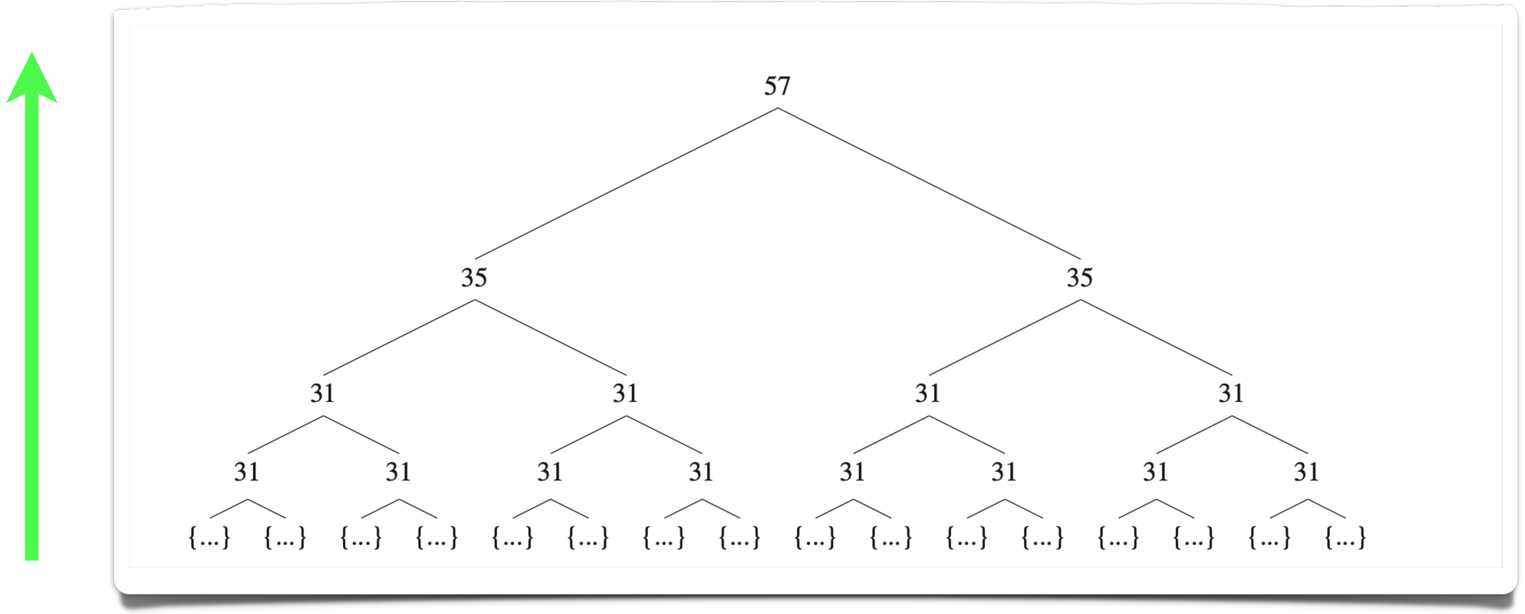

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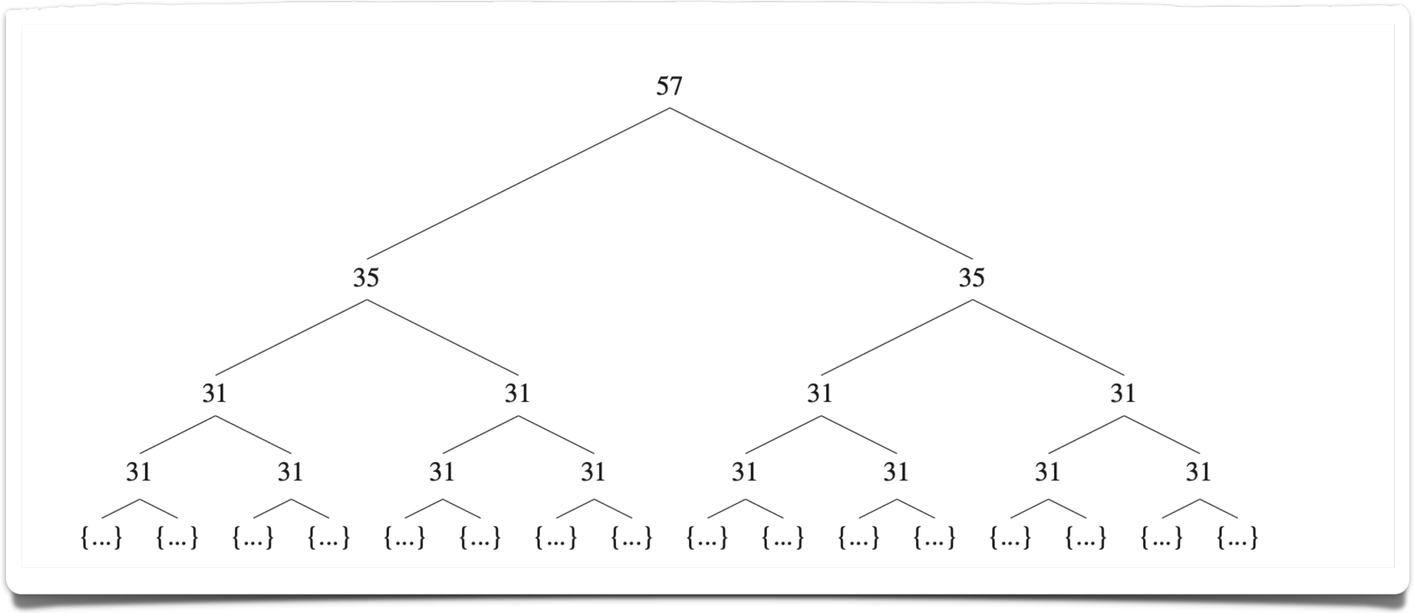
Dynamic Programming



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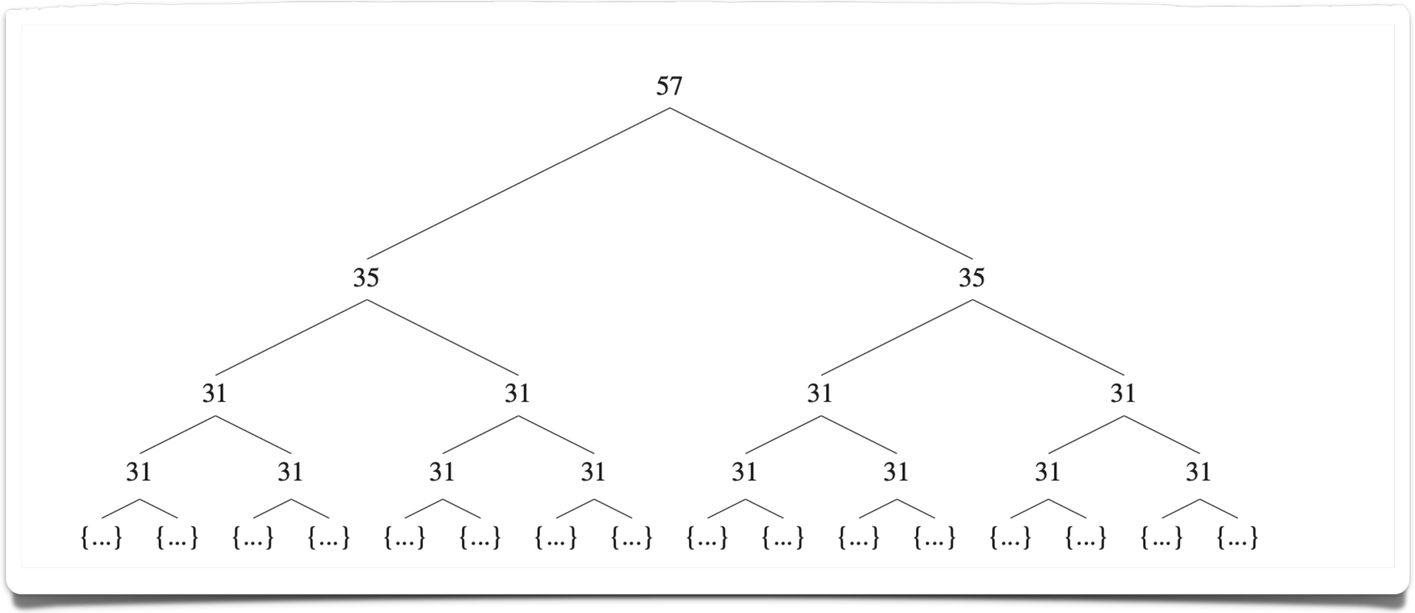

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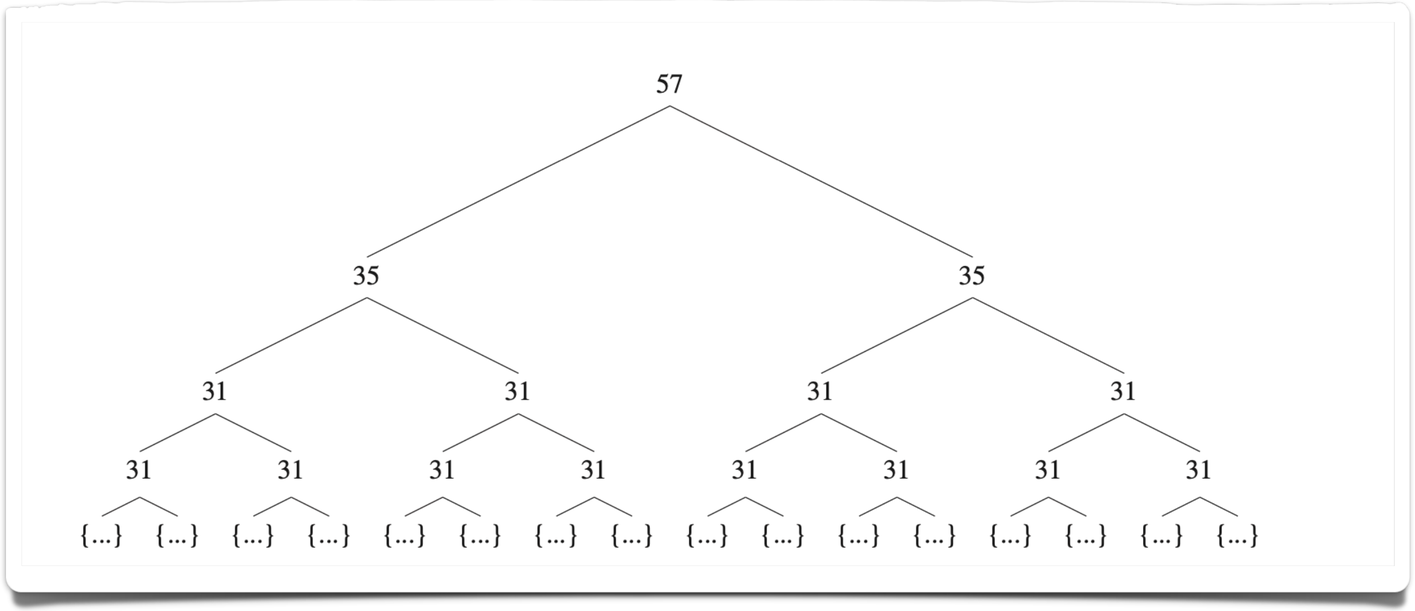
Dynamic Programming



Branch and Bound

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Zur Erinnerung: Subset Sum

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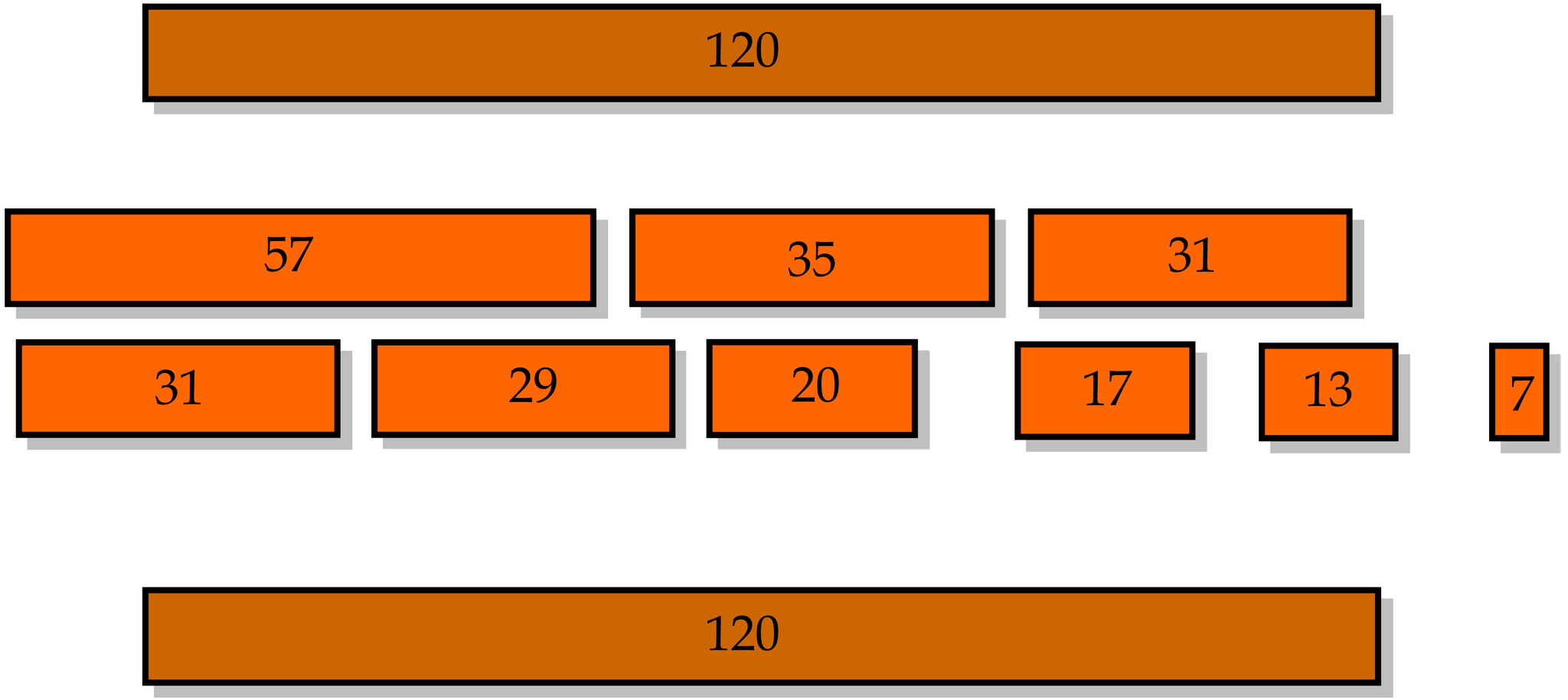

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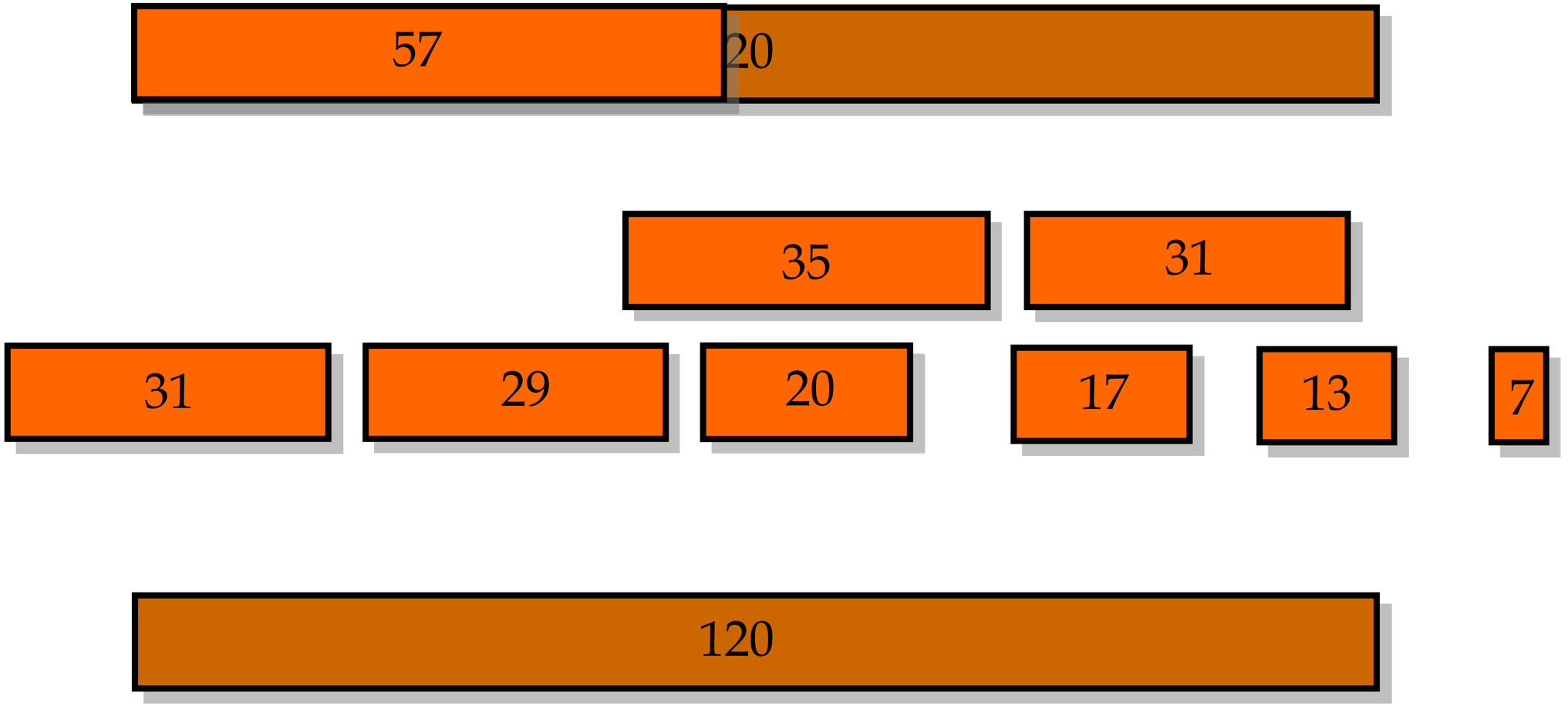

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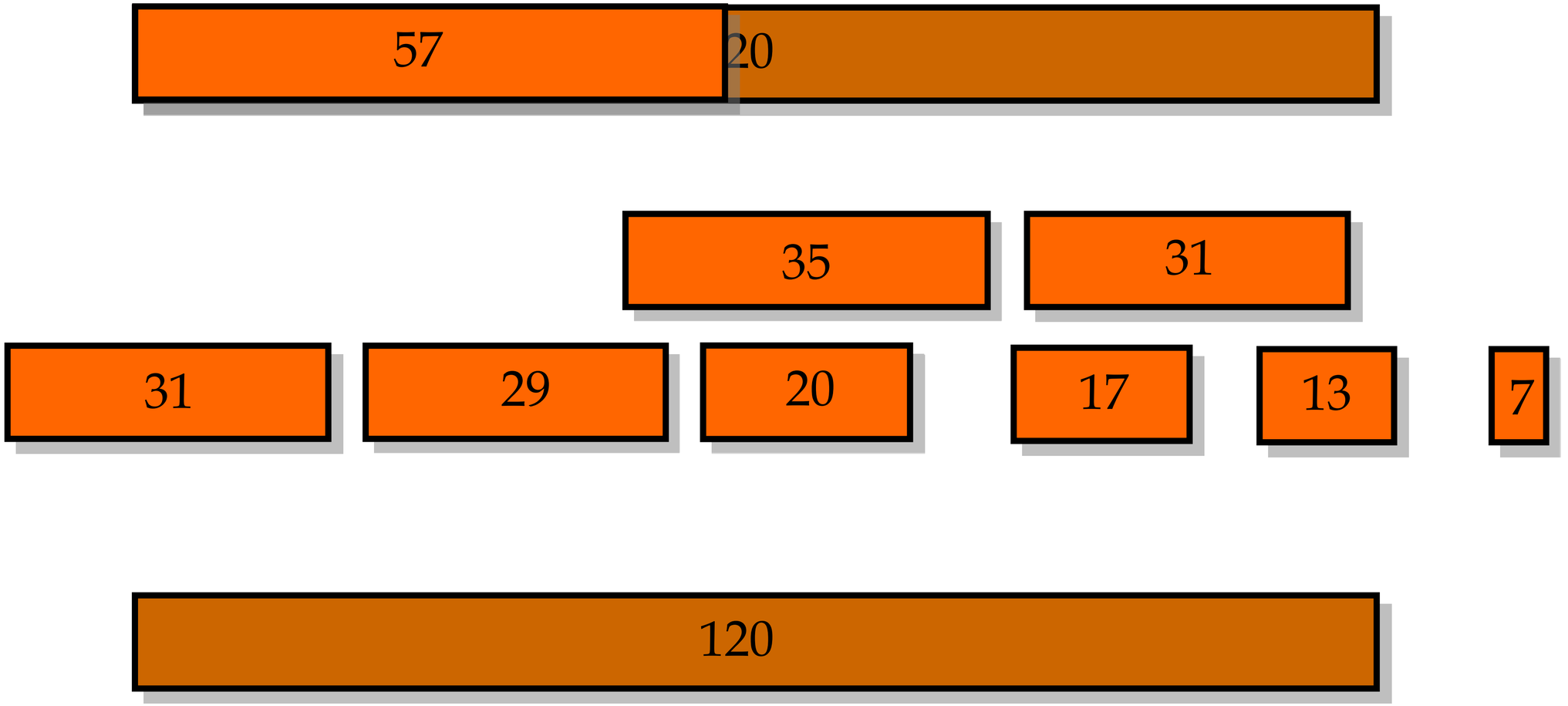

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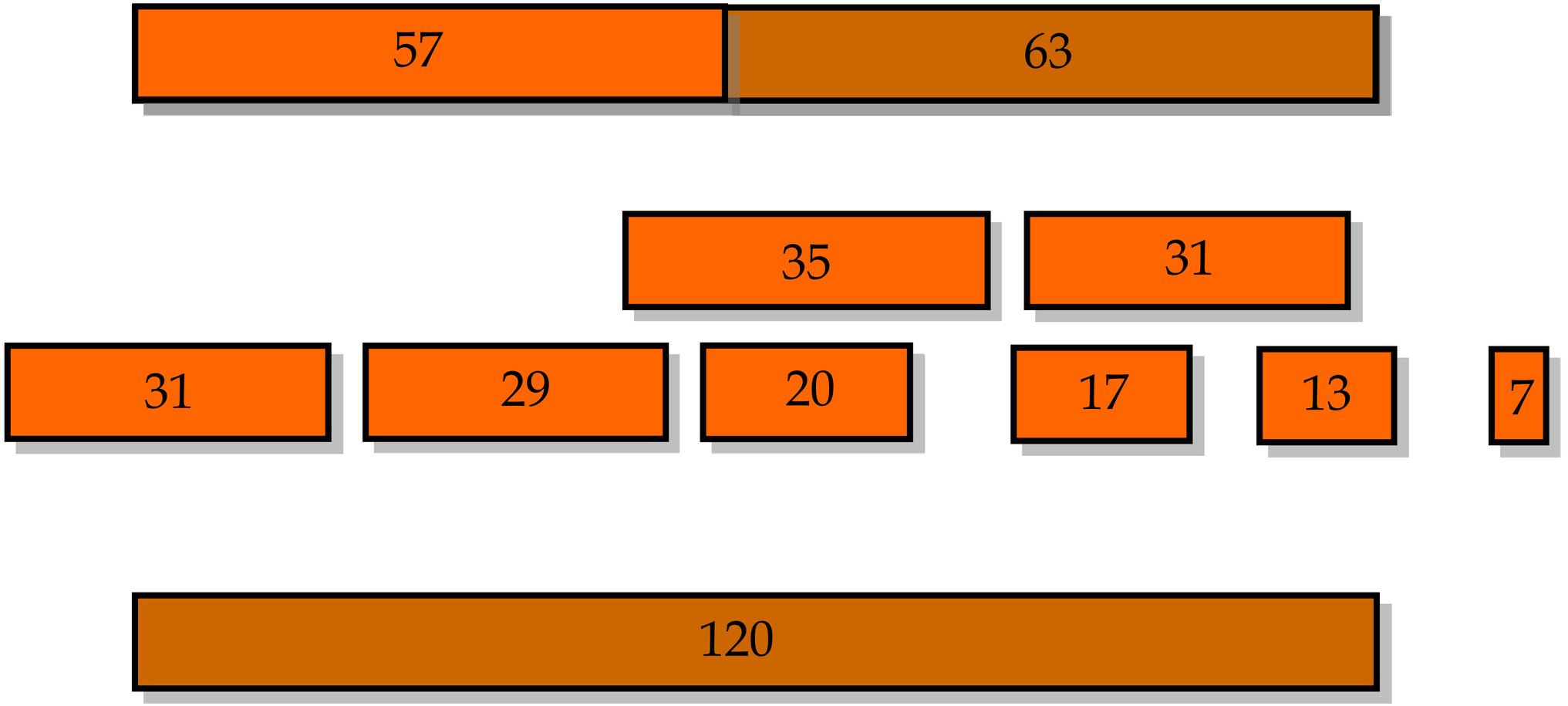

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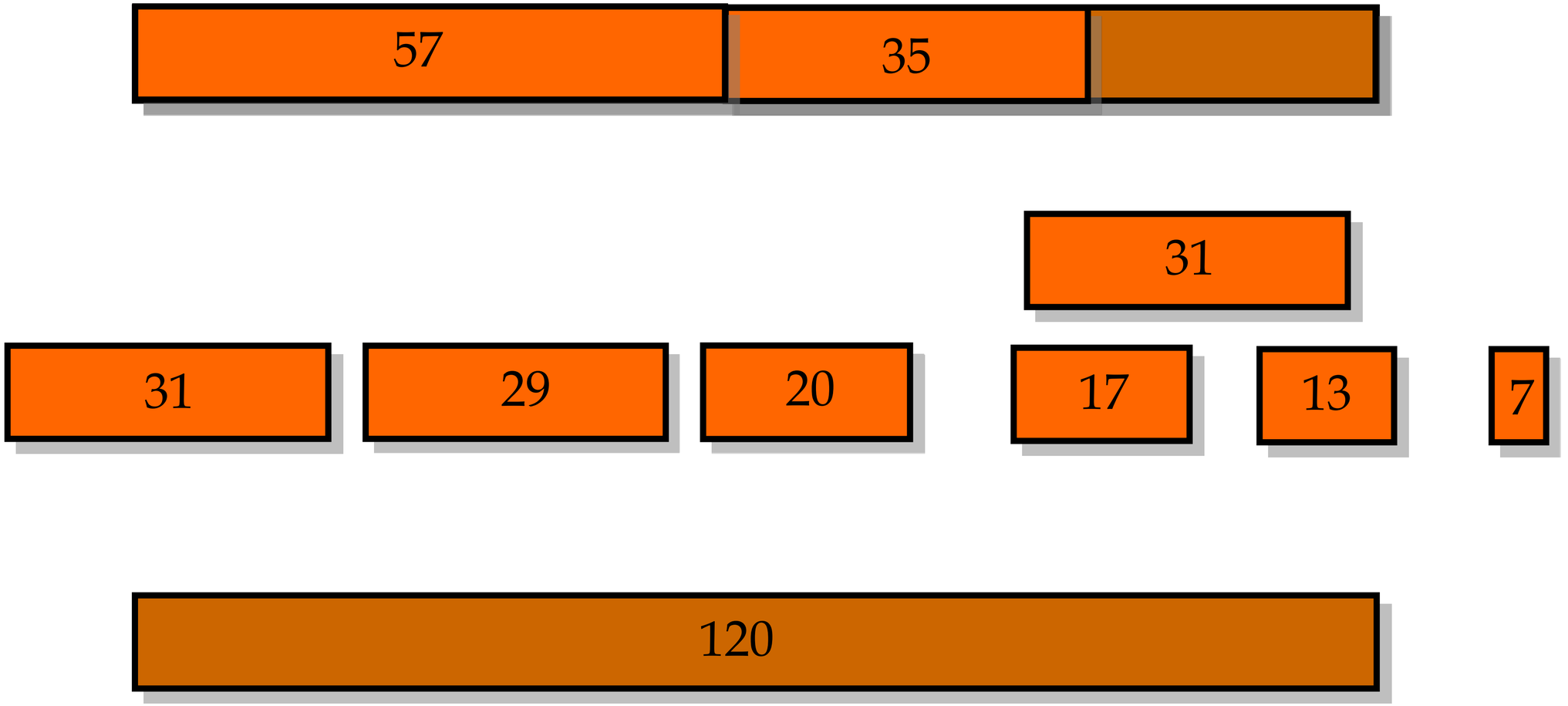

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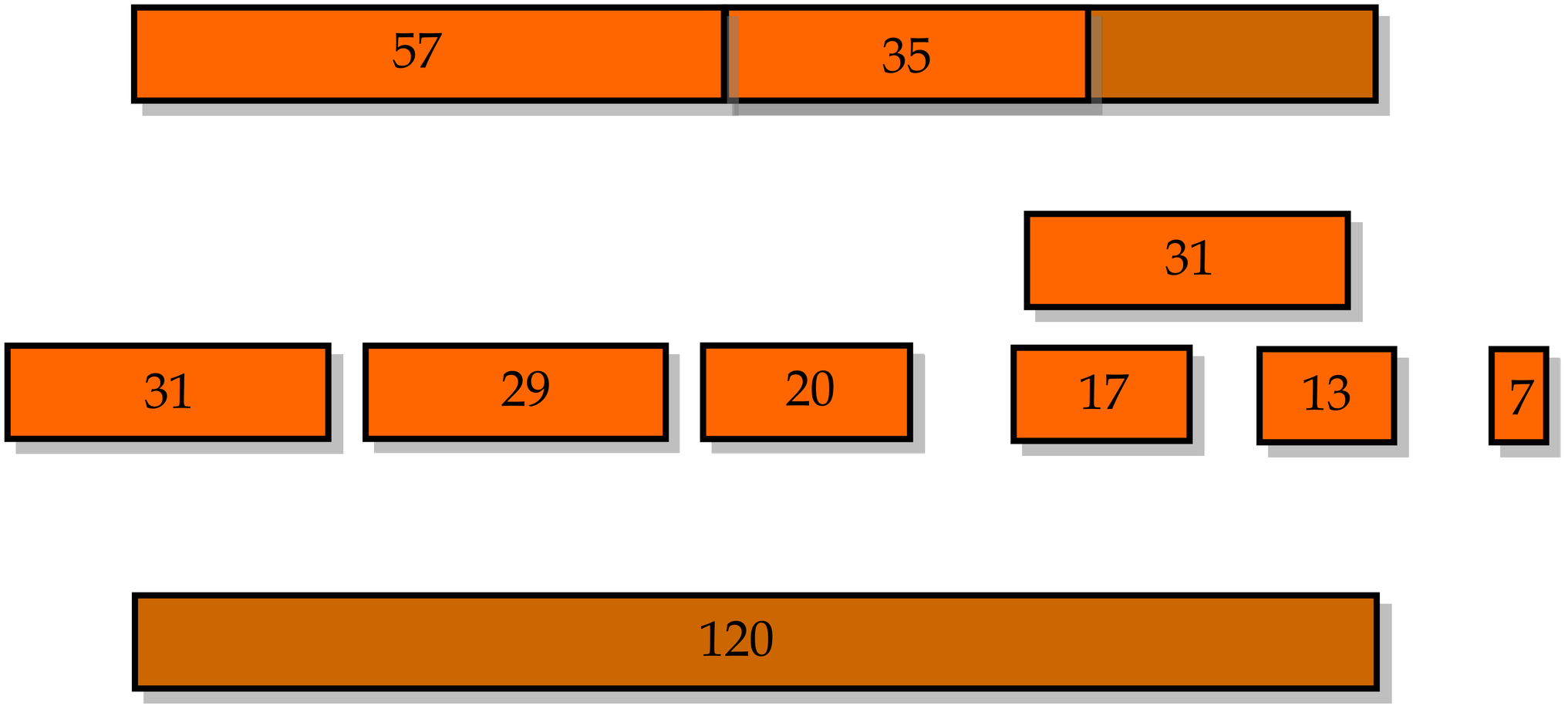

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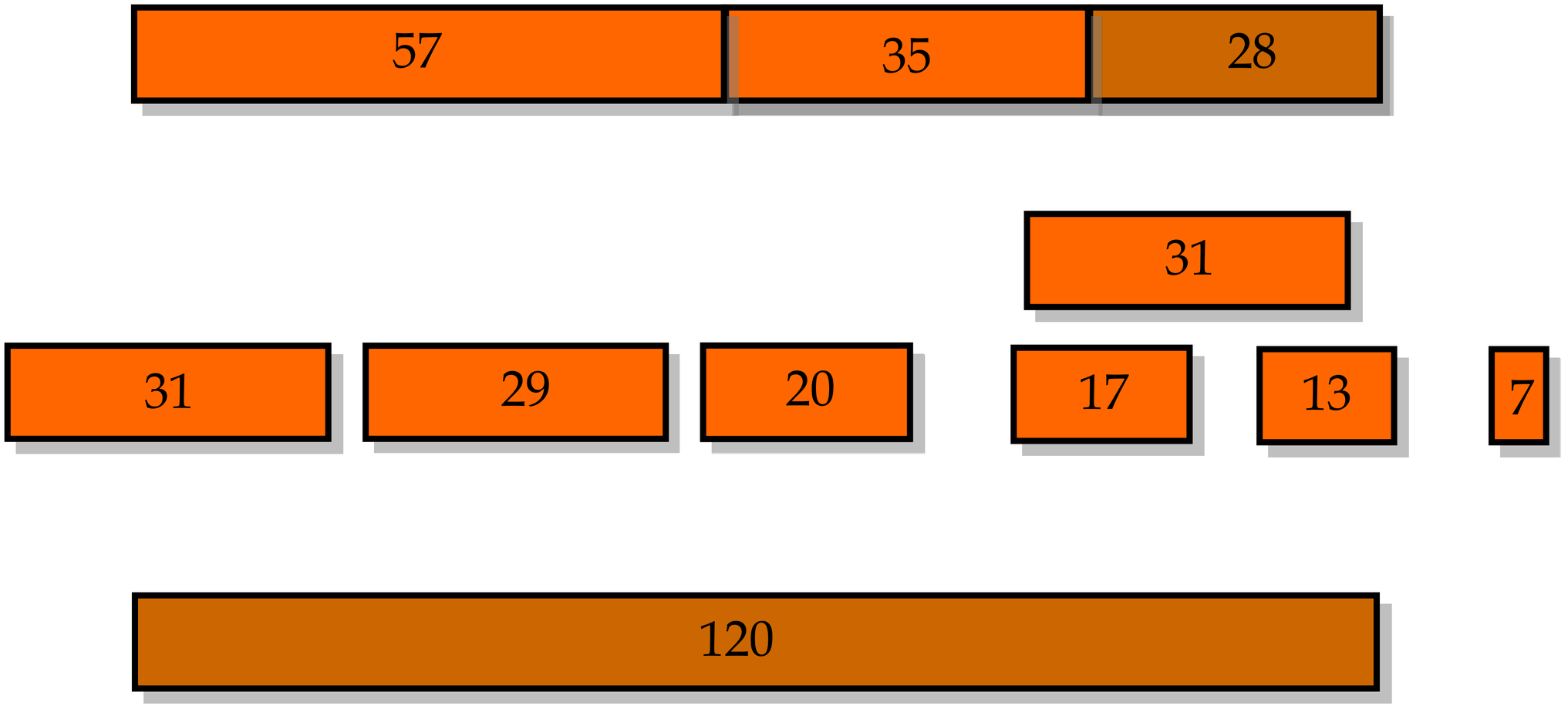

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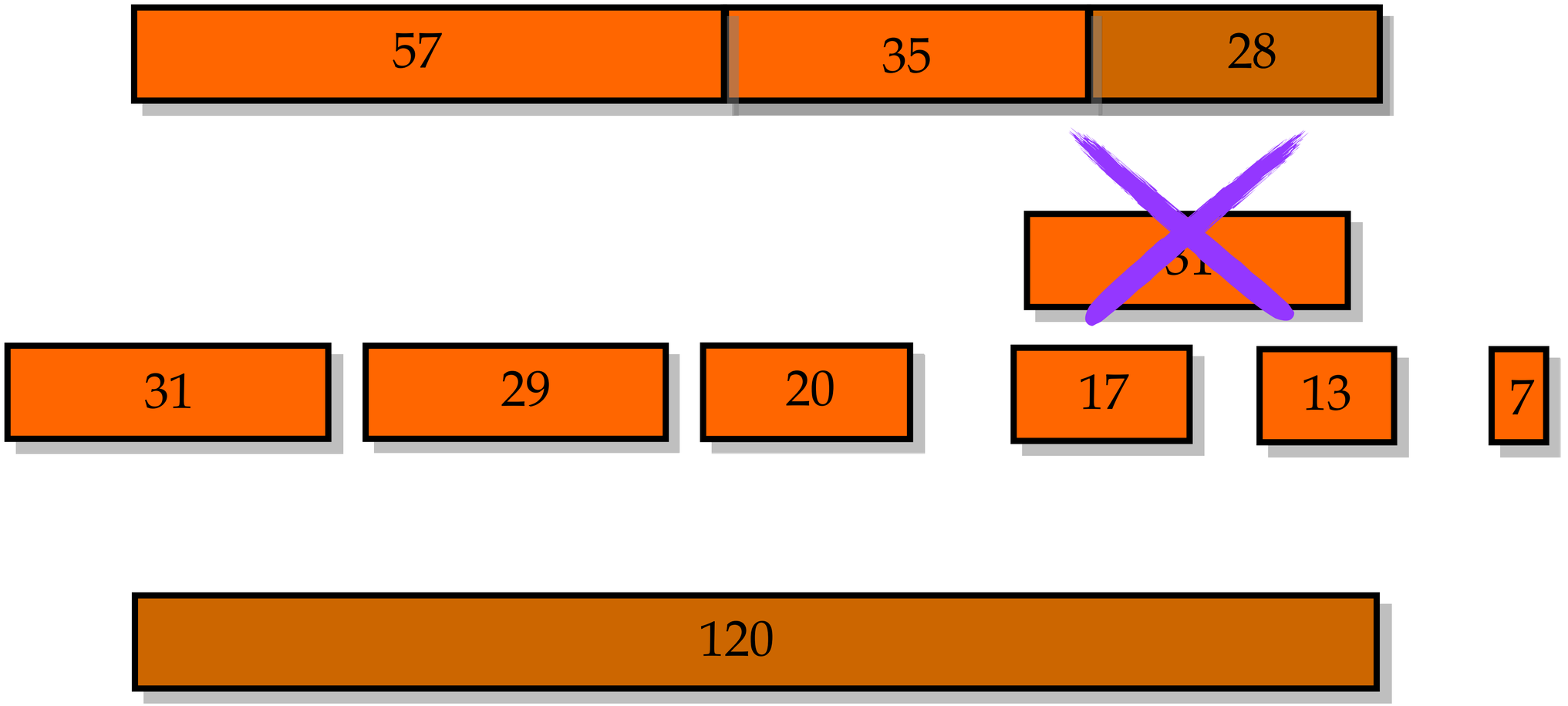

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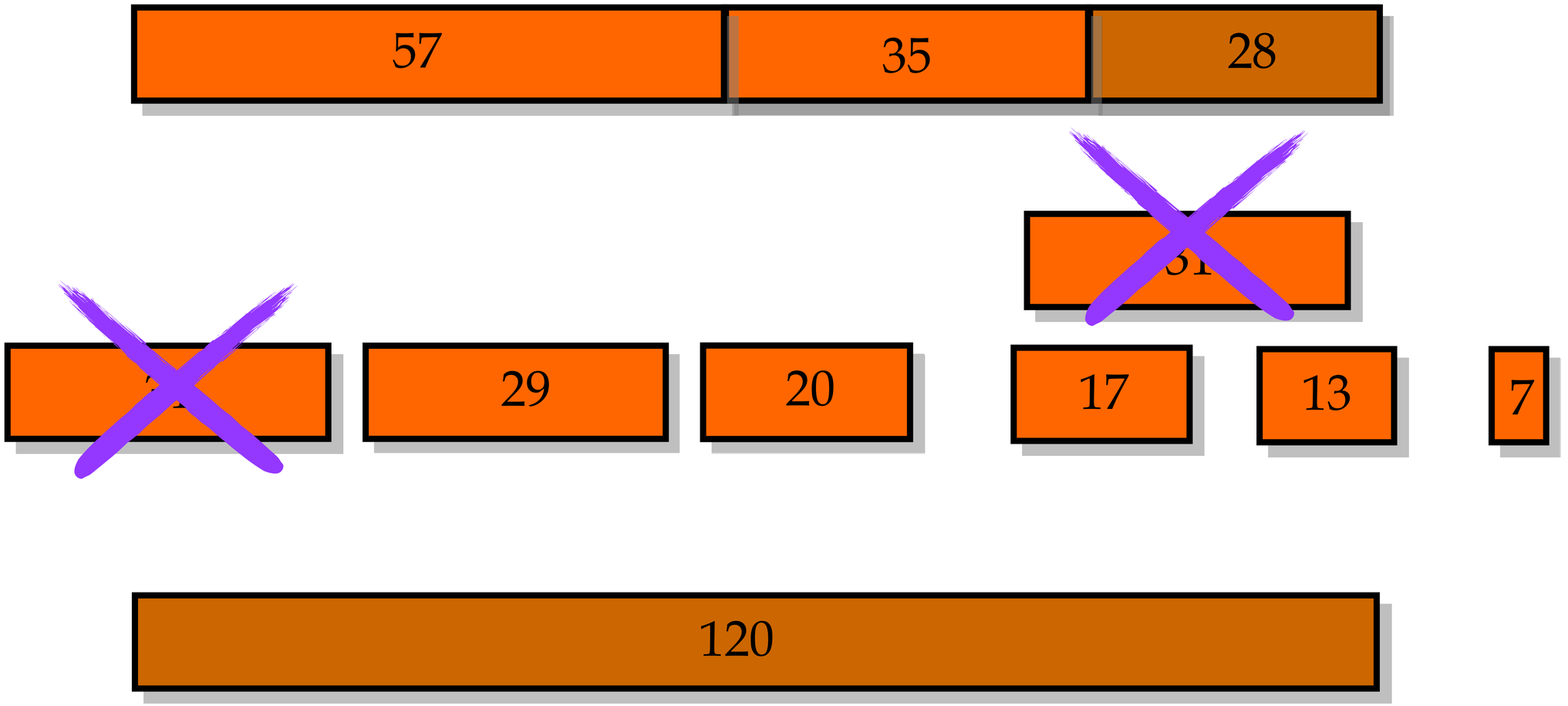

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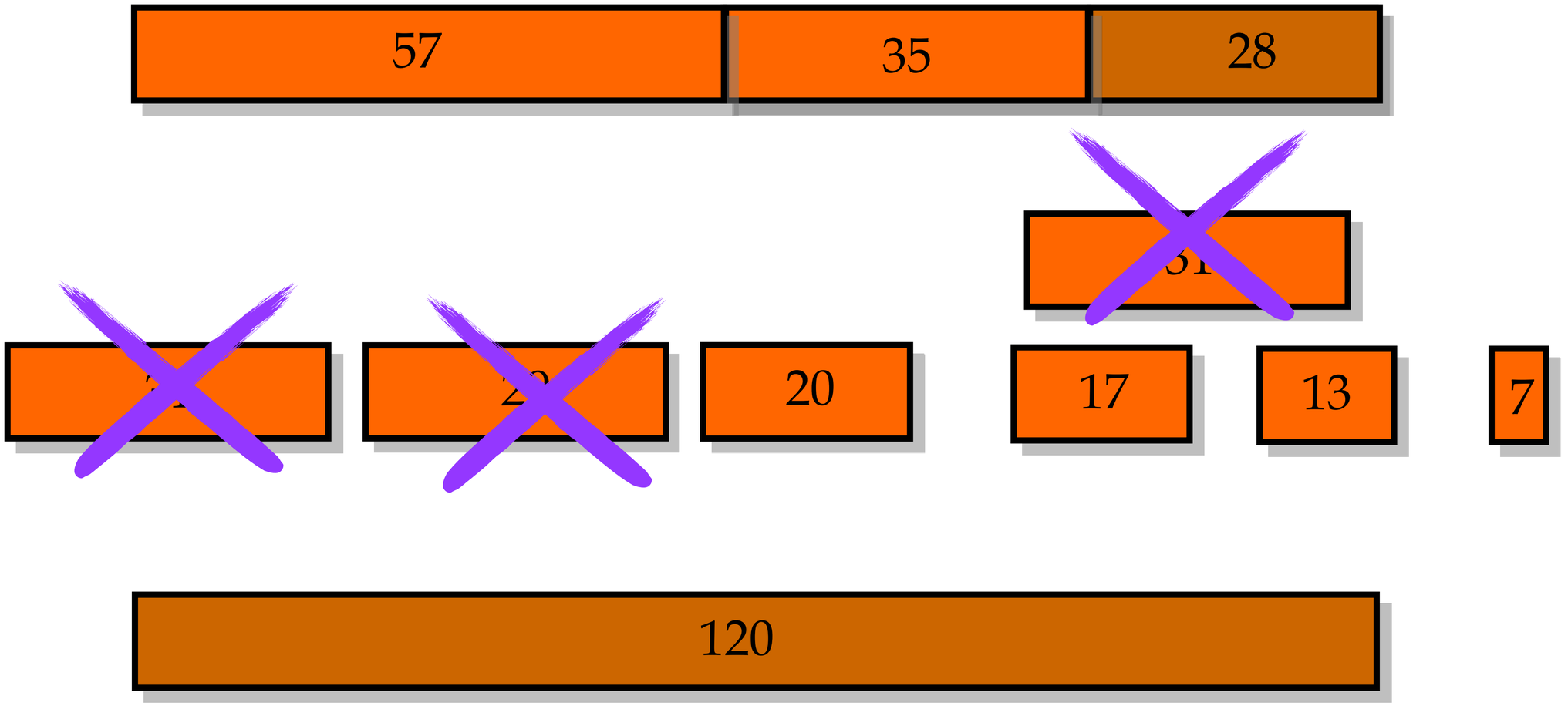

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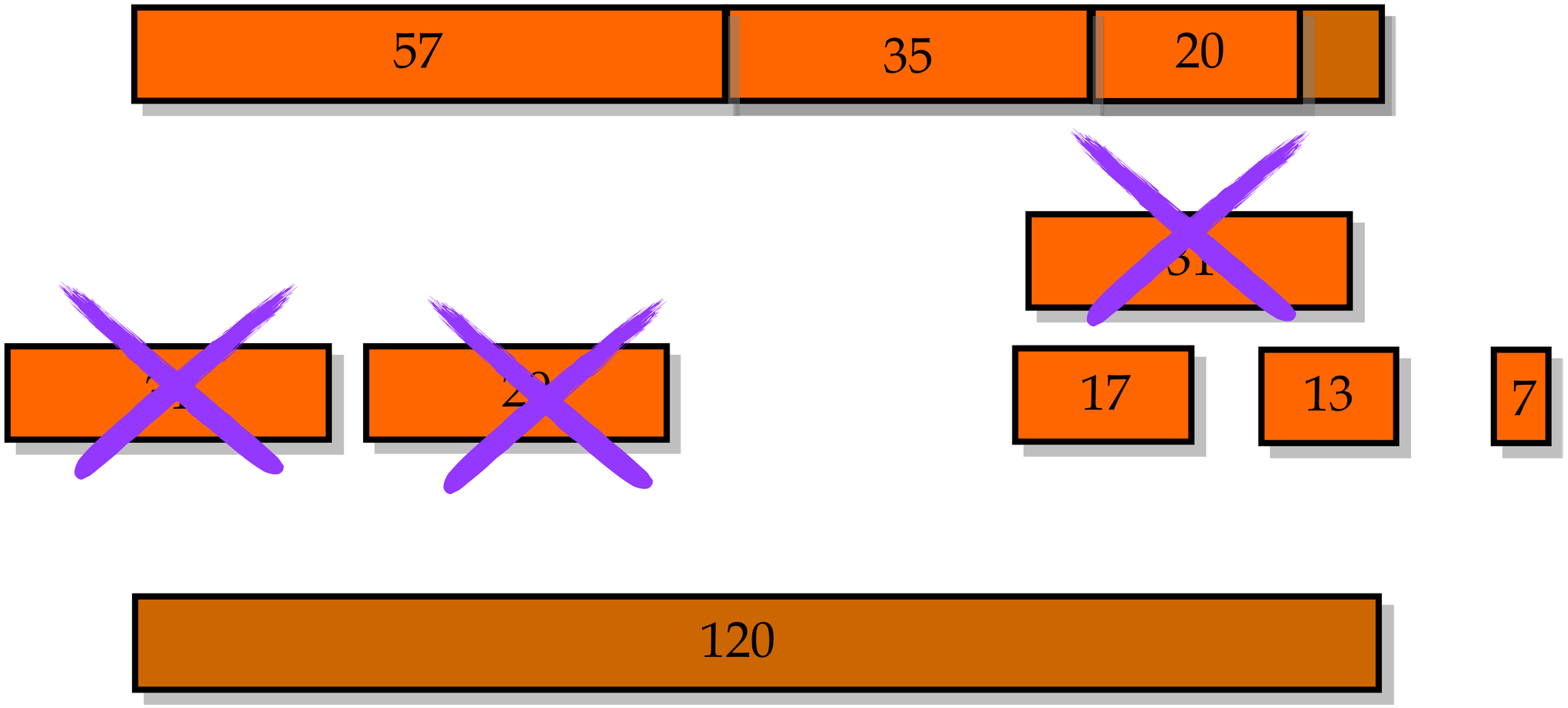

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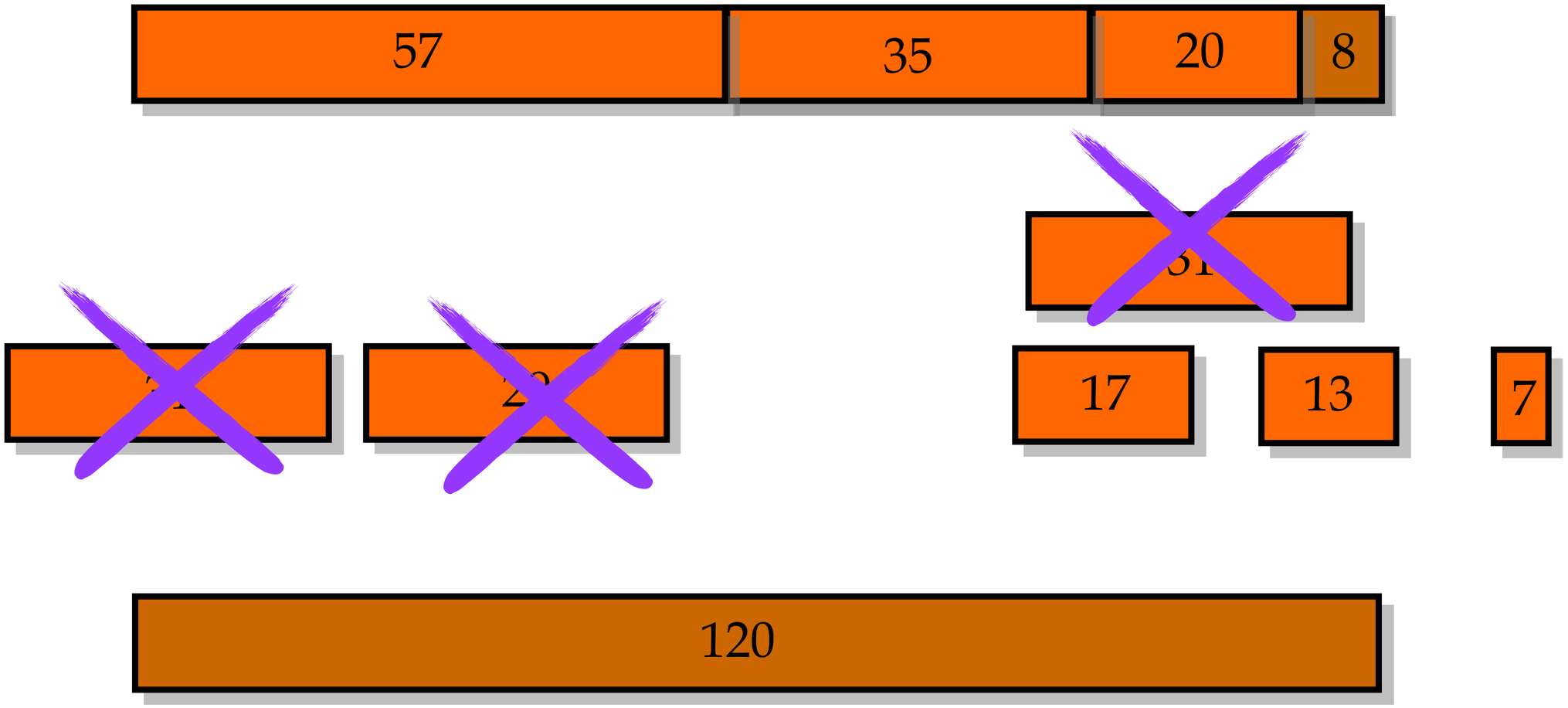

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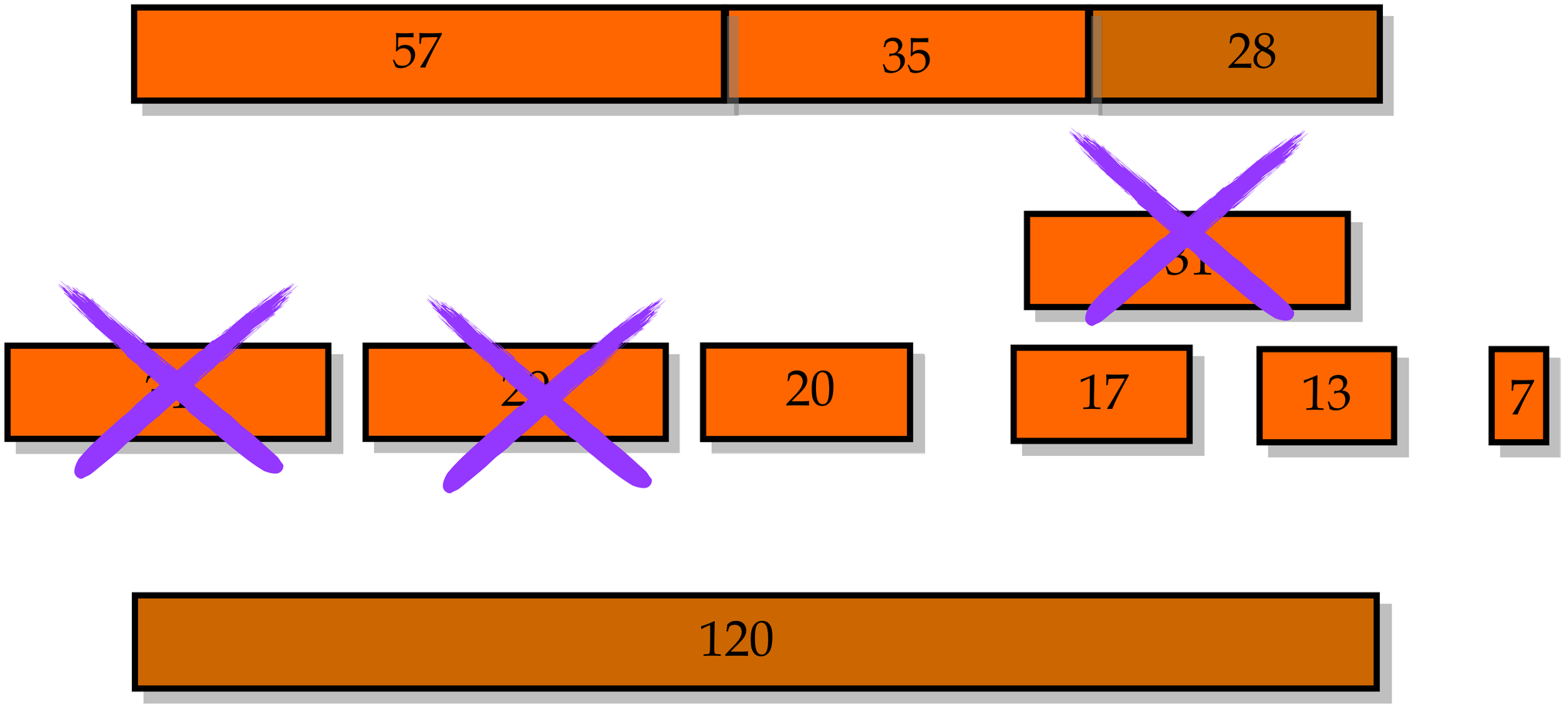

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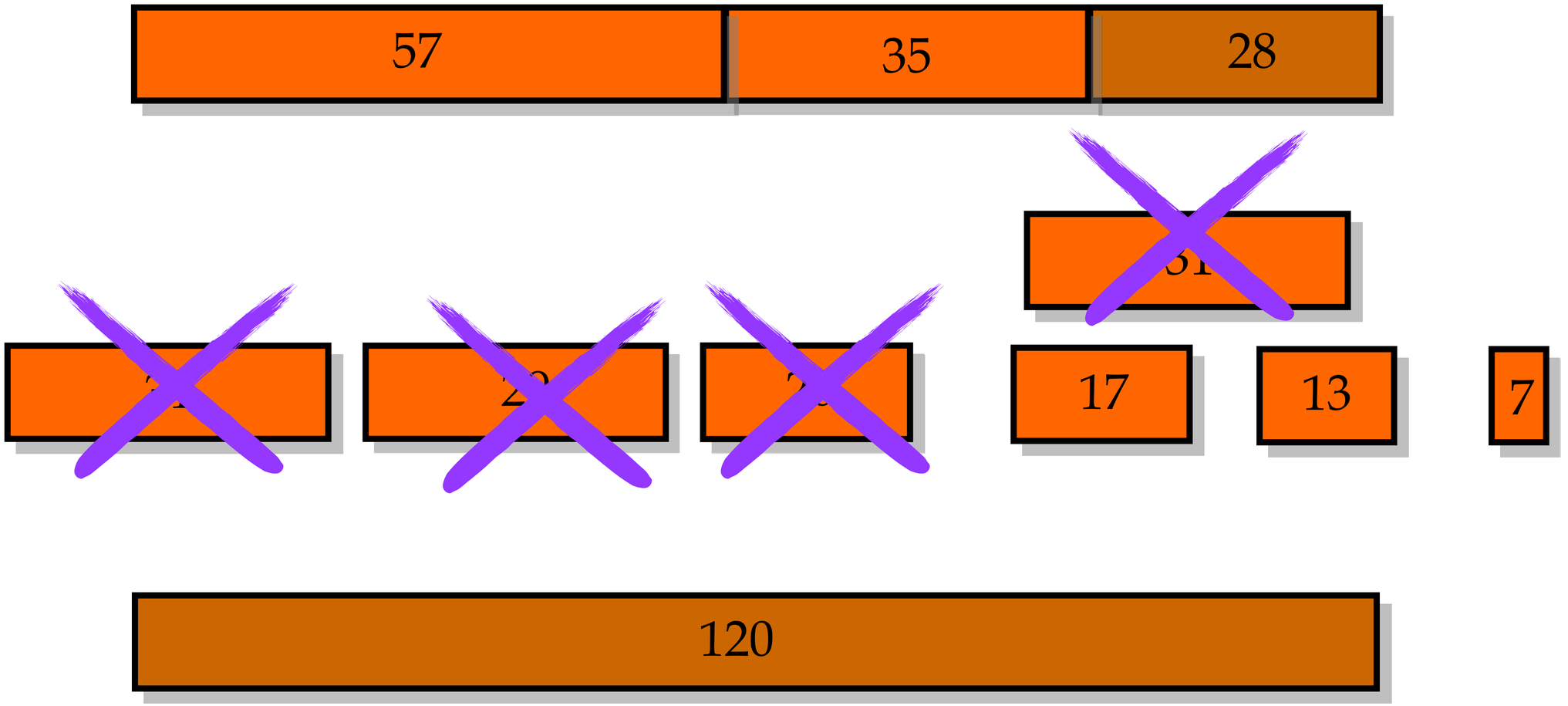

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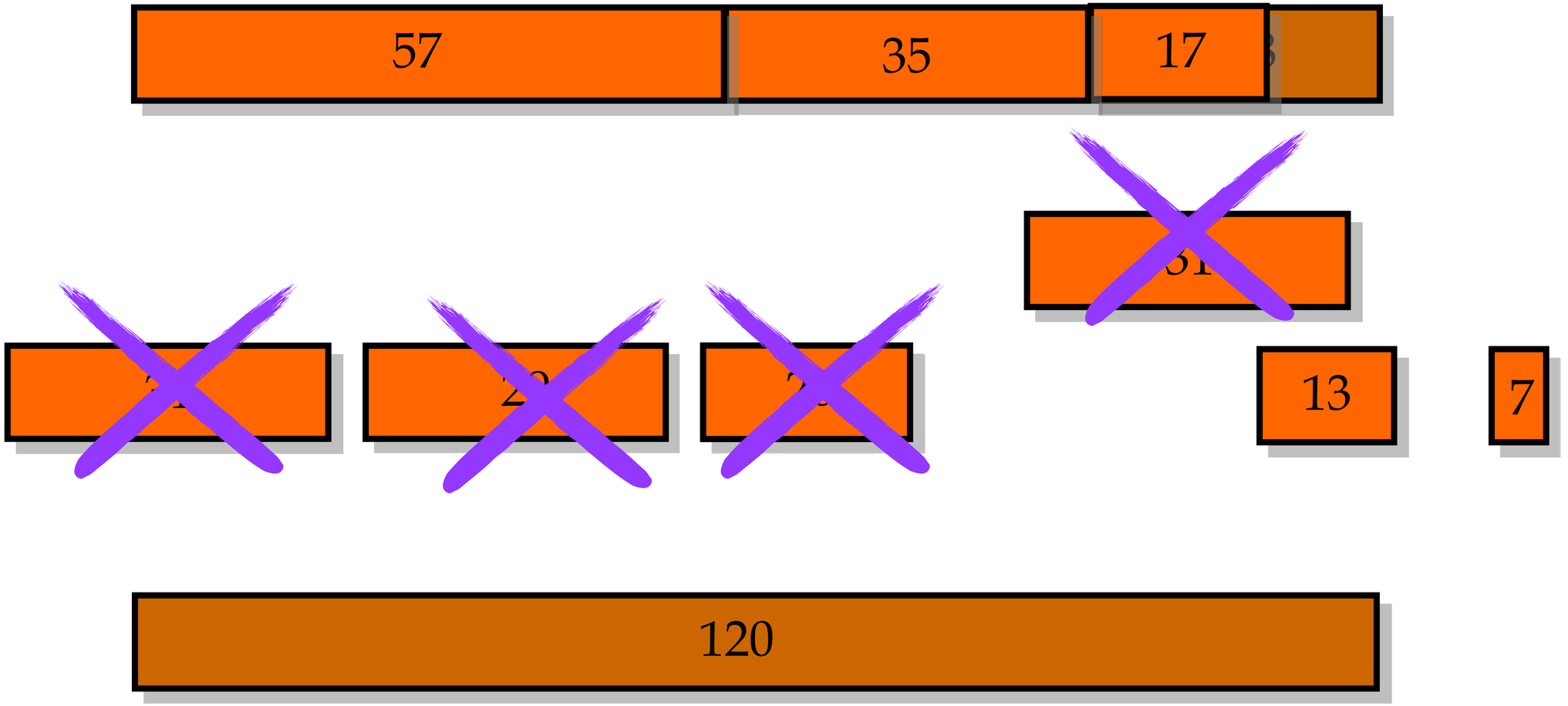

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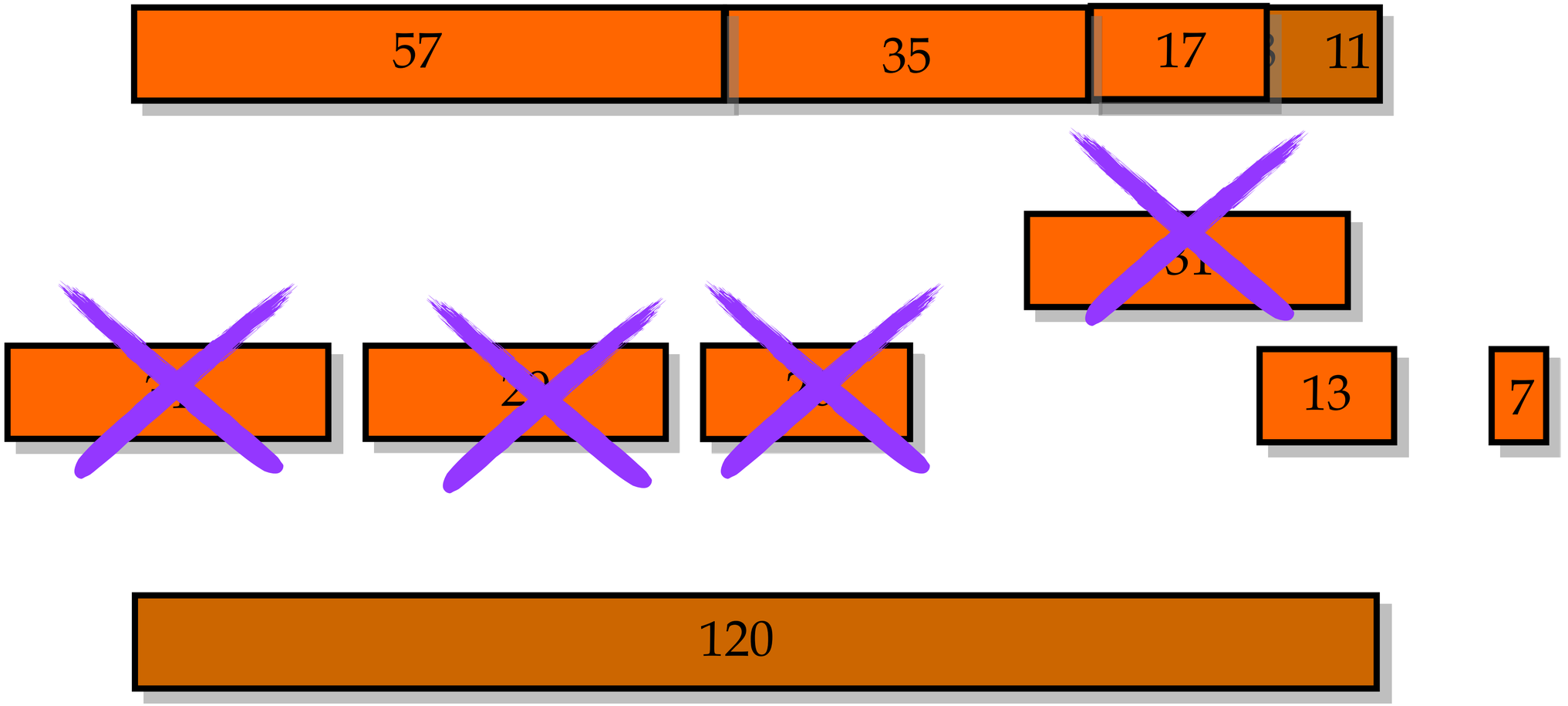

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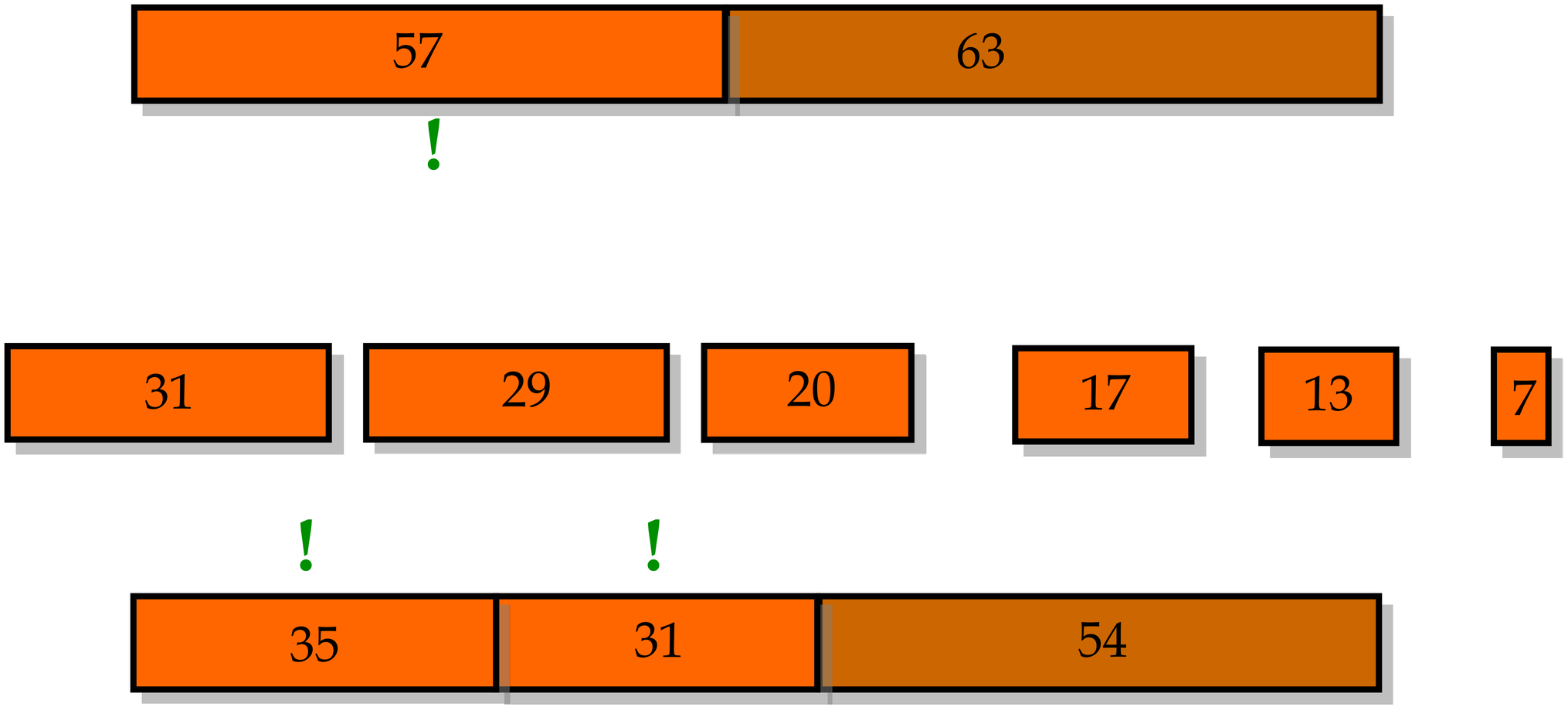

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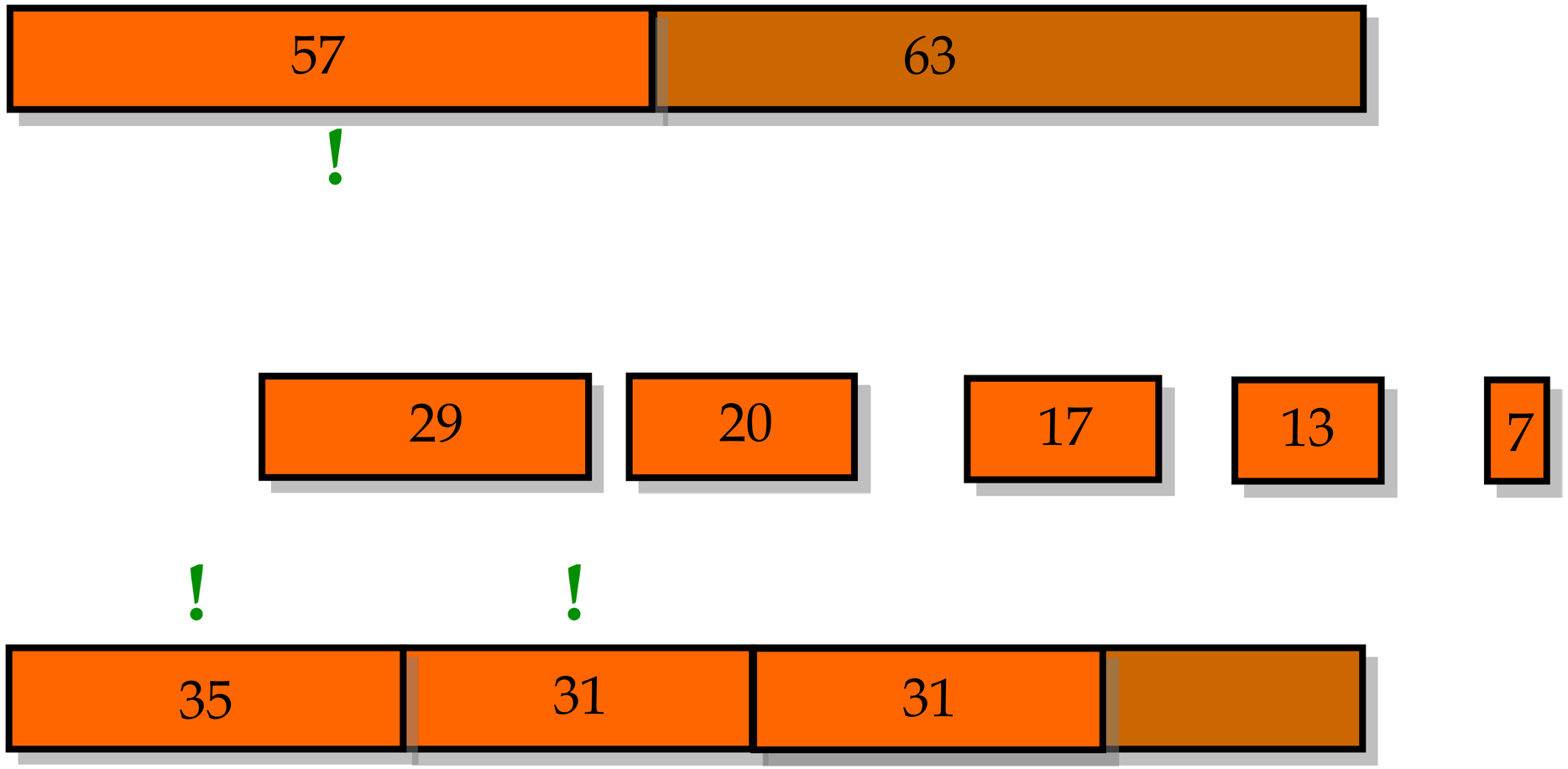

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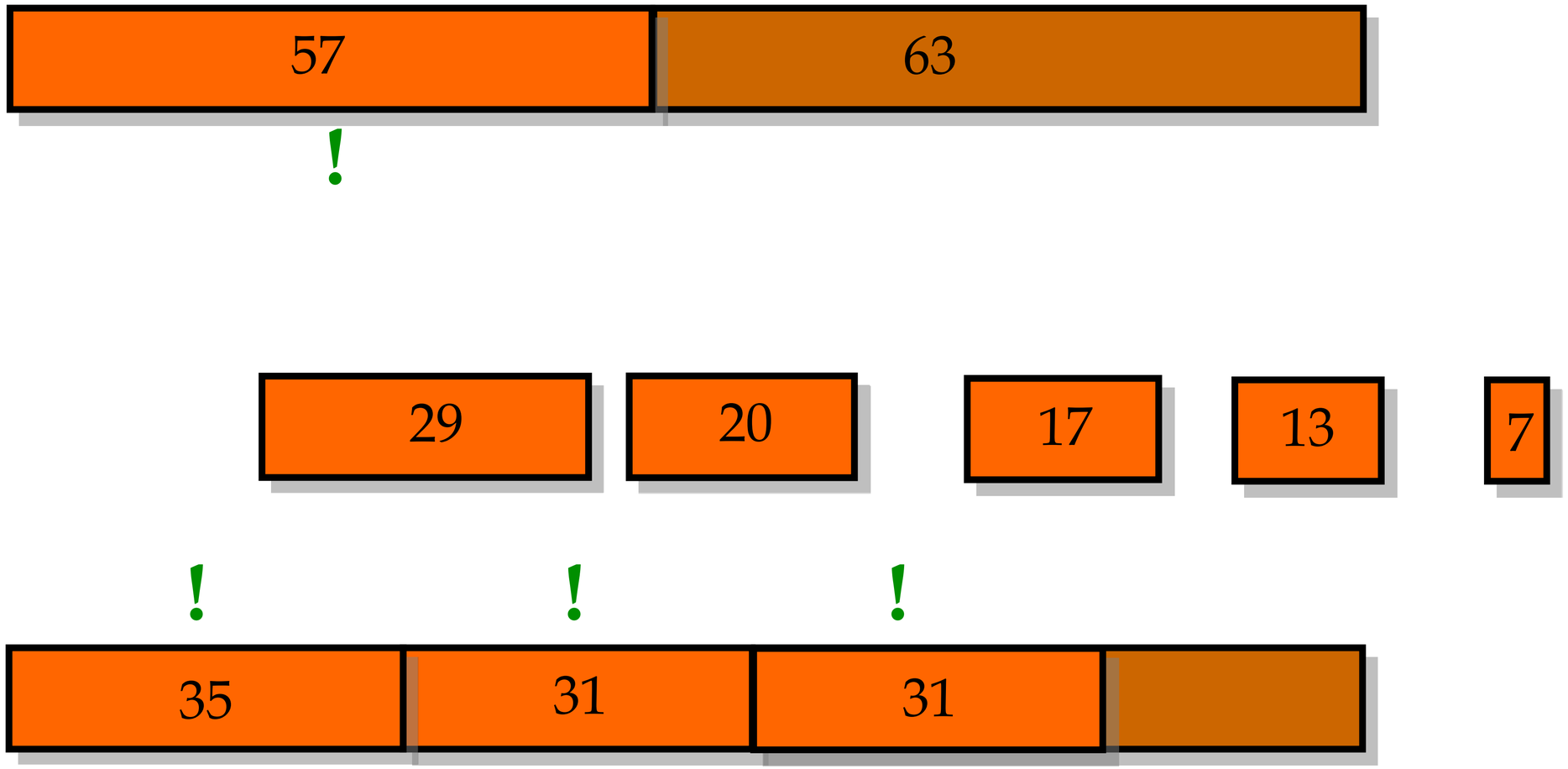

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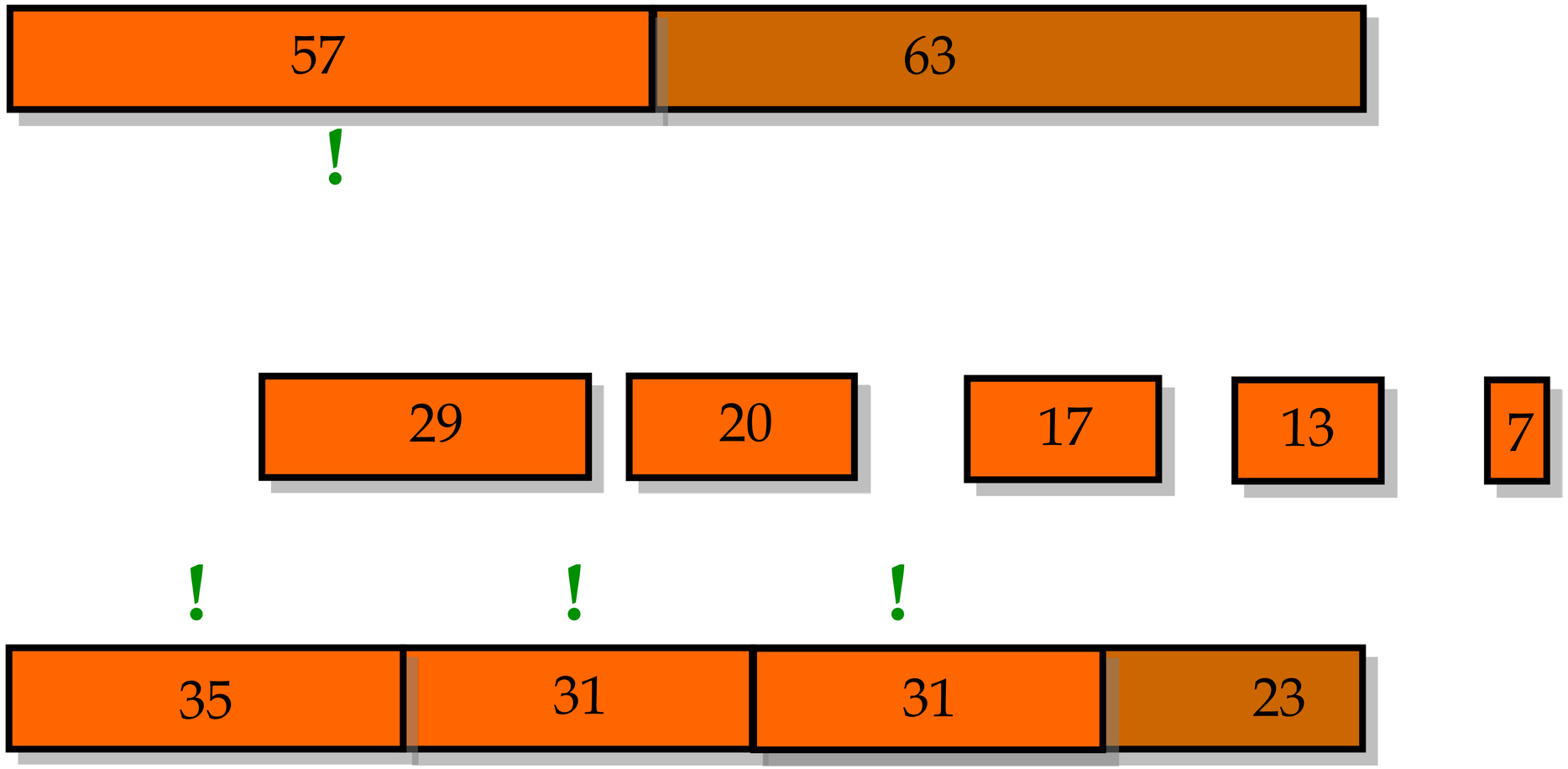

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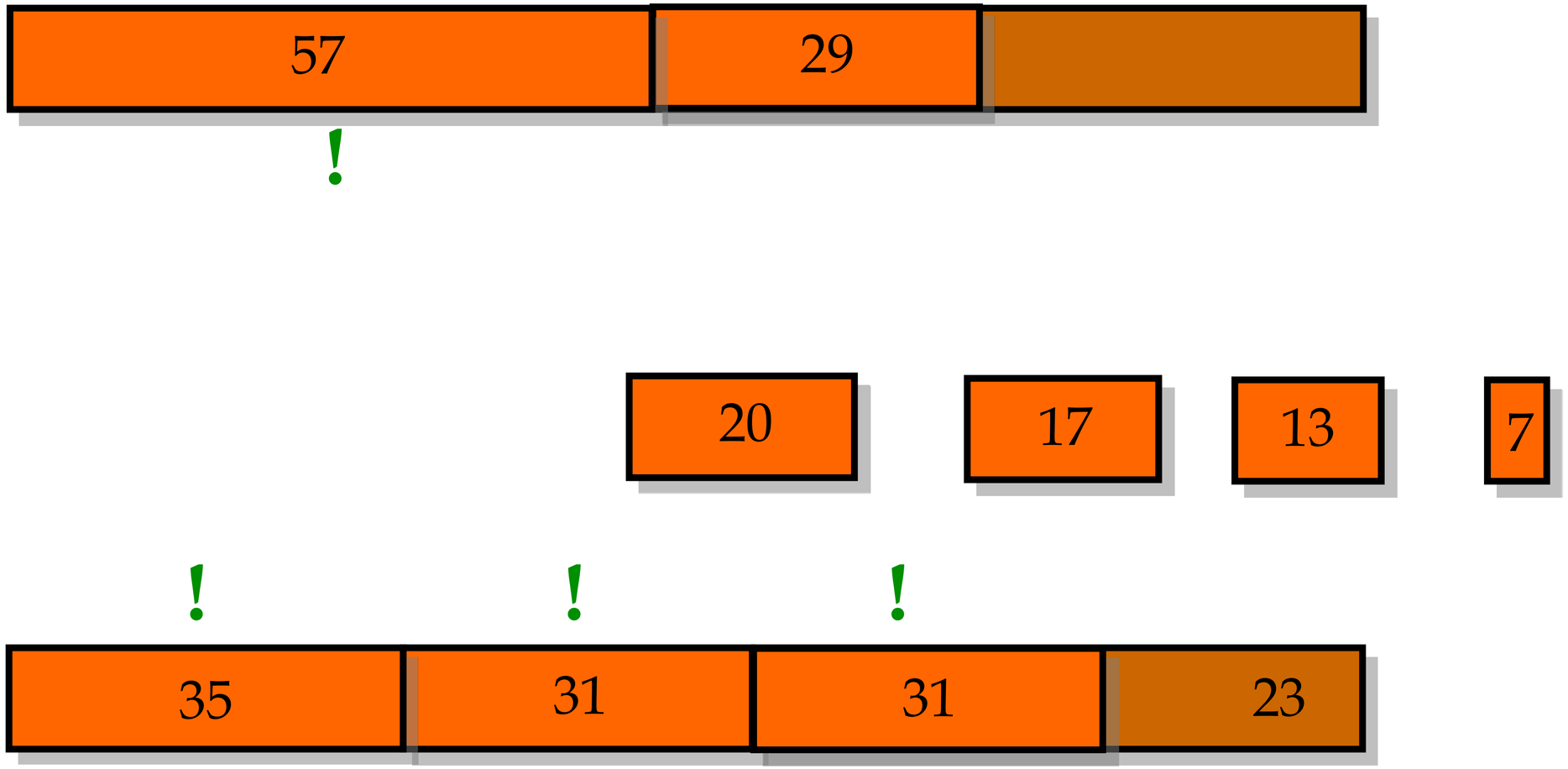

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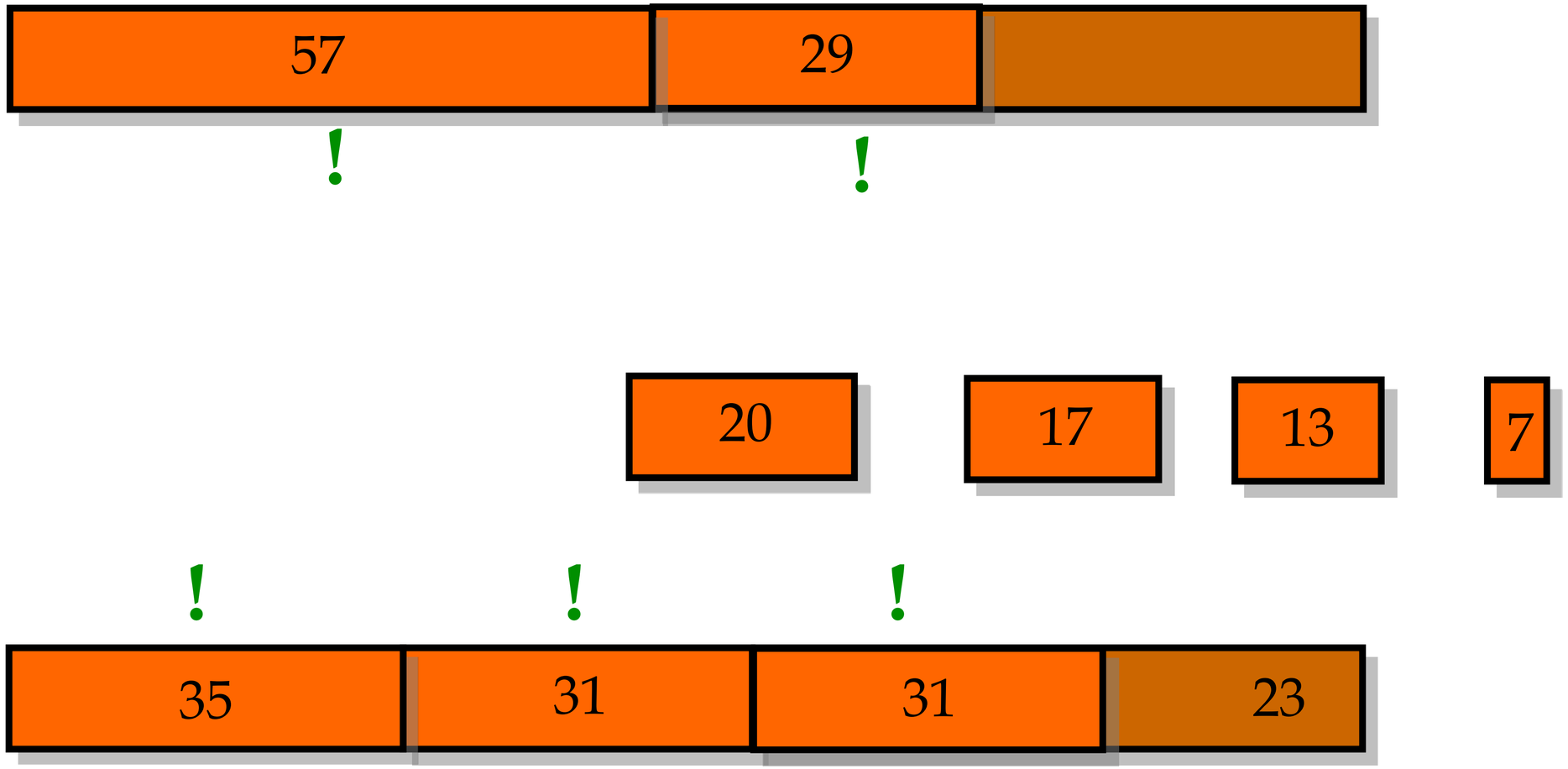

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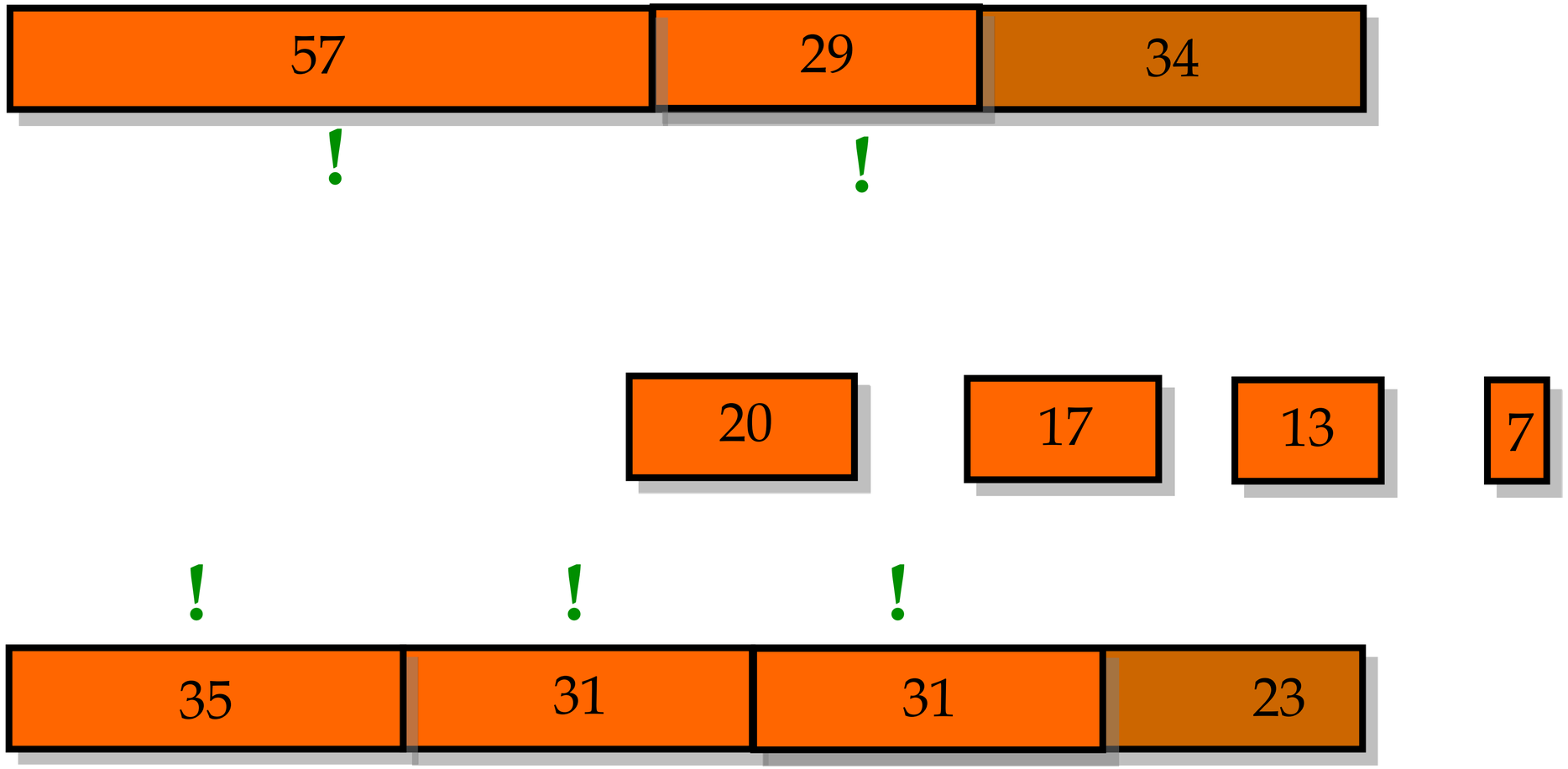

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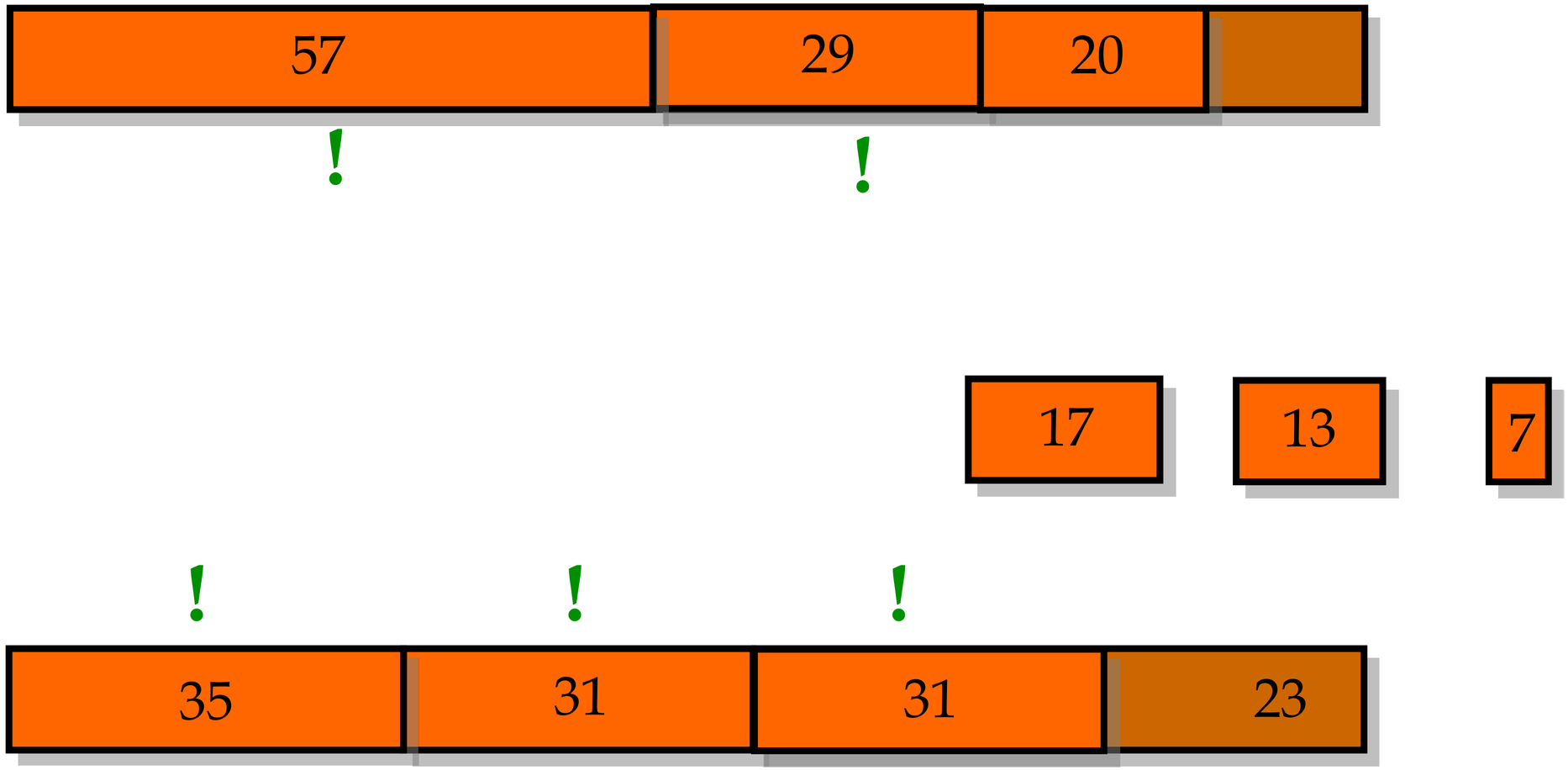

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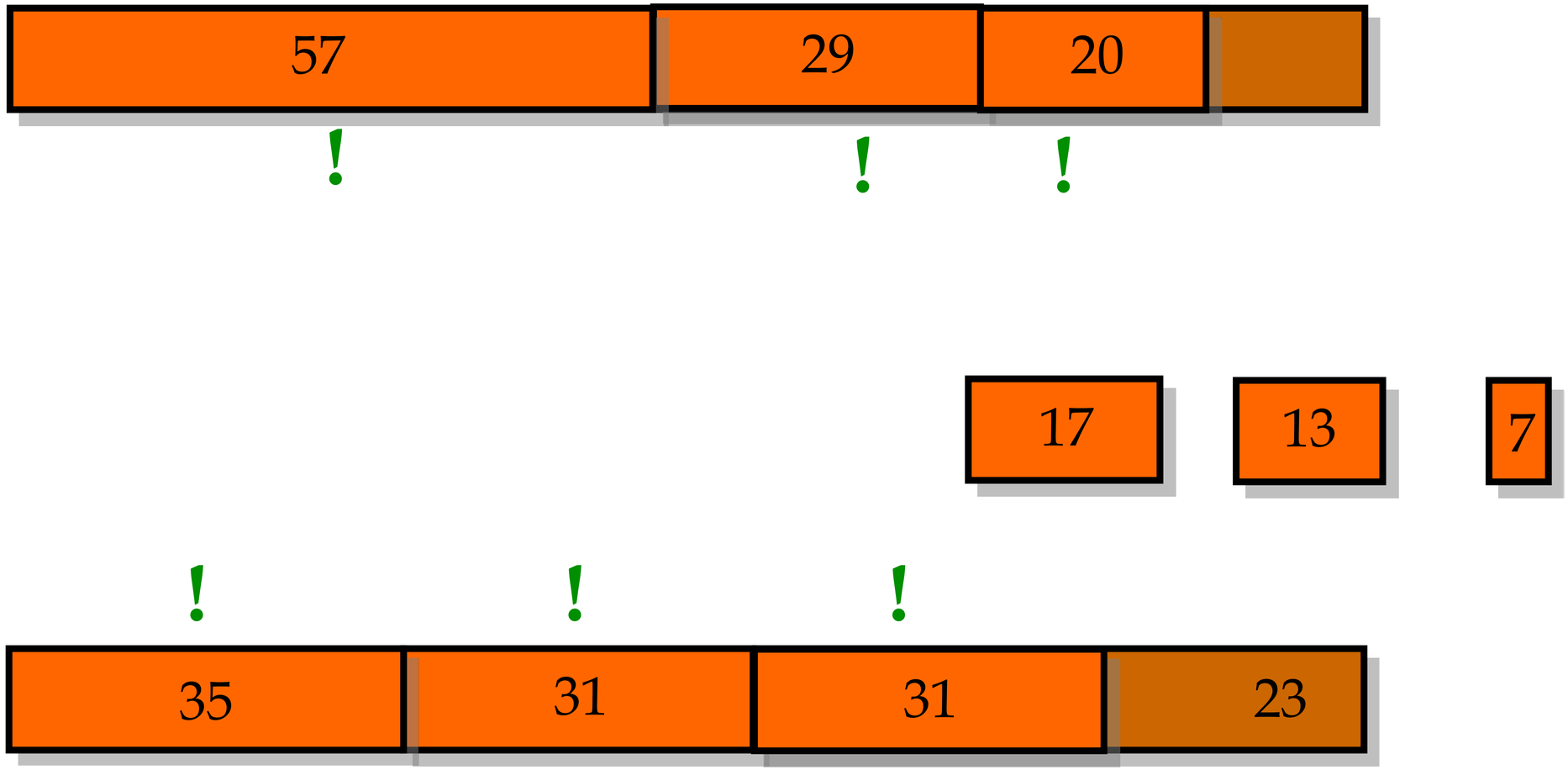

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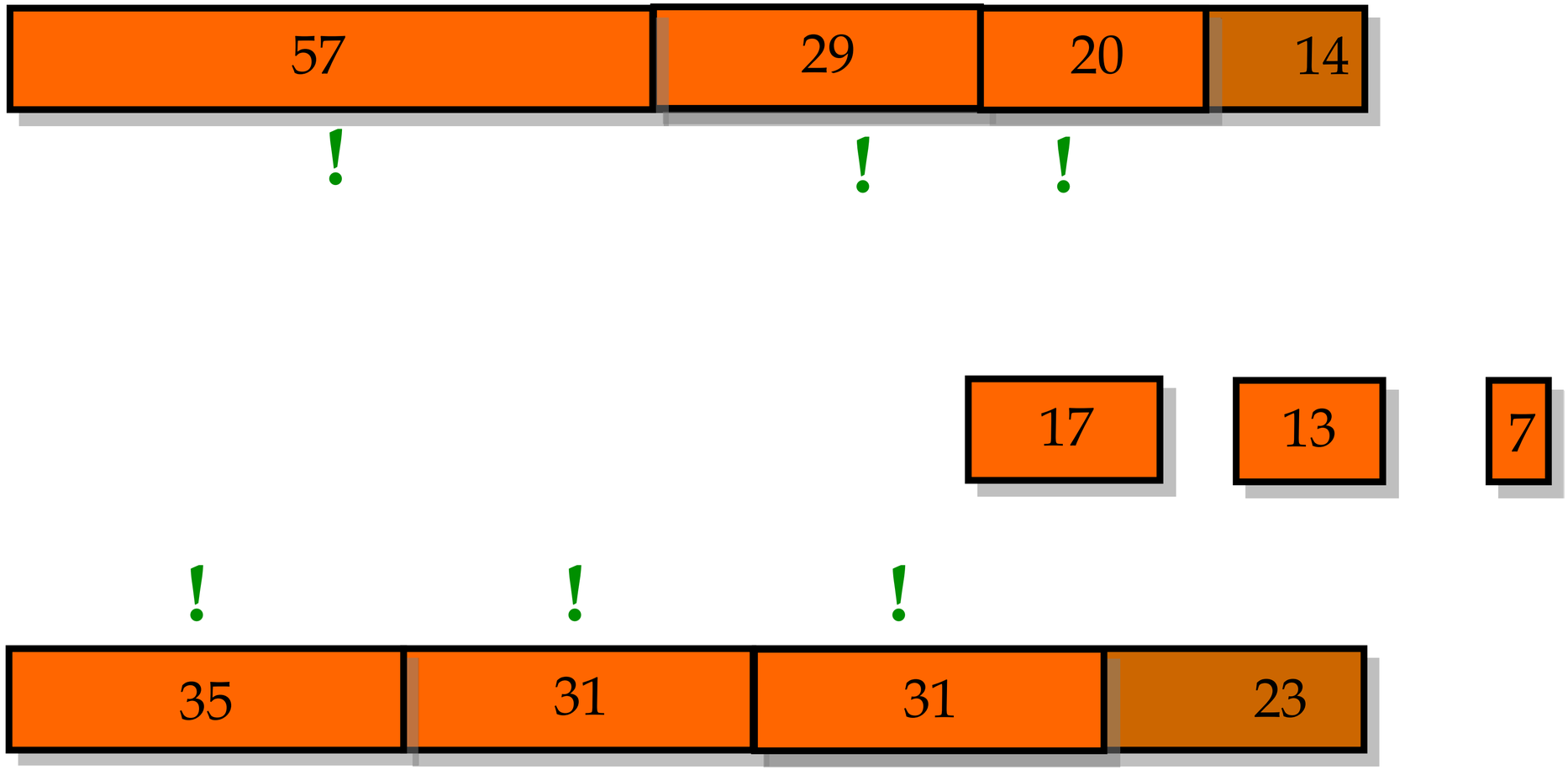

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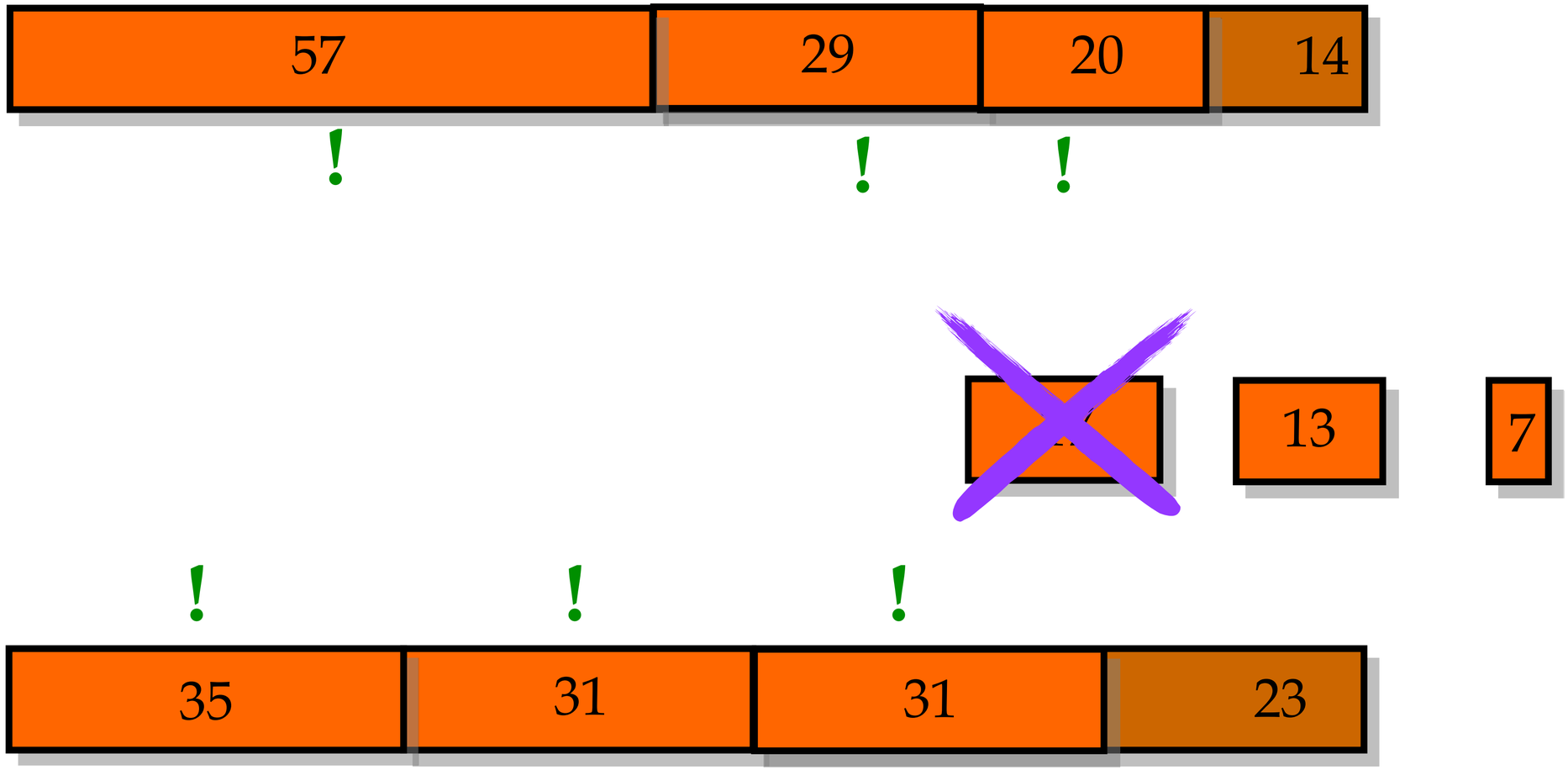

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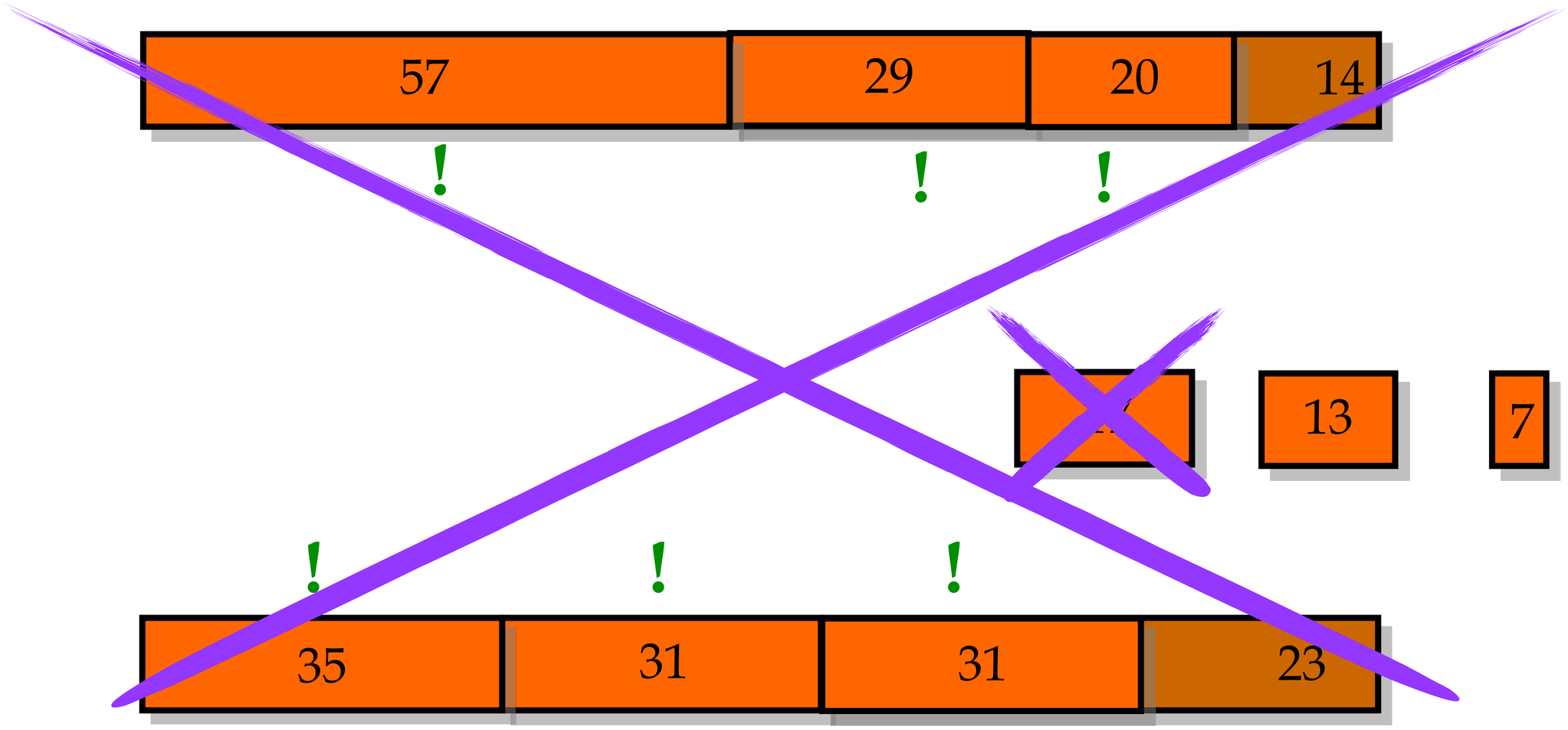


Dynamic Programming

Satz 2.6 *Algorithmus 2.5 berechnet besten Lösungswert für das Rucksackproblem in einer Laufzeit $O(nZ)$.*

Algorithmus 2.5 *(Dynamic Programming für Knapsack)*

Eingabe: $z_1, ..., z_n, Z, p_1, ...p_n$

Ausgabe: Funktion $P : \{1, ..., Z\} \times \{1, ..., n\} \to \mathbb{R};\ (x, i) \mapsto P(x, i)$

$$\text{mit } P(x,i) = max \sum_{i=1}^{i} p_j y_j \text{ mit } \sum_{j=1}^{i} z_j y_j \leq x, \text{ für } y_j \in \{0,1\}$$

```
1:  for (x = 0) to Z do
2:      P(x,0) := 0
3:  for (i = 1) to n do
4:      for (x = 0) to (z_i − 1) do
5:          P(x,i) := P(x,i−1)
6:      for (x = z_i) to Z do
7:          if ((P(x − z_i, i−1) + p_i) > P(x,i−1)) then
8:              P(x,i) := P(x − z_i, i−1) + p_i
9:          else
0:              P(x,i) := P(x,i−1)
```

$O(nZ)$

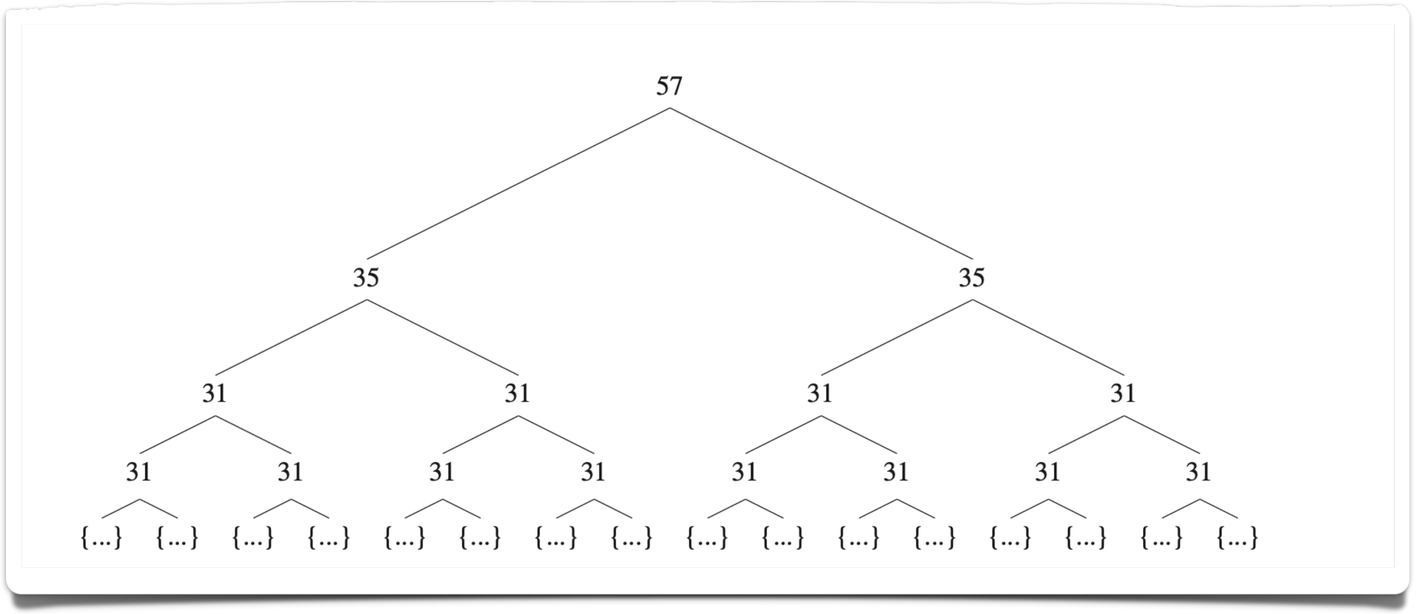
i \ x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	6	6	6	6	6	6	6	6
2	0	0	6	6	6	11	11	11	11	11
3	0	0	6	6	6	11	11	11	14	14
4	0	0	6	6	6	11	11	11	14	15
5	0	0	6	6	6	11	11	12	14	15
6	0	0	6	6	6	11	11	12	14	15
7	0	0	6	6	6	11	11	12	14	15

Induktion!

1. Objekt i ist zu teuer für Kostenschranke x, falls $z_i > x$
2. Objekt i ist nicht zu teuer, bringt aber keine Verbesserung
3. Objekt i ist nicht zu teuer und bringt eine Verbesserung

Branch-and-Bound

Branch-and-Bound



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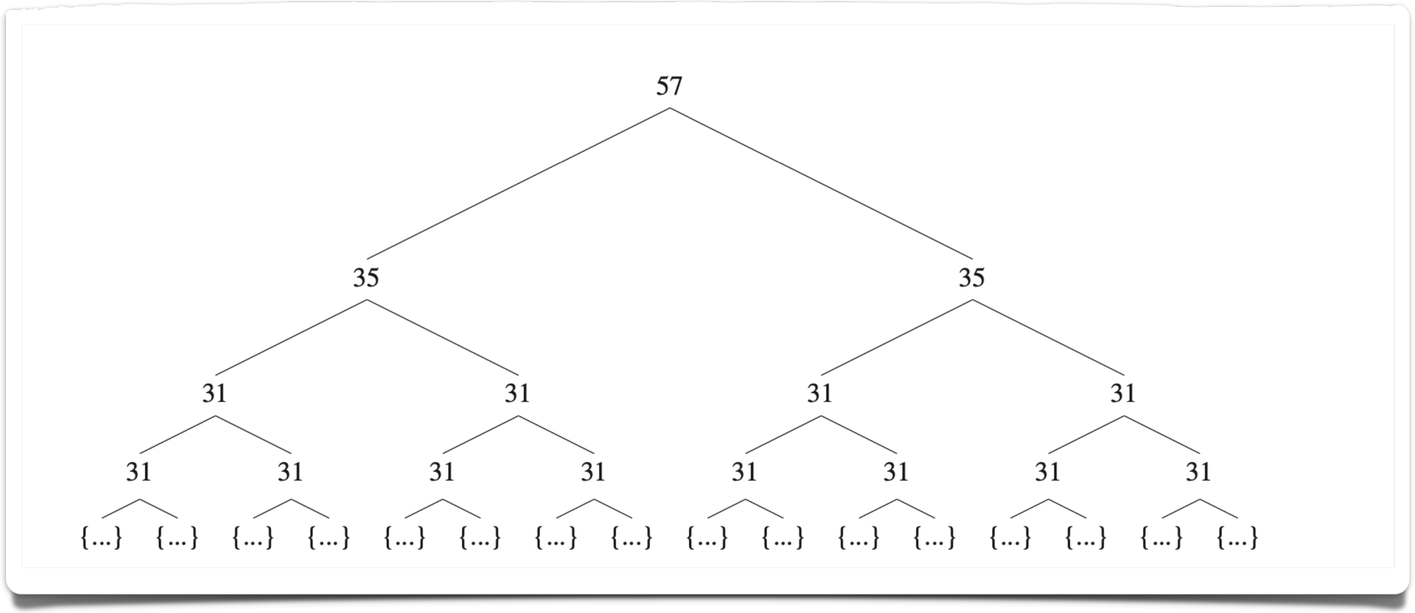

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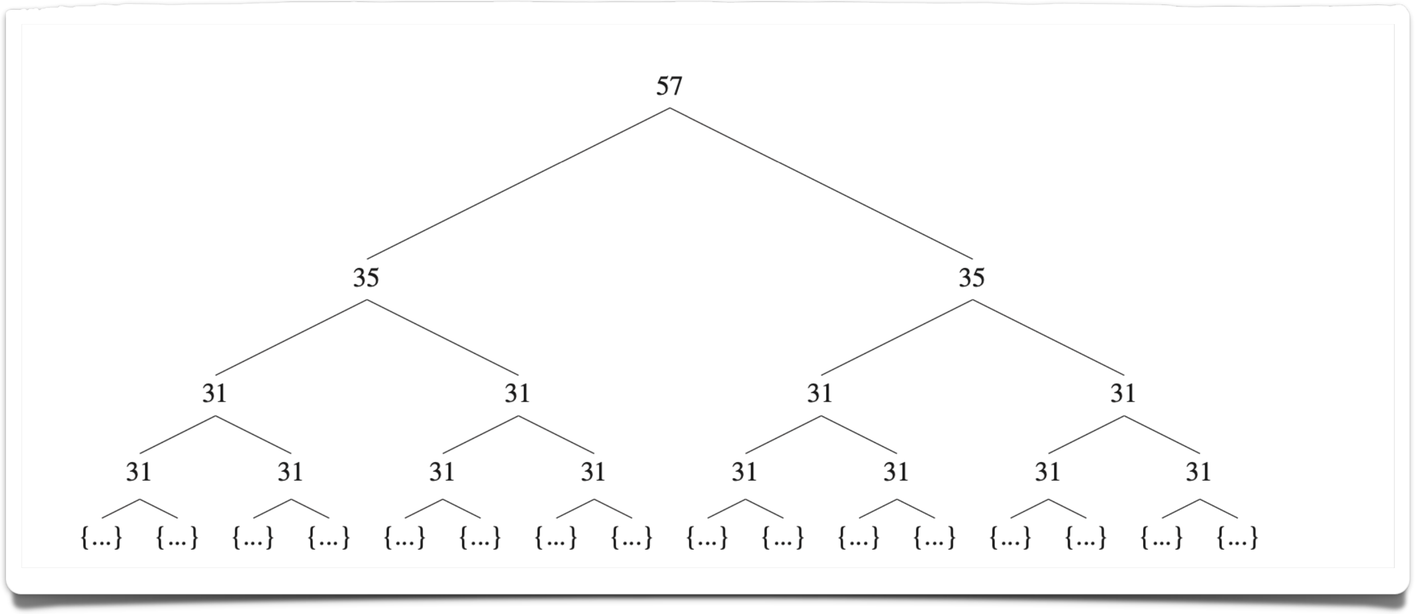

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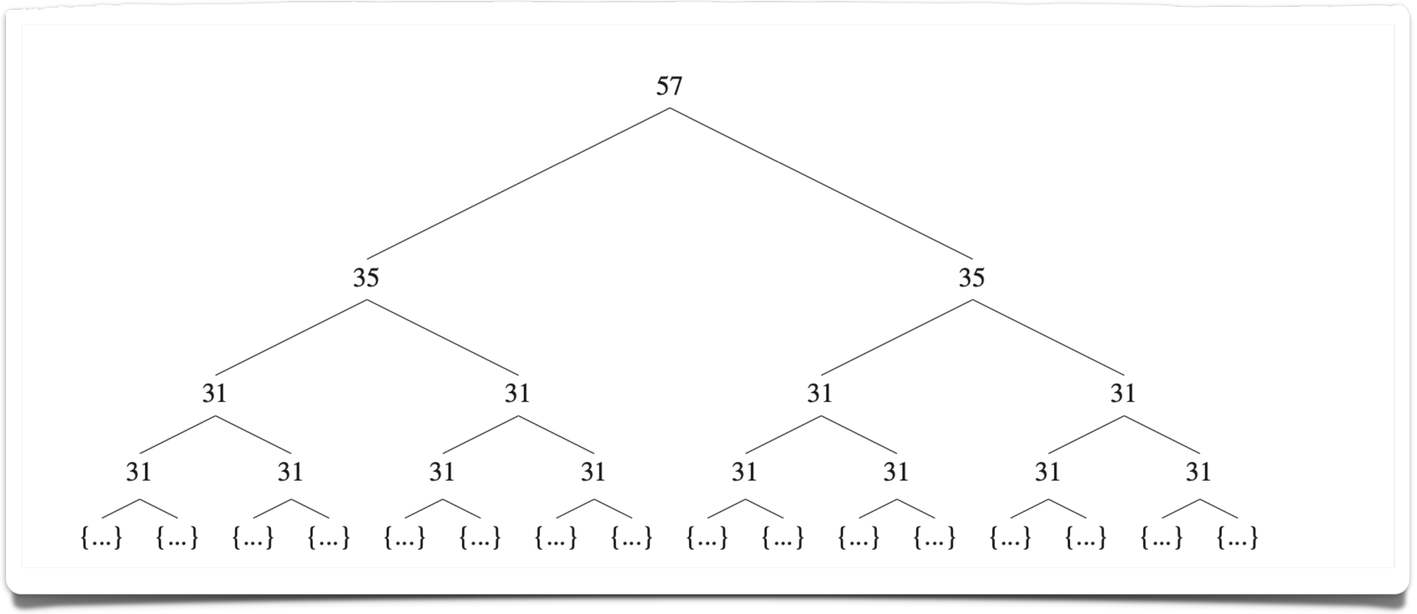

Branch and Bound

- Beschneide Enumerationsbaum

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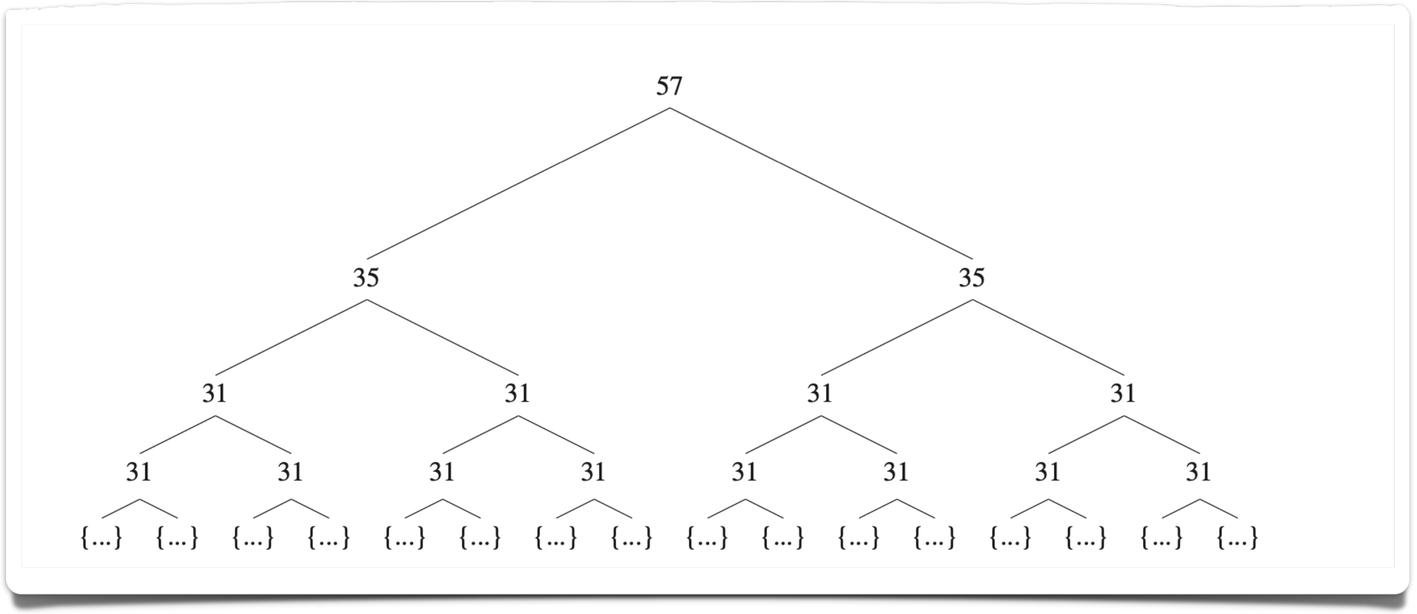
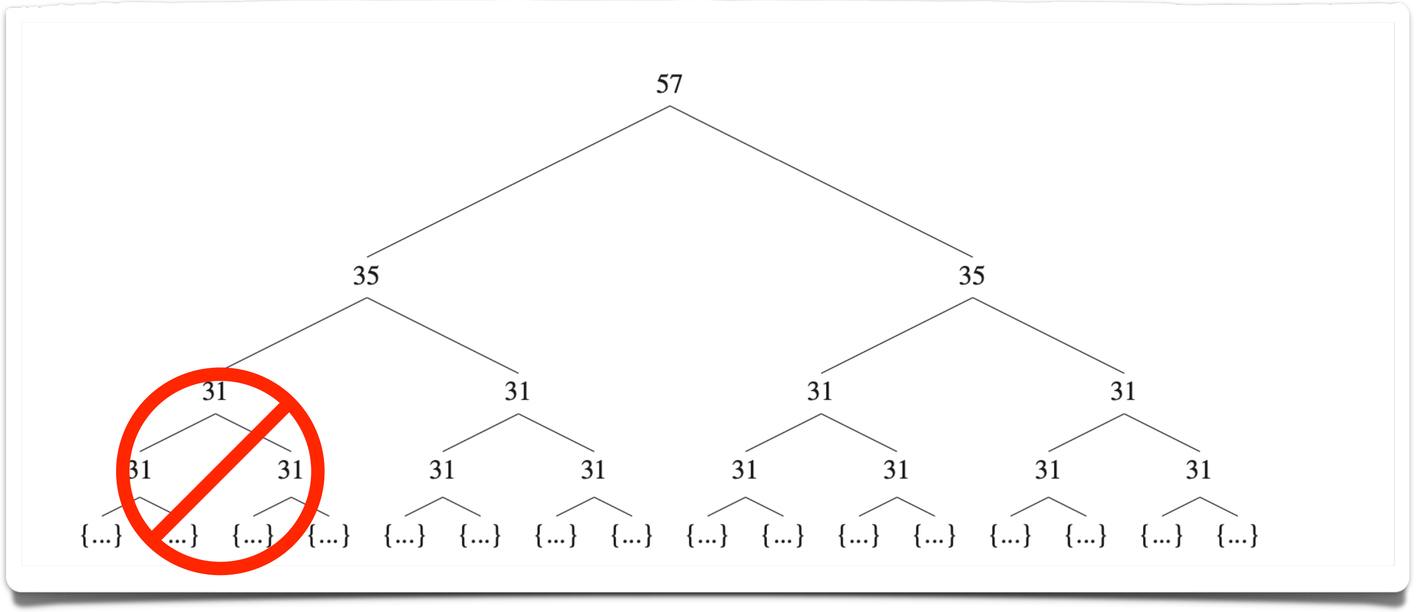


Branch and Bound

- Beschneide Enumerationsbaum
- Verfolge Mindest- und Maximalwerte

Branch-and-Bound



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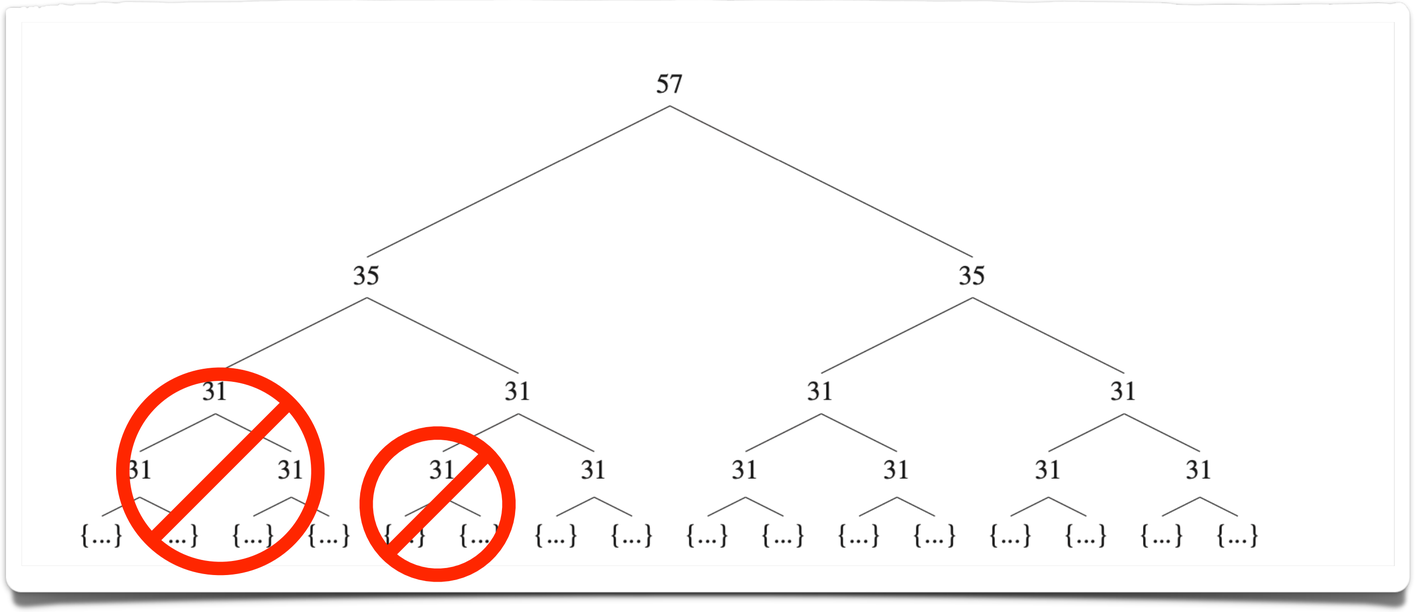

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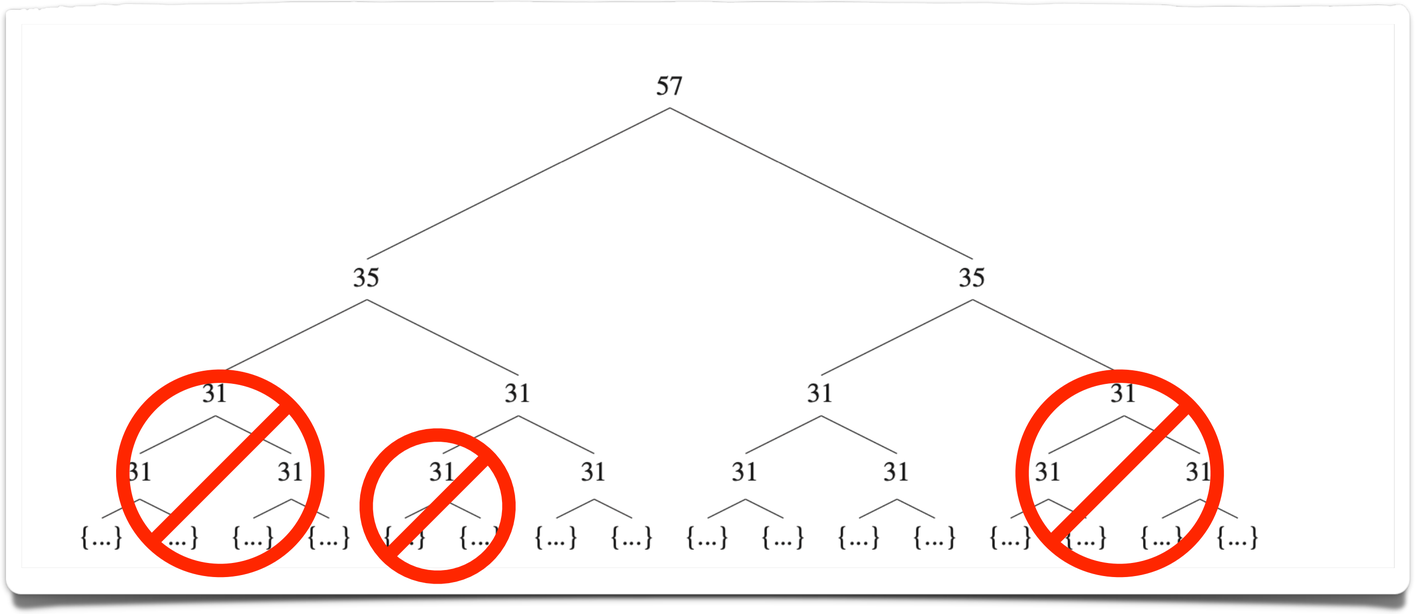

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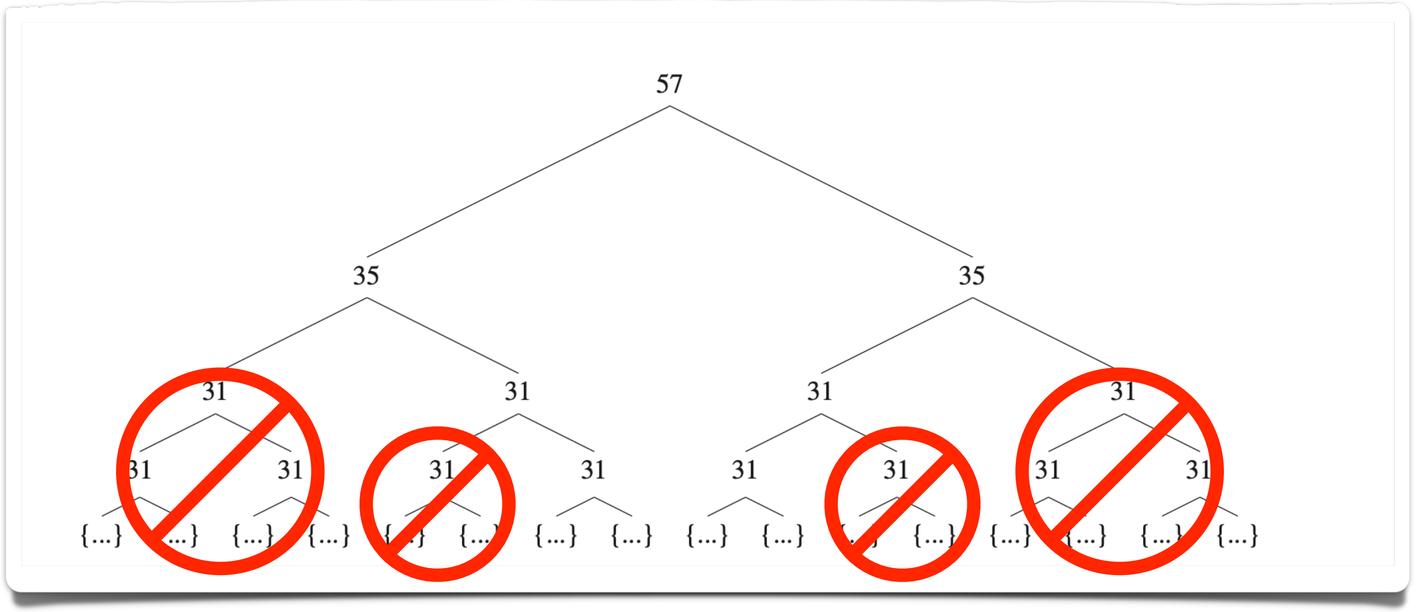
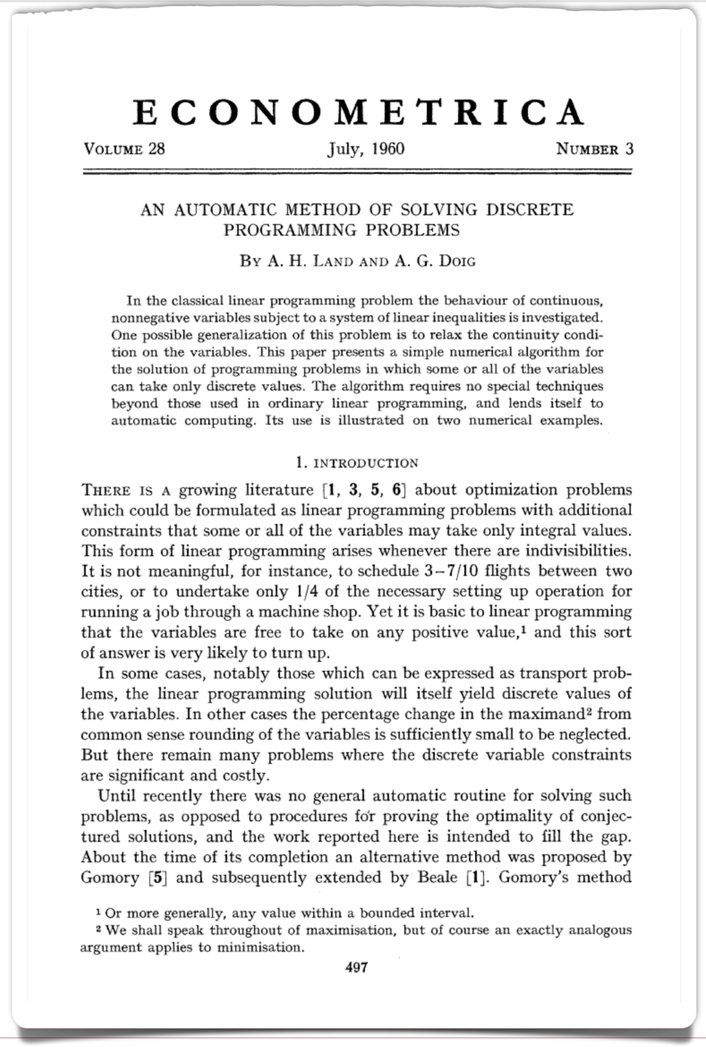


Branch and Bound

- Beschneide Enumerationsbaum
- Verfolge Mindest- und Maximalwerte

Historie

Historie



ECONOMETRICA

VOLUME 28 July, 1960 NUMBER 3

AN AUTOMATIC METHOD OF SOLVING DISCRETE PROGRAMMING PROBLEMS

BY A. H. LAND AND A. G. DOIG

In the classical linear programming problem the behaviour of continuous, nonnegative variables subject to a system of linear inequalities is investigated. One possible generalization of this problem is to relax the continuity condition on the variables. This paper presents a simple numerical algorithm for the solution of programming problems in which some or all of the variables can take only discrete values. The algorithm requires no special techniques beyond those used in ordinary linear programming, and lends itself to automatic computing. Its use is illustrated on two numerical examples.

1. INTRODUCTION

THERE IS A growing literature [**1**, **3**, **5**, **6**] about optimization problems which could be formulated as linear programming problems with additional constraints that some or all of the variables may take only integral values. This form of linear programming arises whenever there are indivisibilities. It is not meaningful, for instance, to schedule 3--7/10 flights between two cities, or to undertake only 1/4 of the necessary setting up operation for running a job through a machine shop. Yet it is basic to linear programming that the variables are free to take on any positive value,[1] and this sort of answer is very likely to turn up.

In some cases, notably those which can be expressed as transport problems, the linear programming solution will itself yield discrete values of the variables. In other cases the percentage change in the maximand[2] from common sense rounding of the variables is sufficiently small to be neglected. But there remain many problems where the discrete variable constraints are significant and costly.

Until recently there was no general automatic routine for solving such problems, as opposed to procedures for proving the optimality of conjectured solutions, and the work reported here is intended to fill the gap. About the time of its completion an alternative method was proposed by Gomory [**5**] and subsequently extended by Beale [**1**]. Gomory's method

[1] Or more generally, any value within a bounded interval.

[2] We shall speak throughout of maximisation, but of course an exactly analogous argument applies to minimisation.

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Historie

1960

ECONOMETRICA

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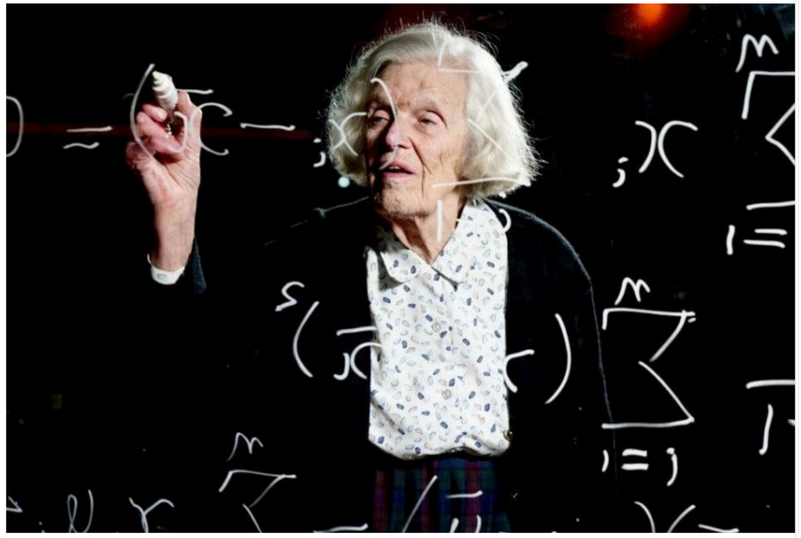

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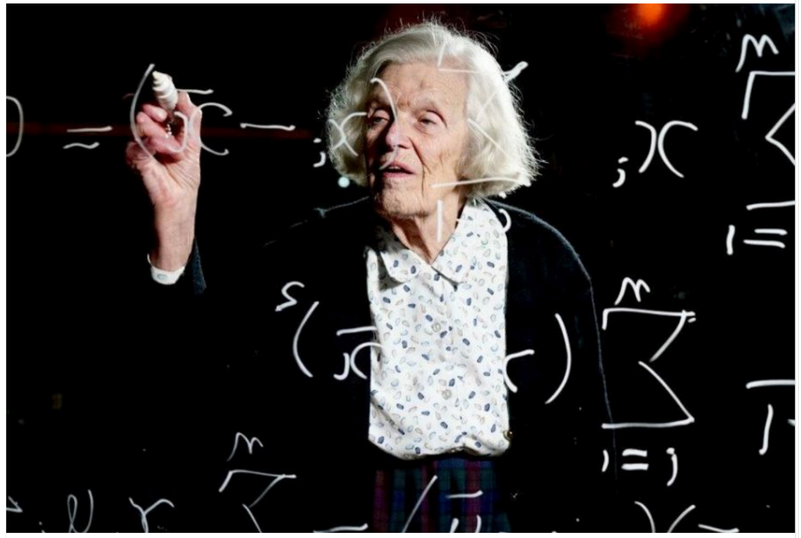

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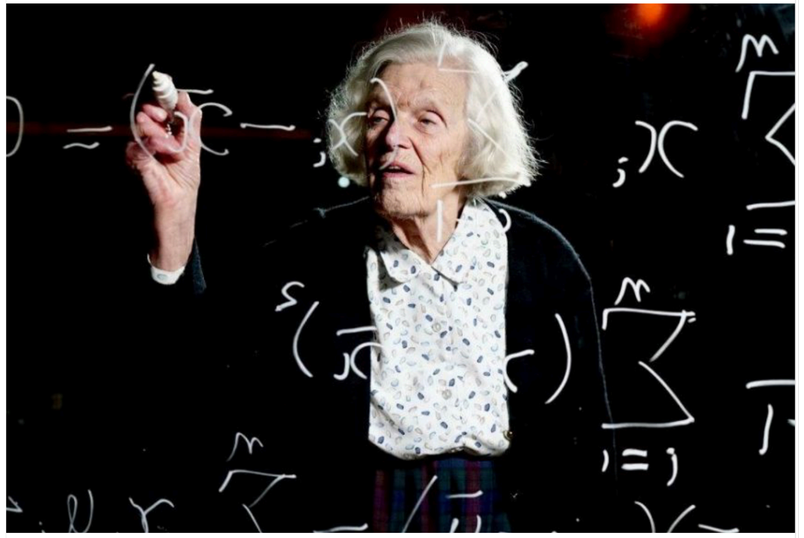

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In June 2019, Harcourt was made an Officer of the Order of Australia

3.2 Methodik

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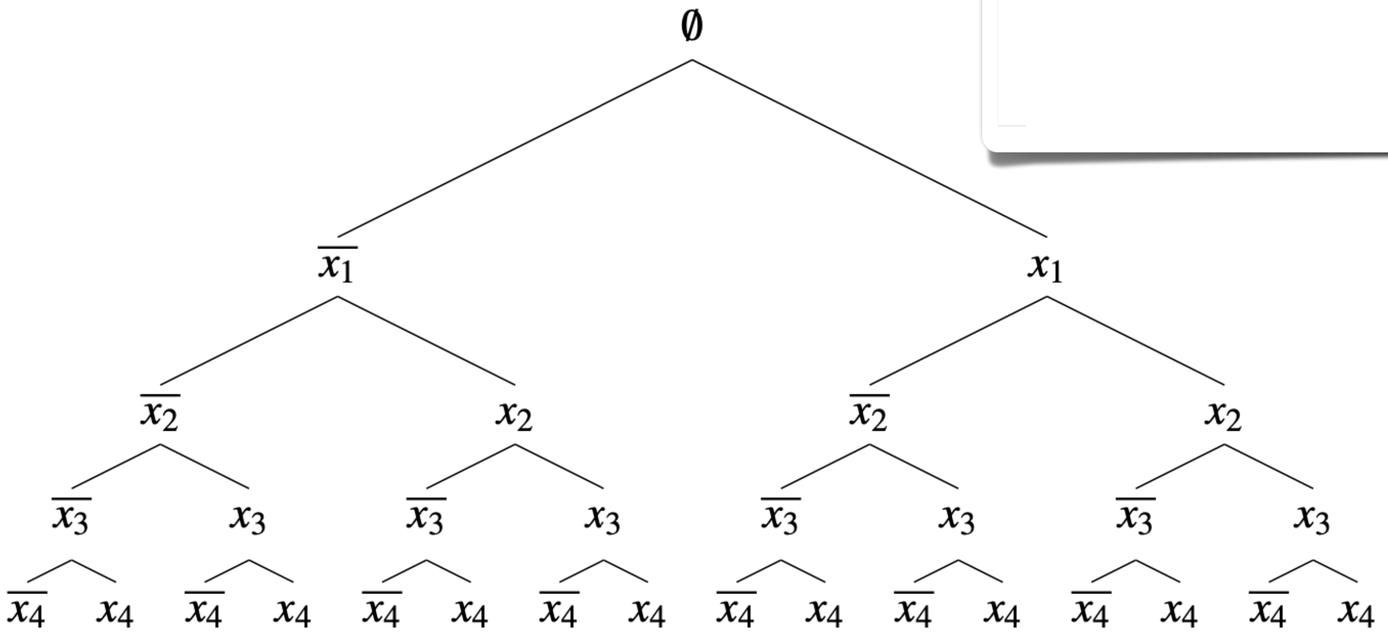

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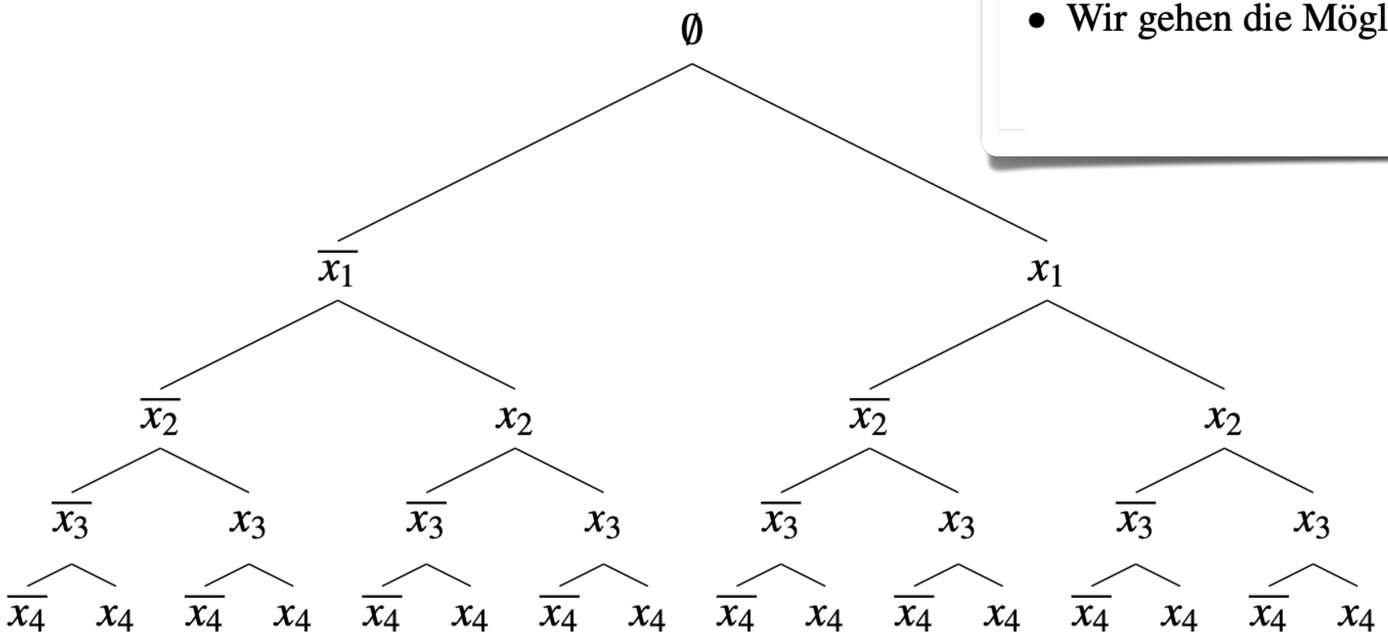

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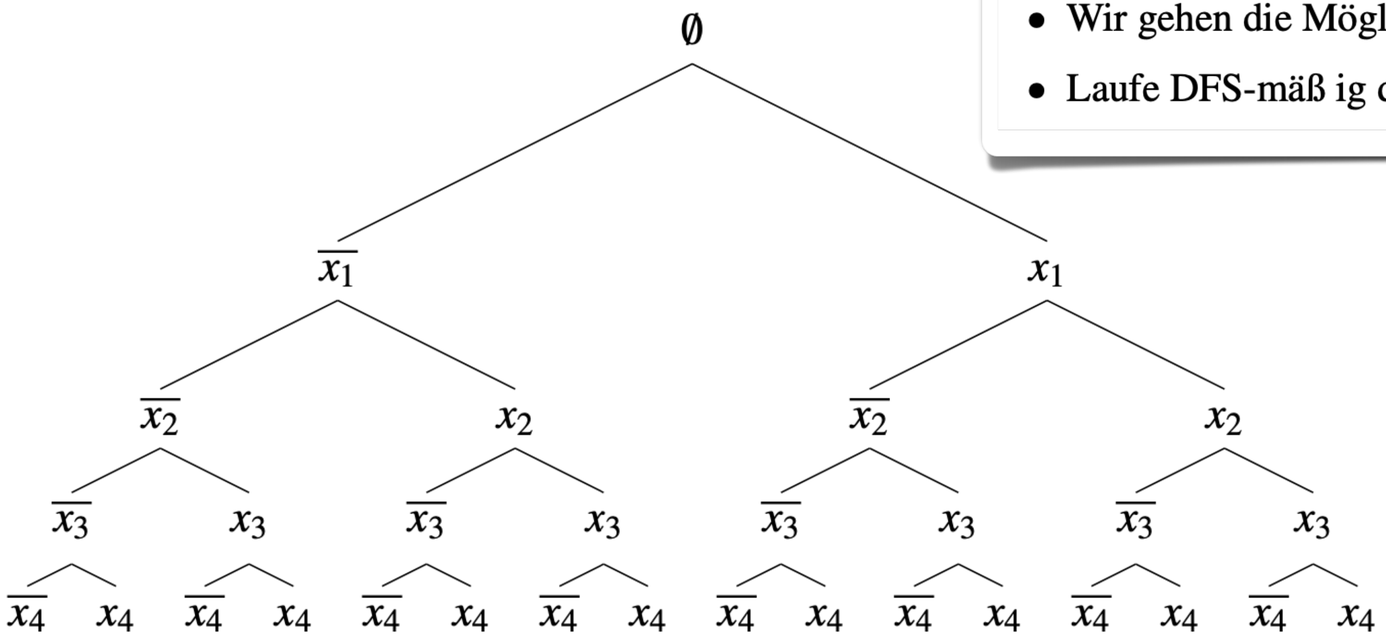

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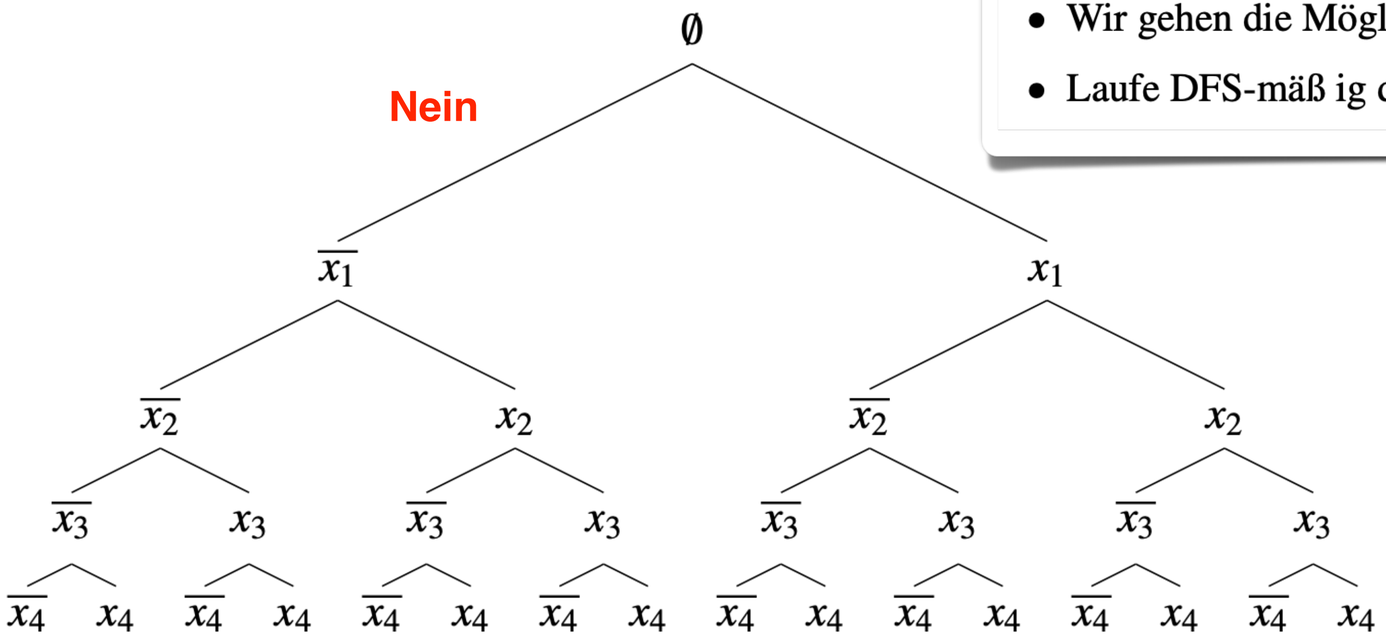

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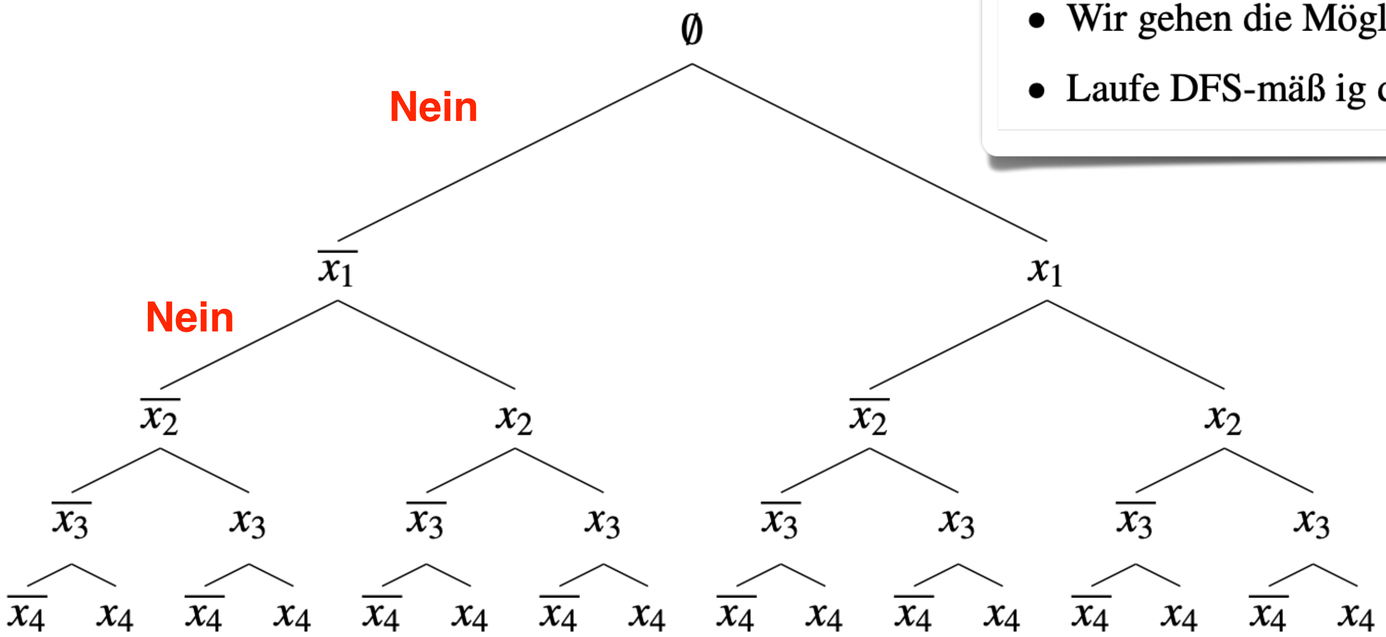

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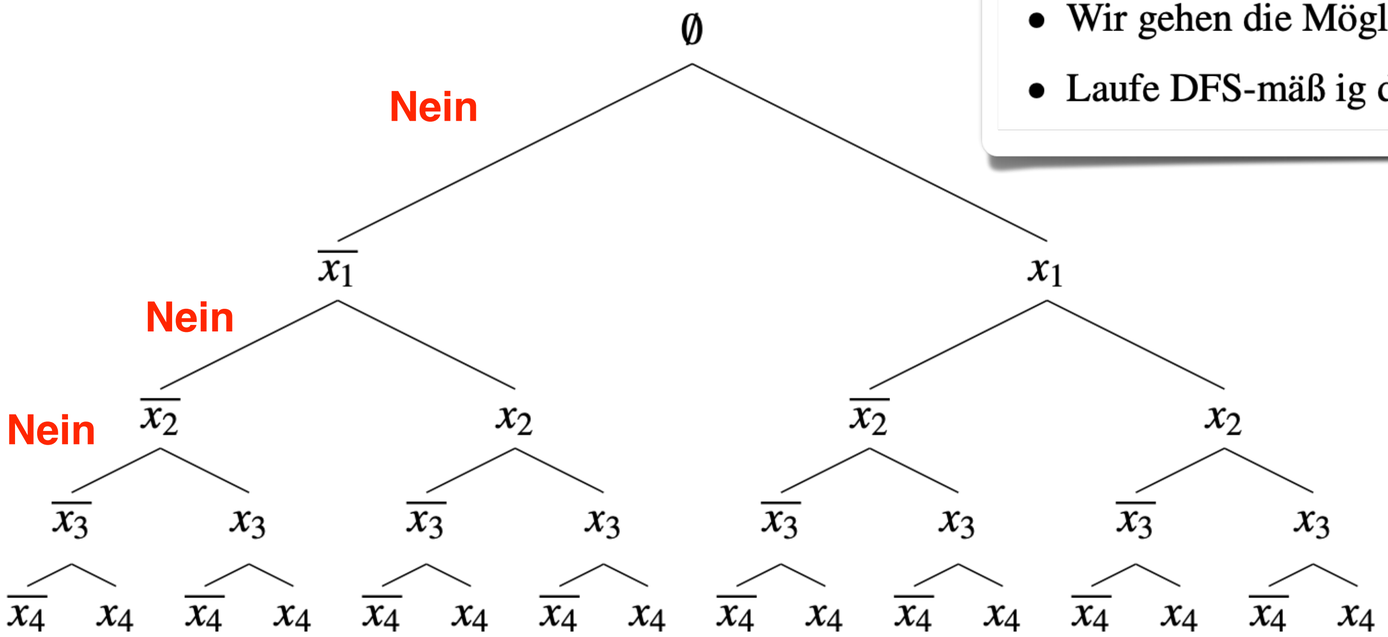

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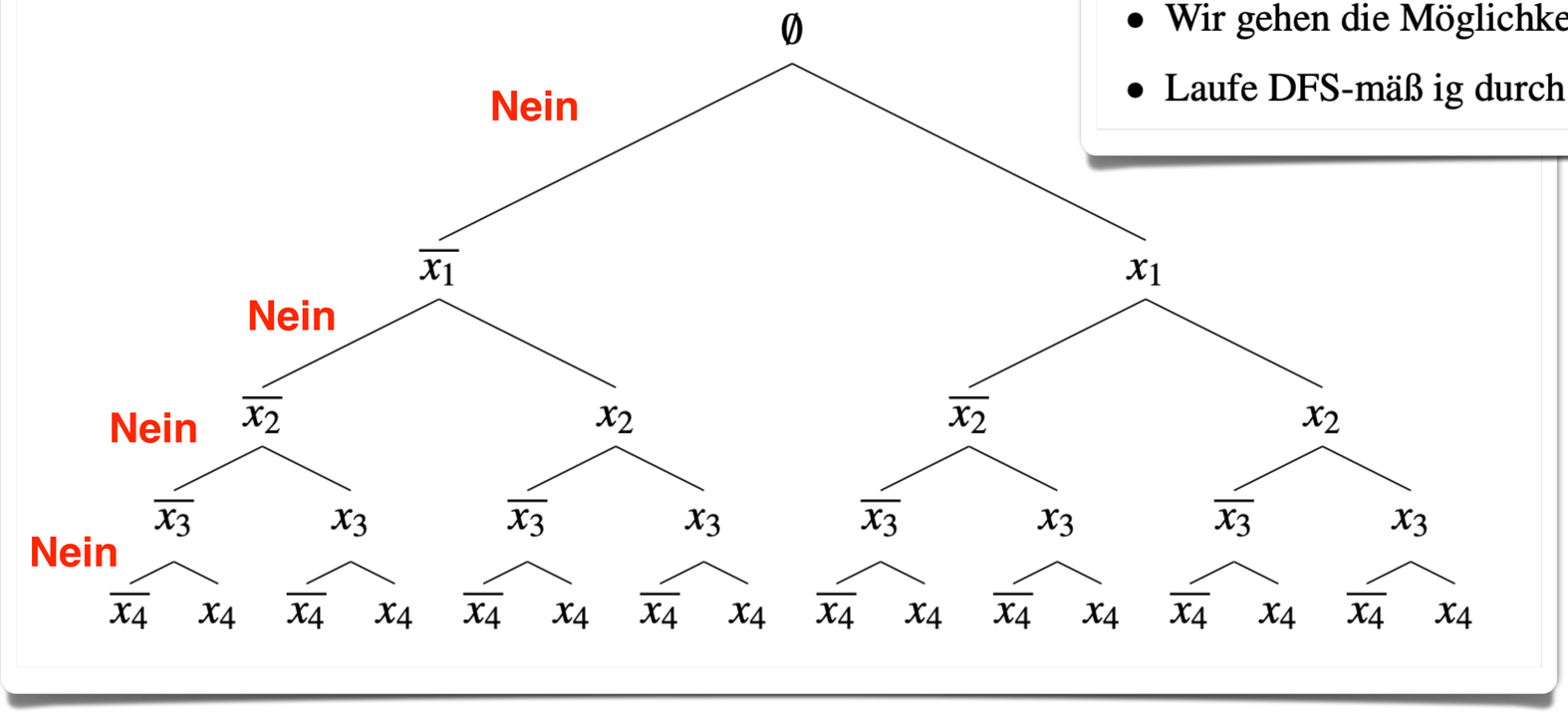

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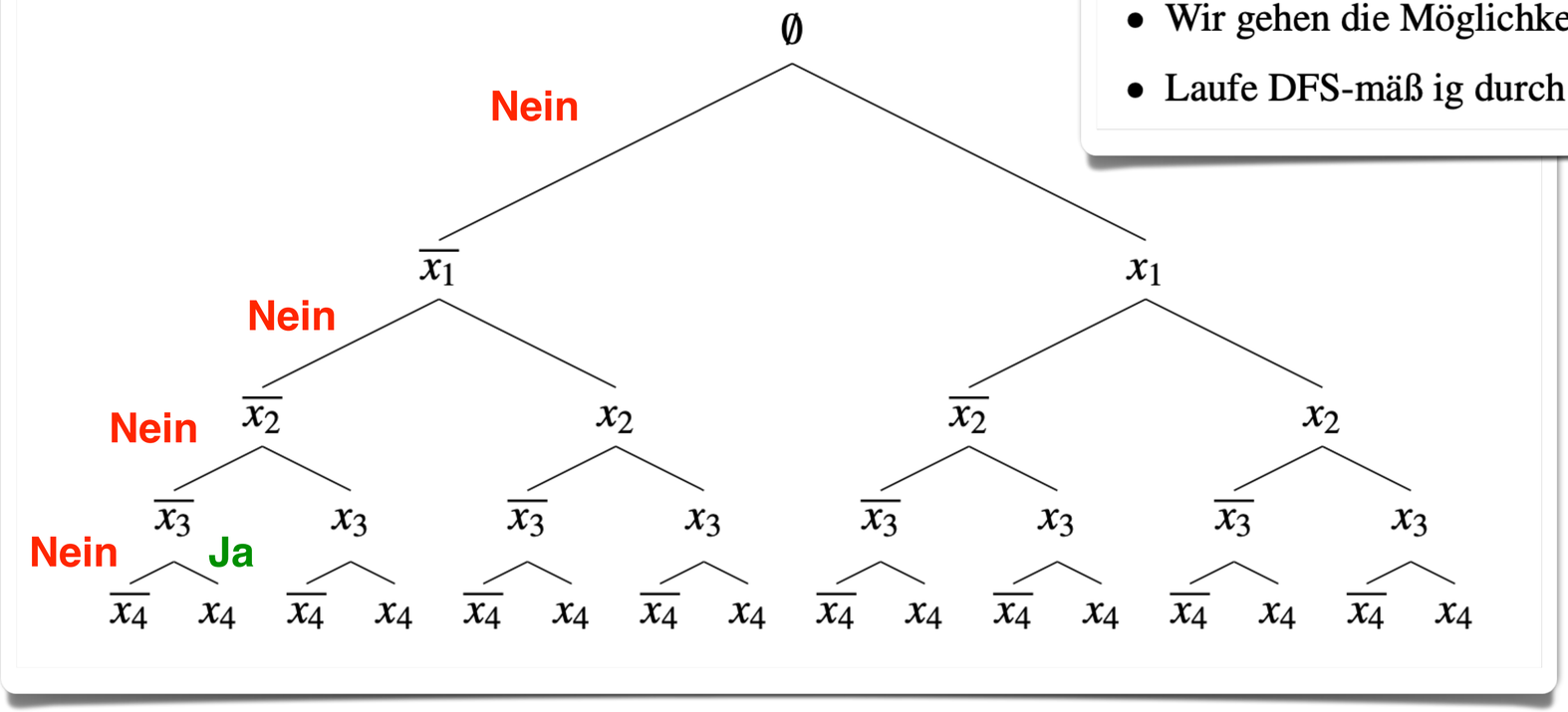

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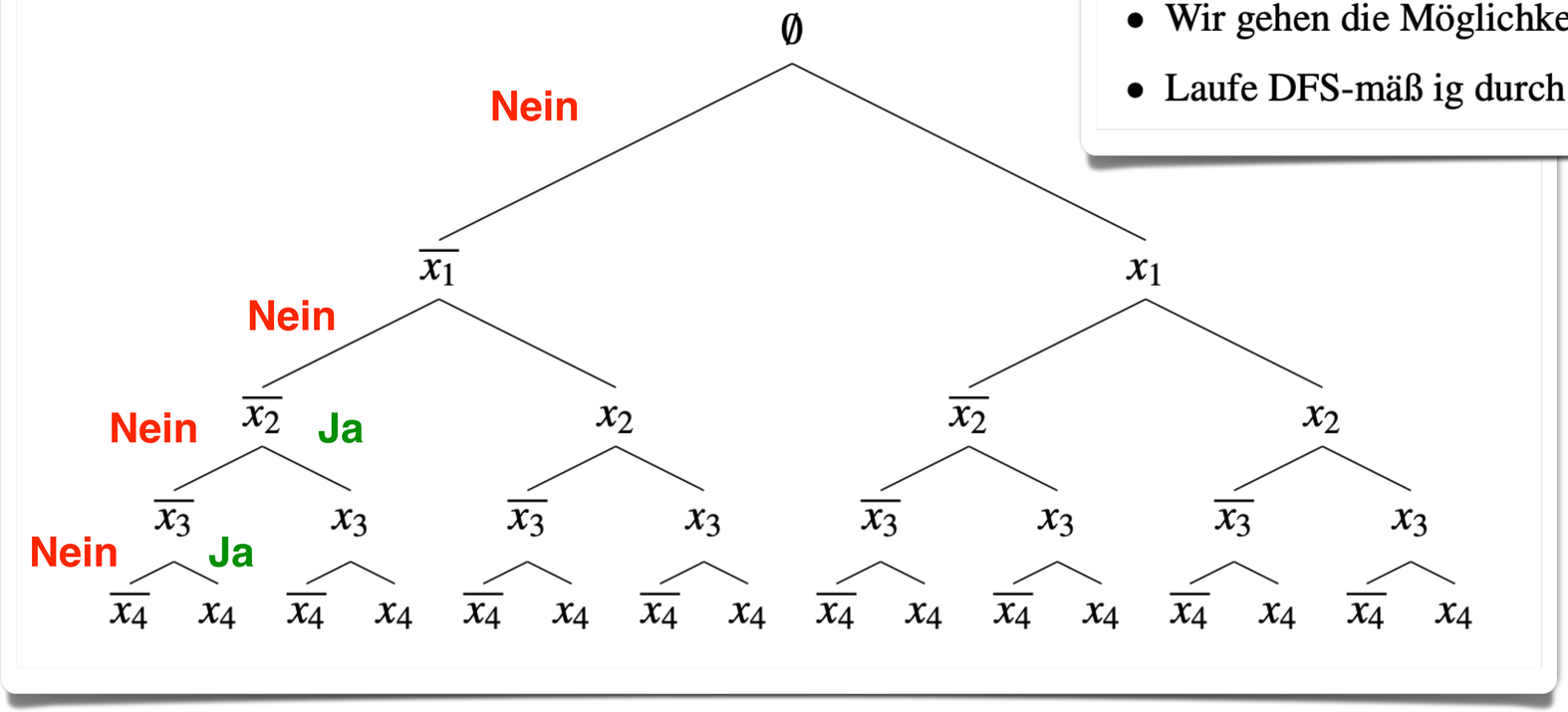

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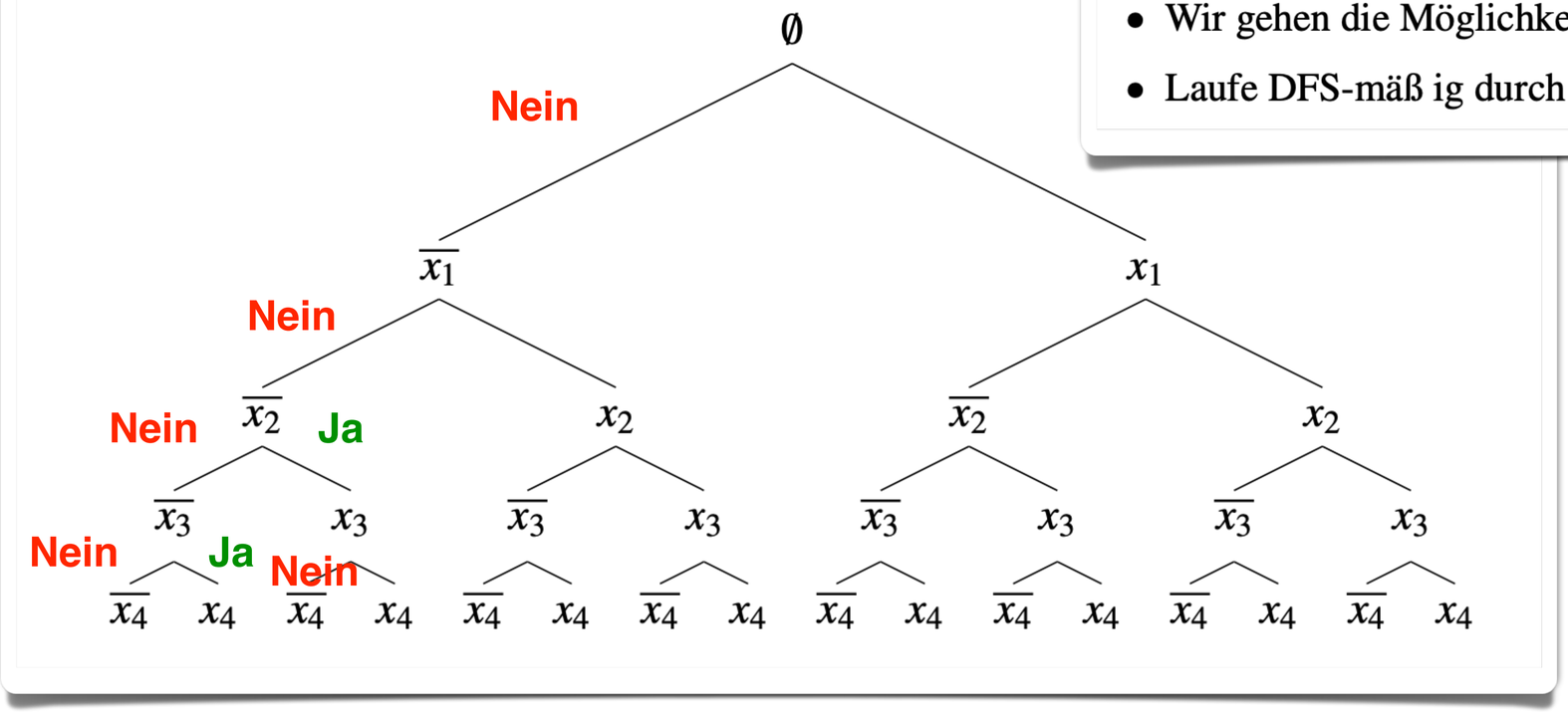

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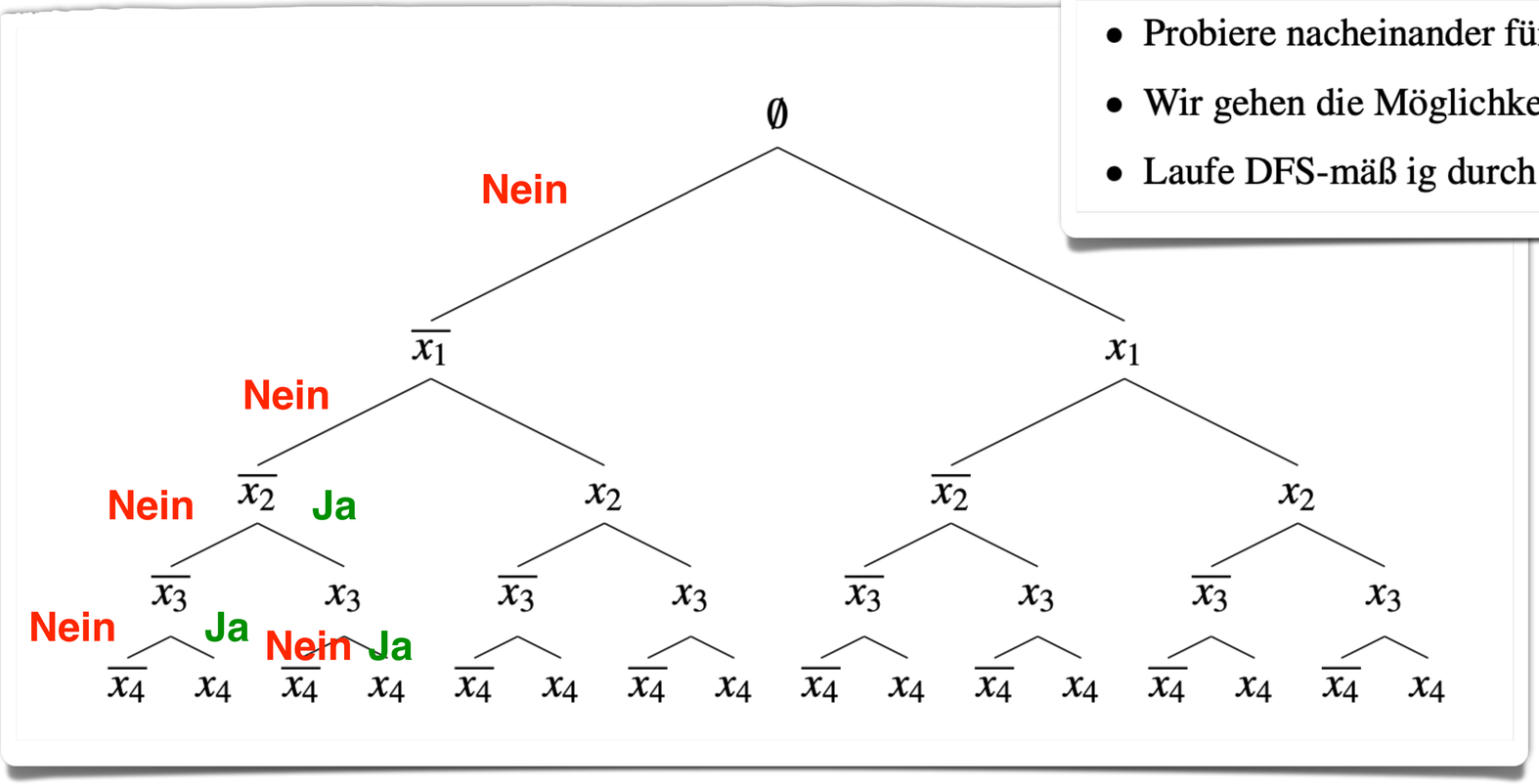

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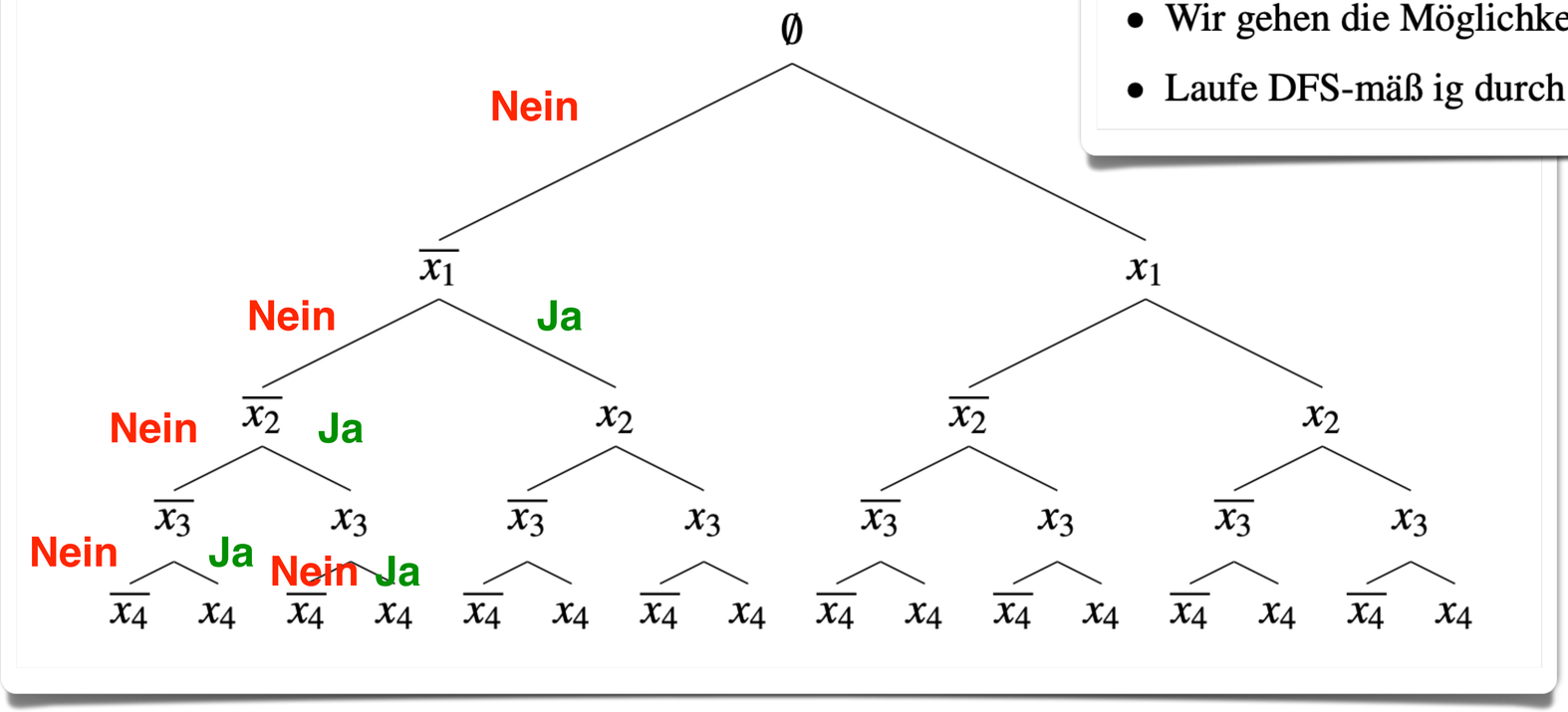

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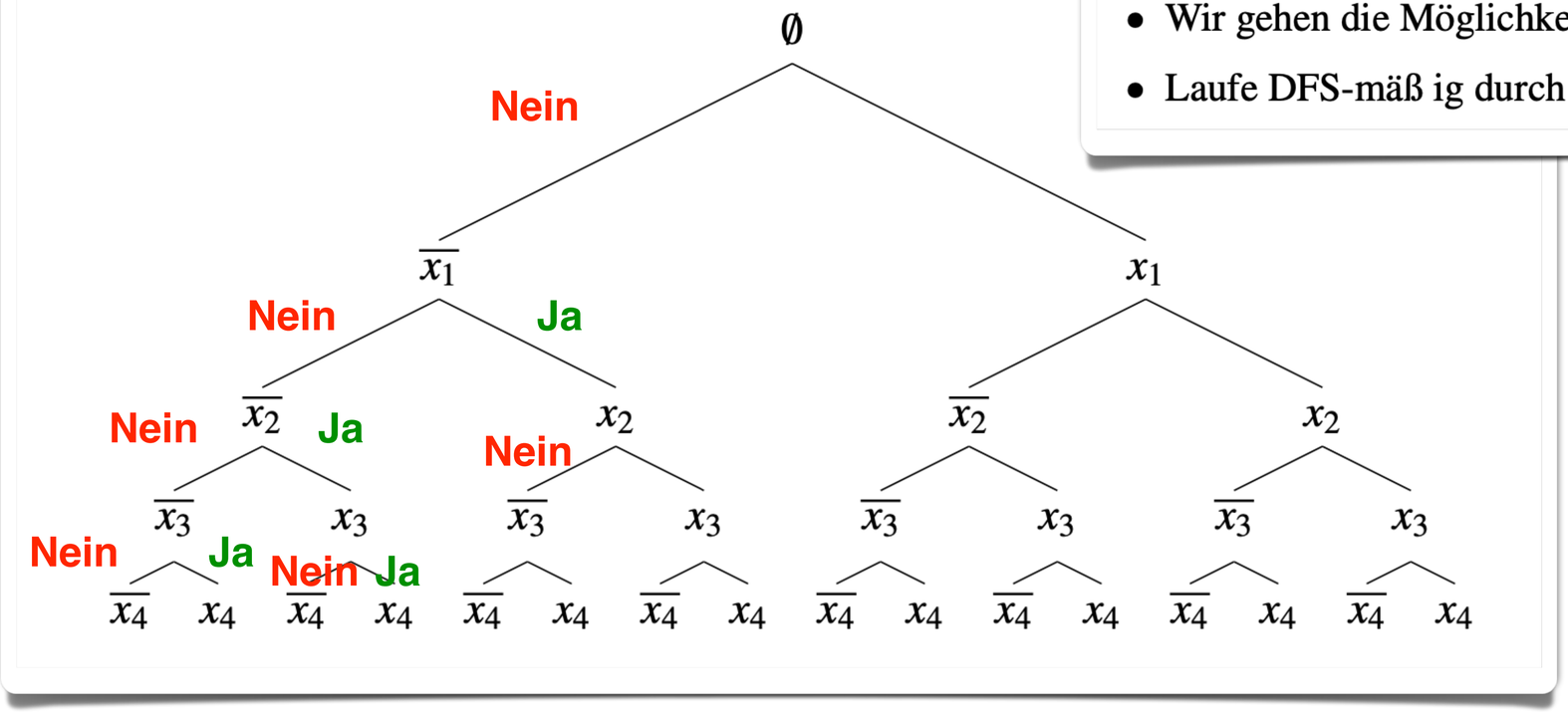

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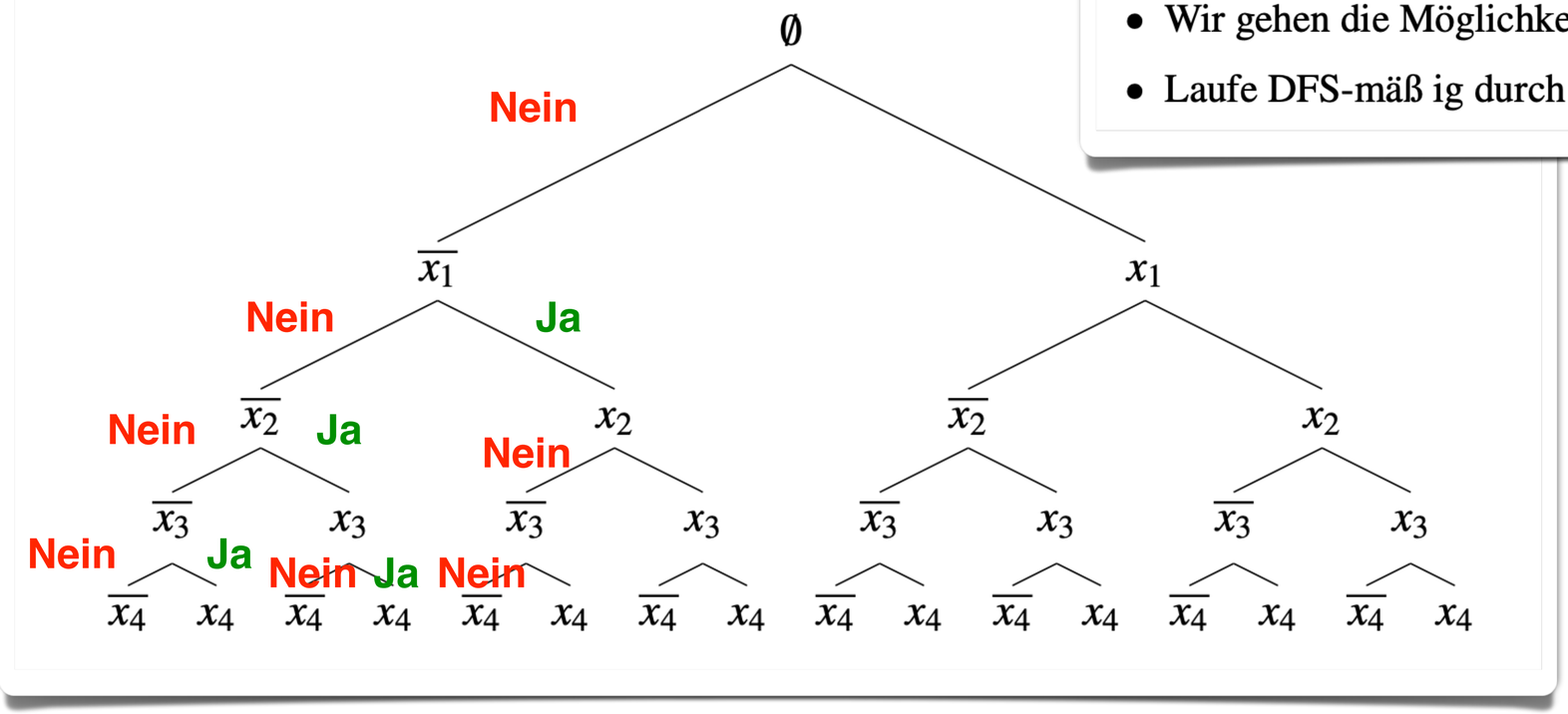

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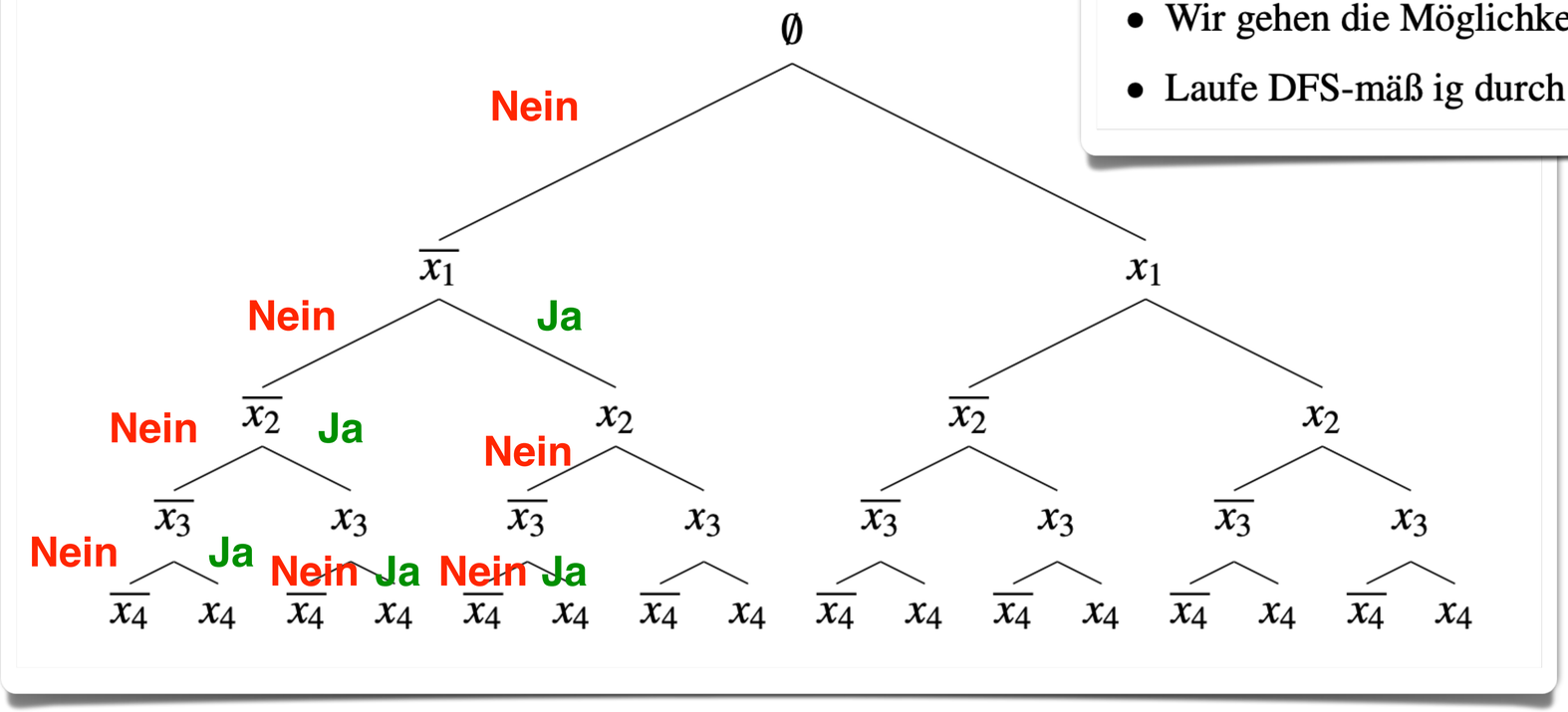

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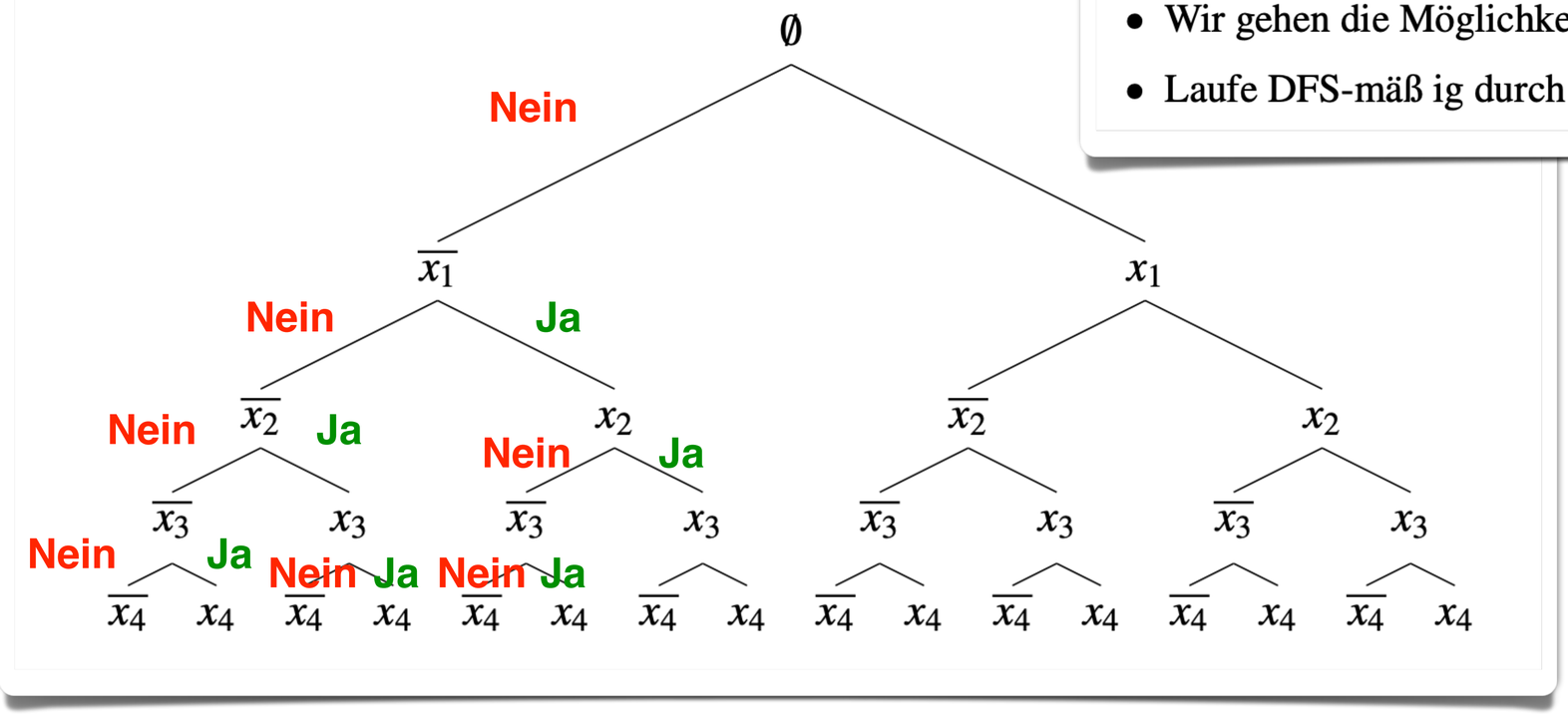

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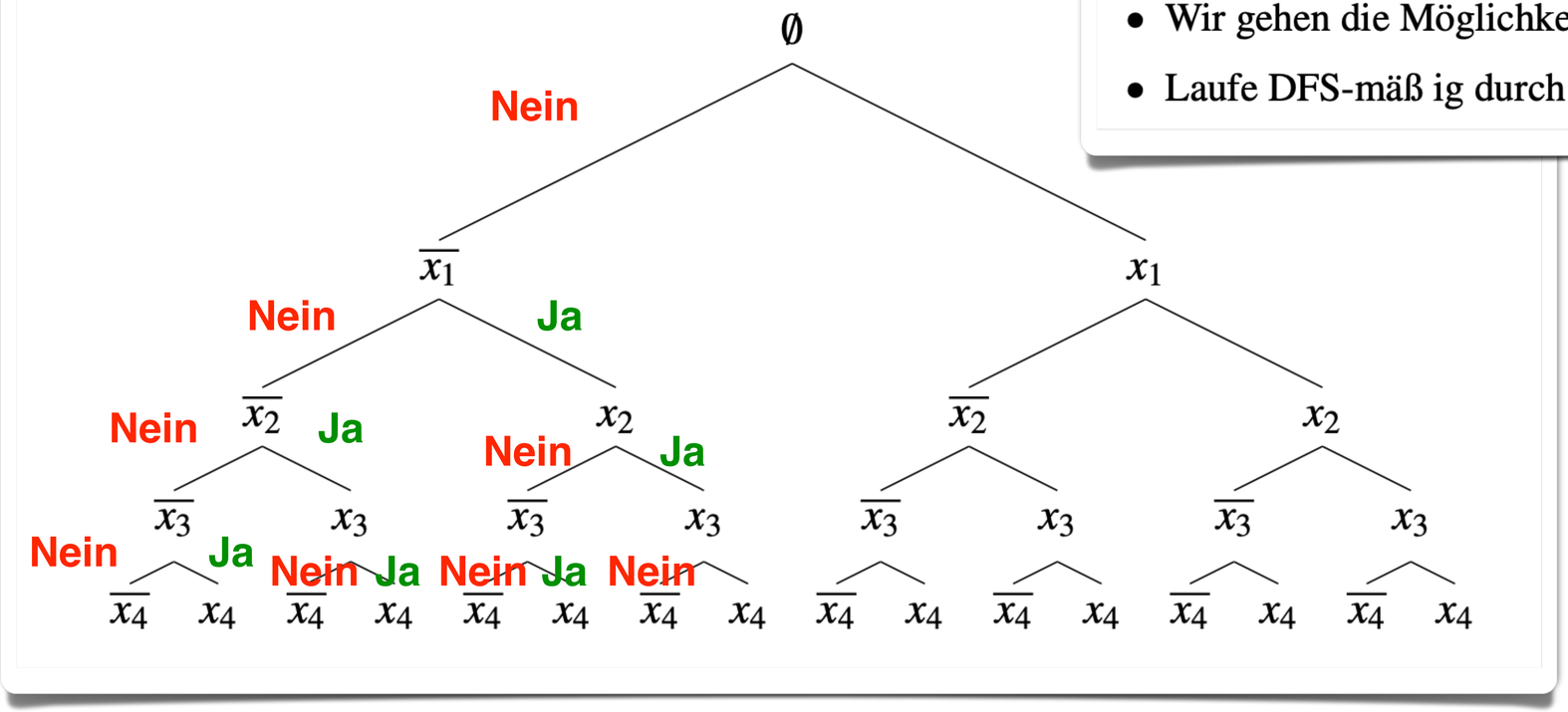

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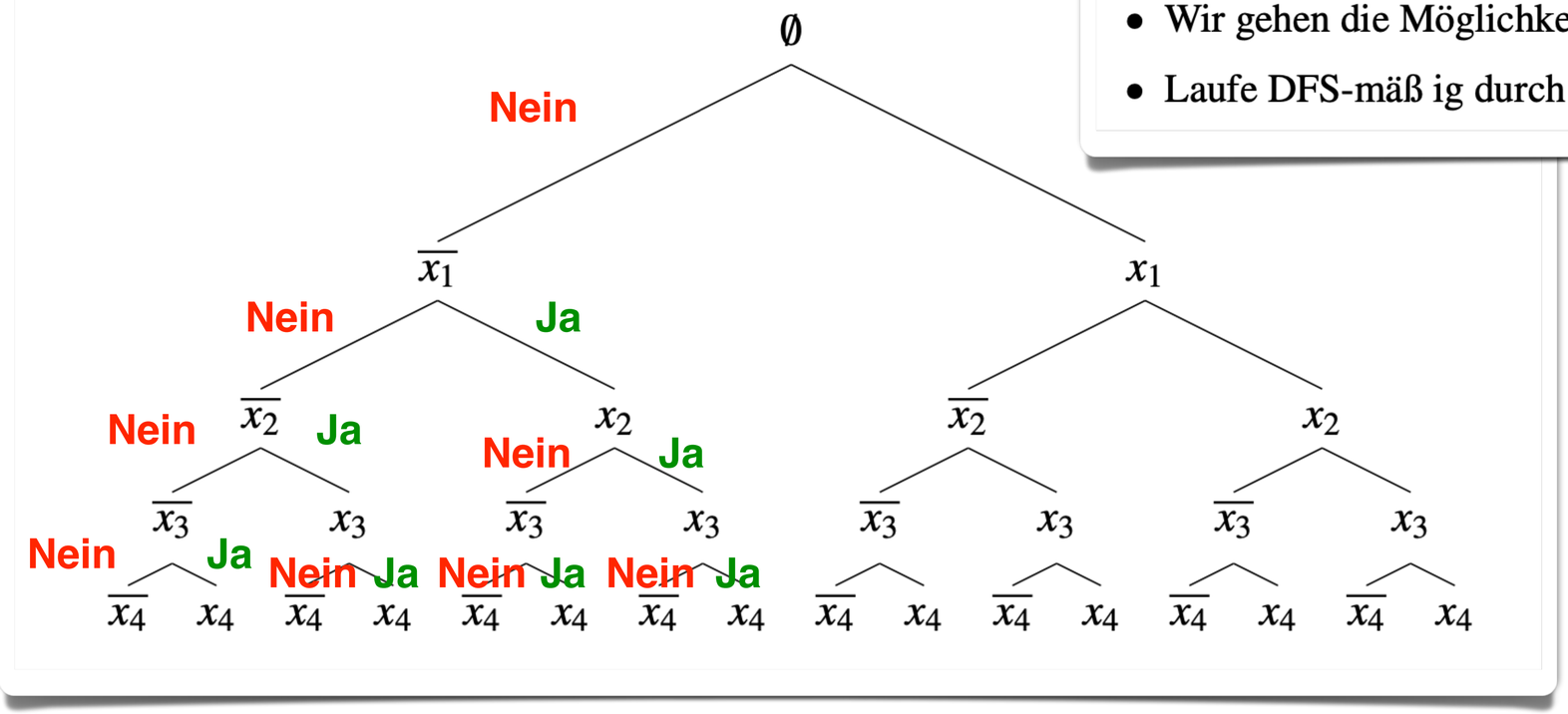

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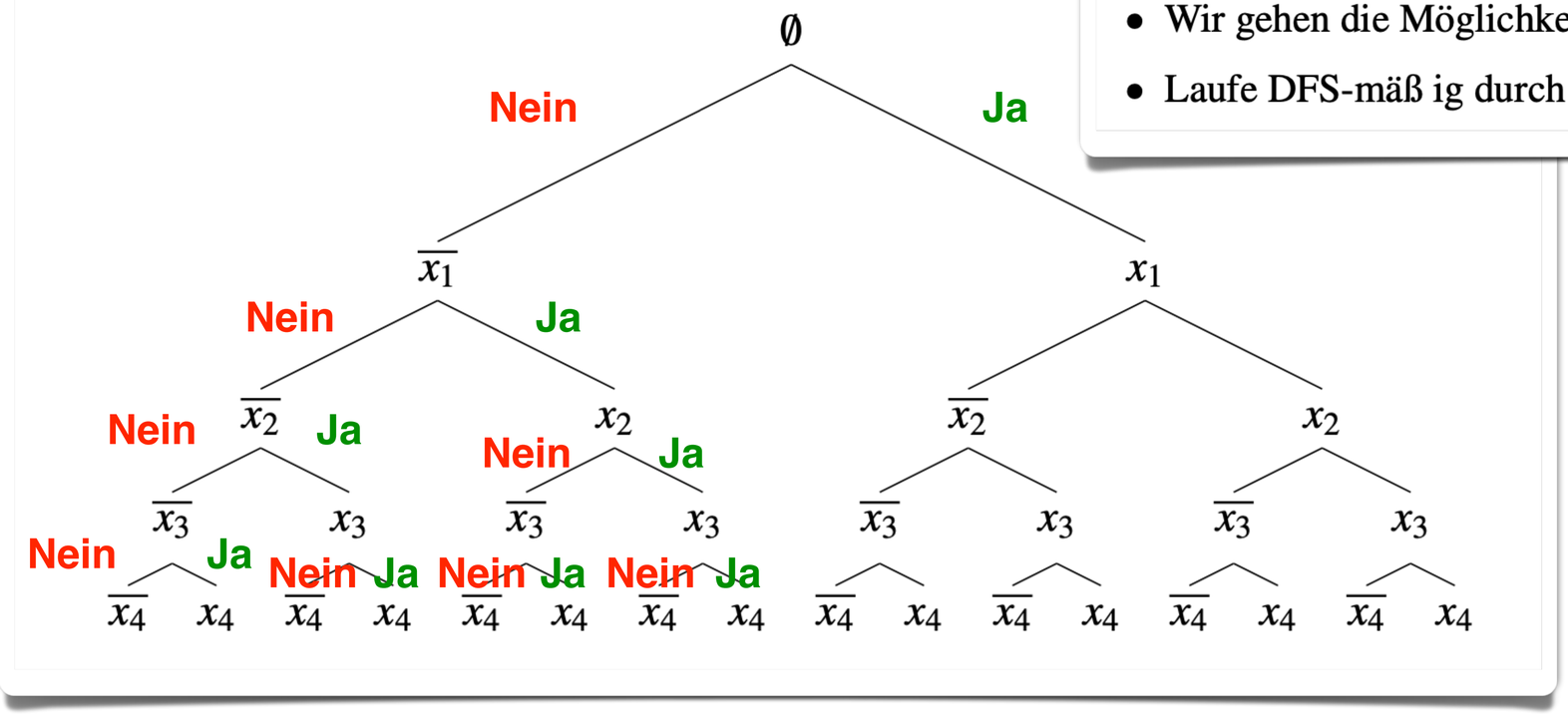

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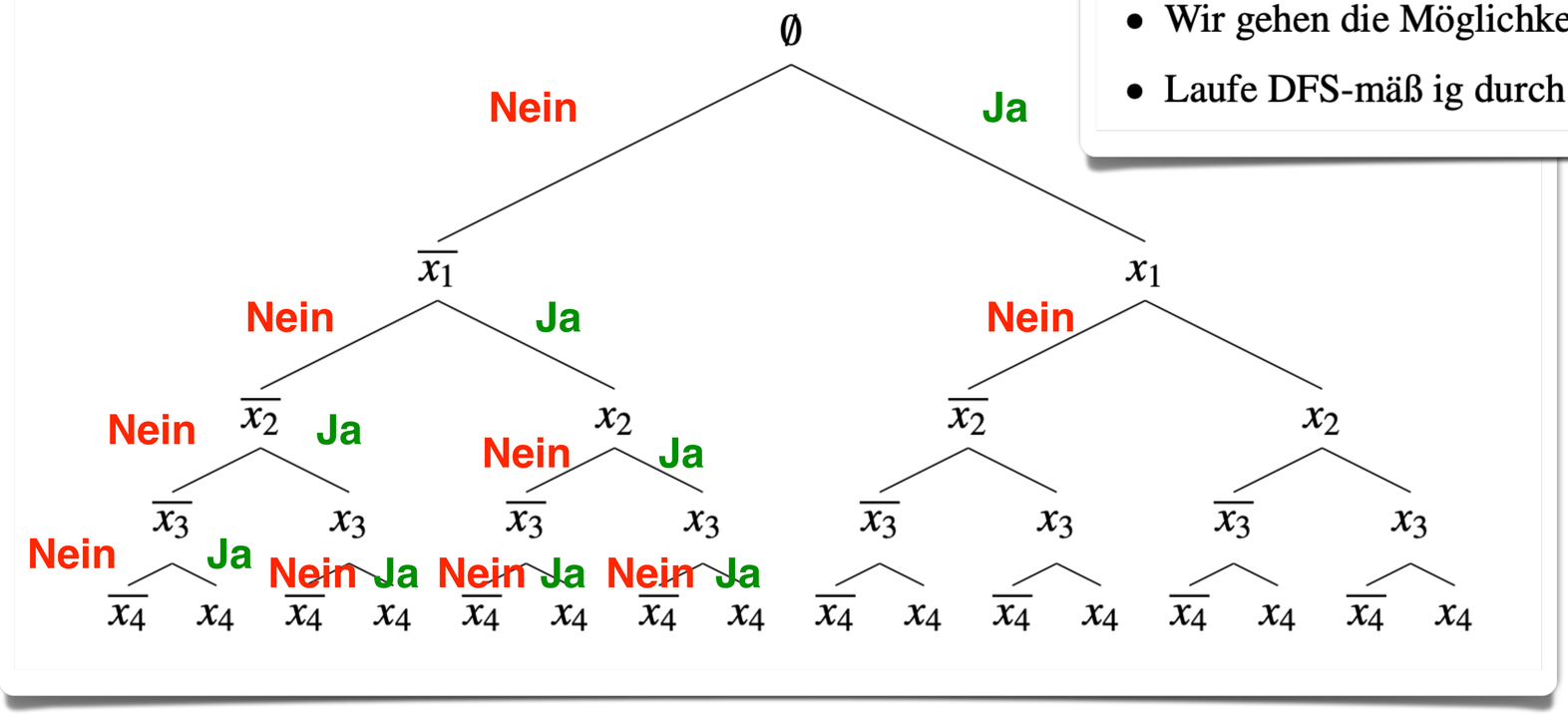

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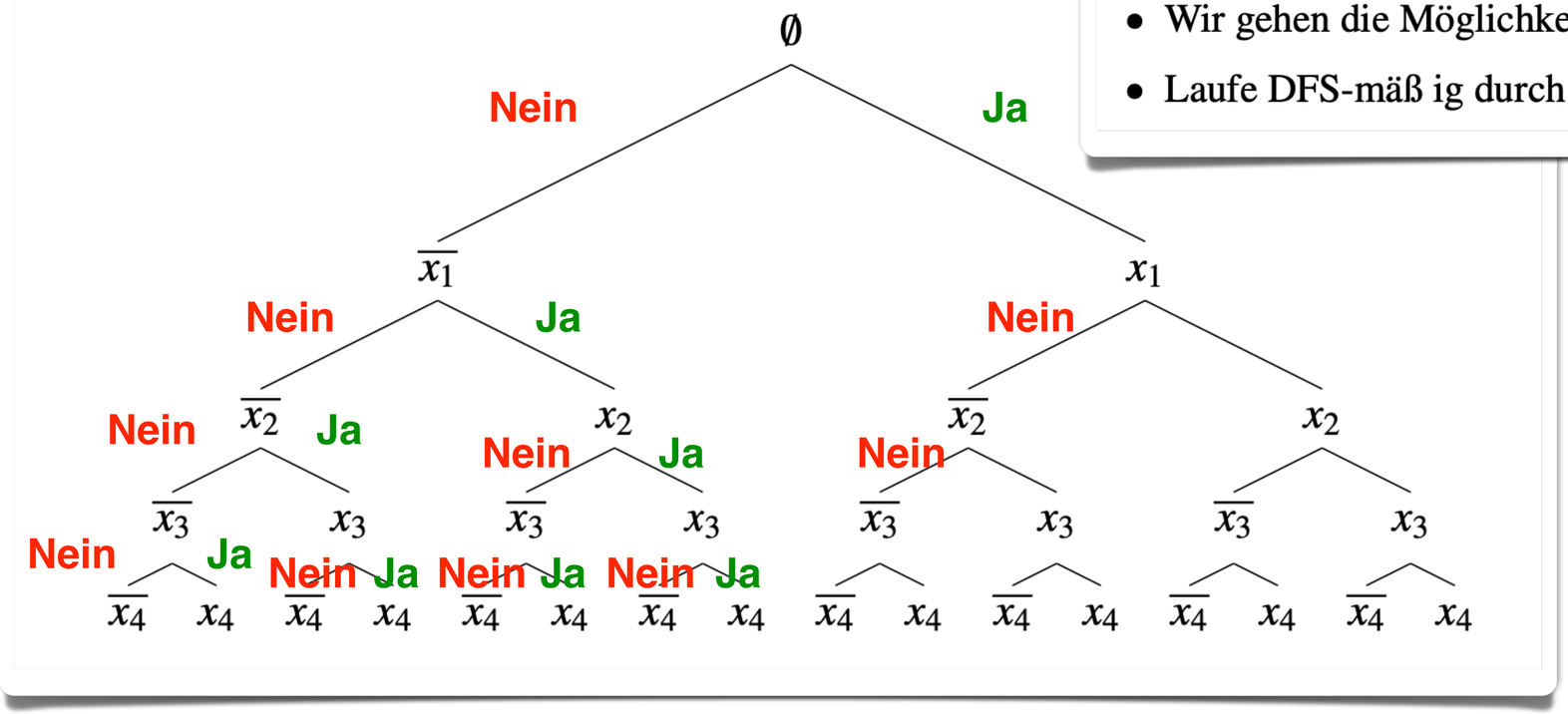

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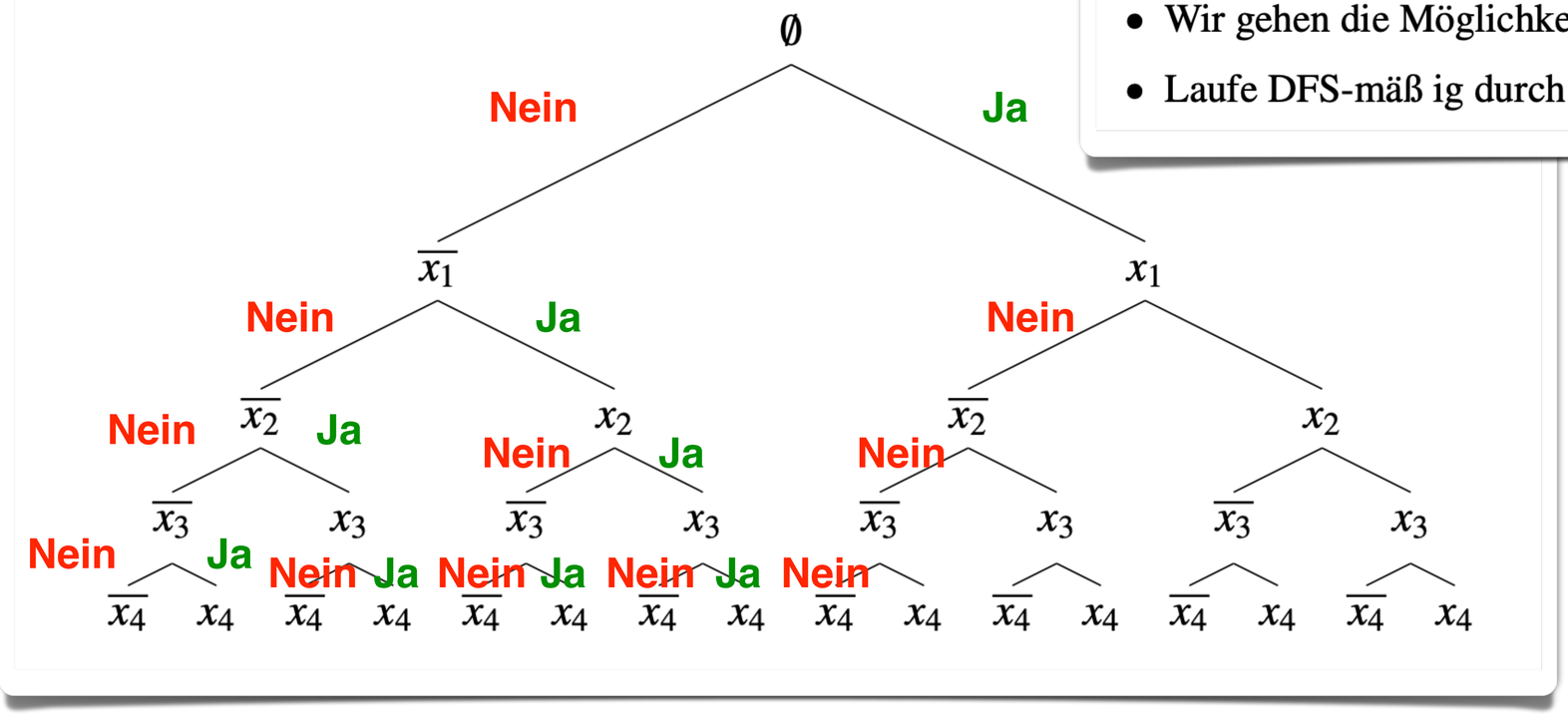

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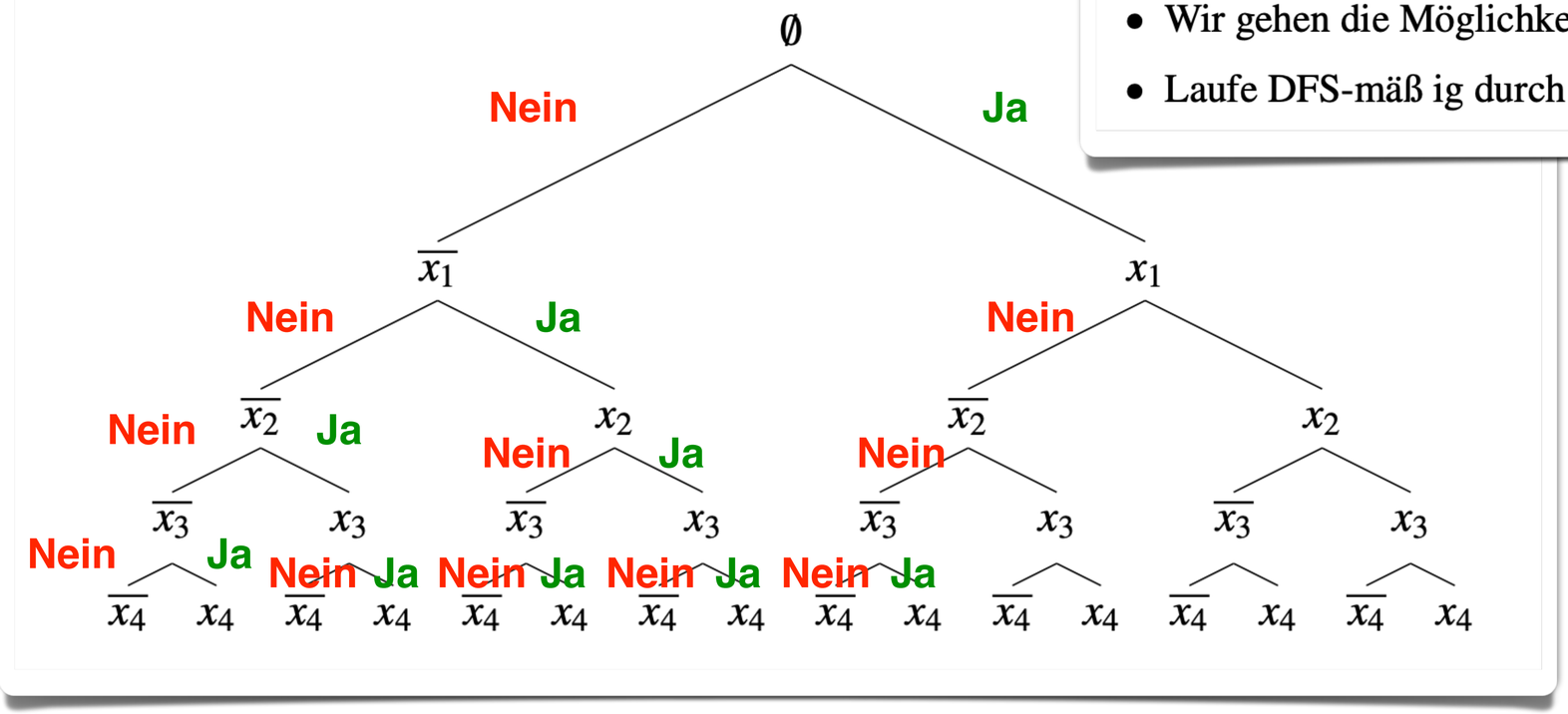

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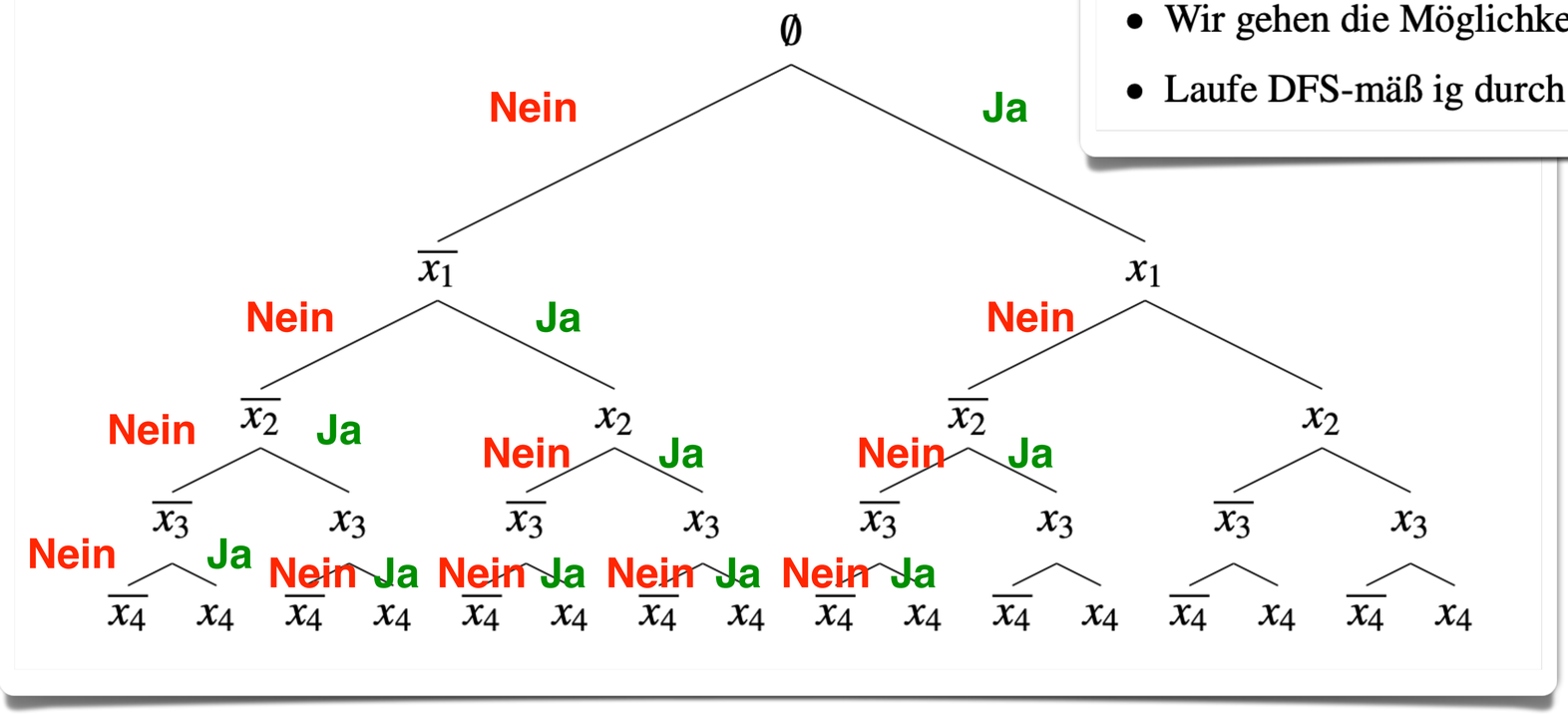

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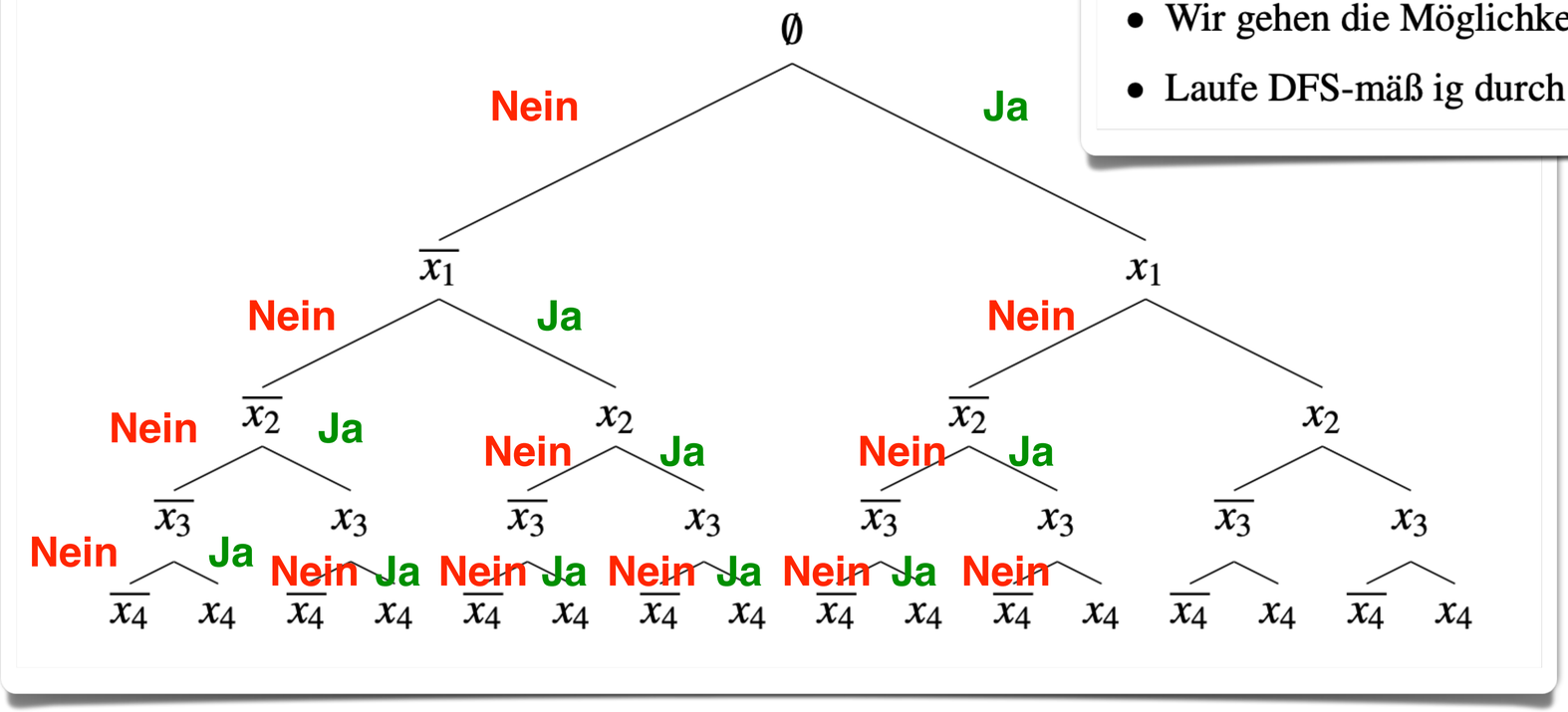

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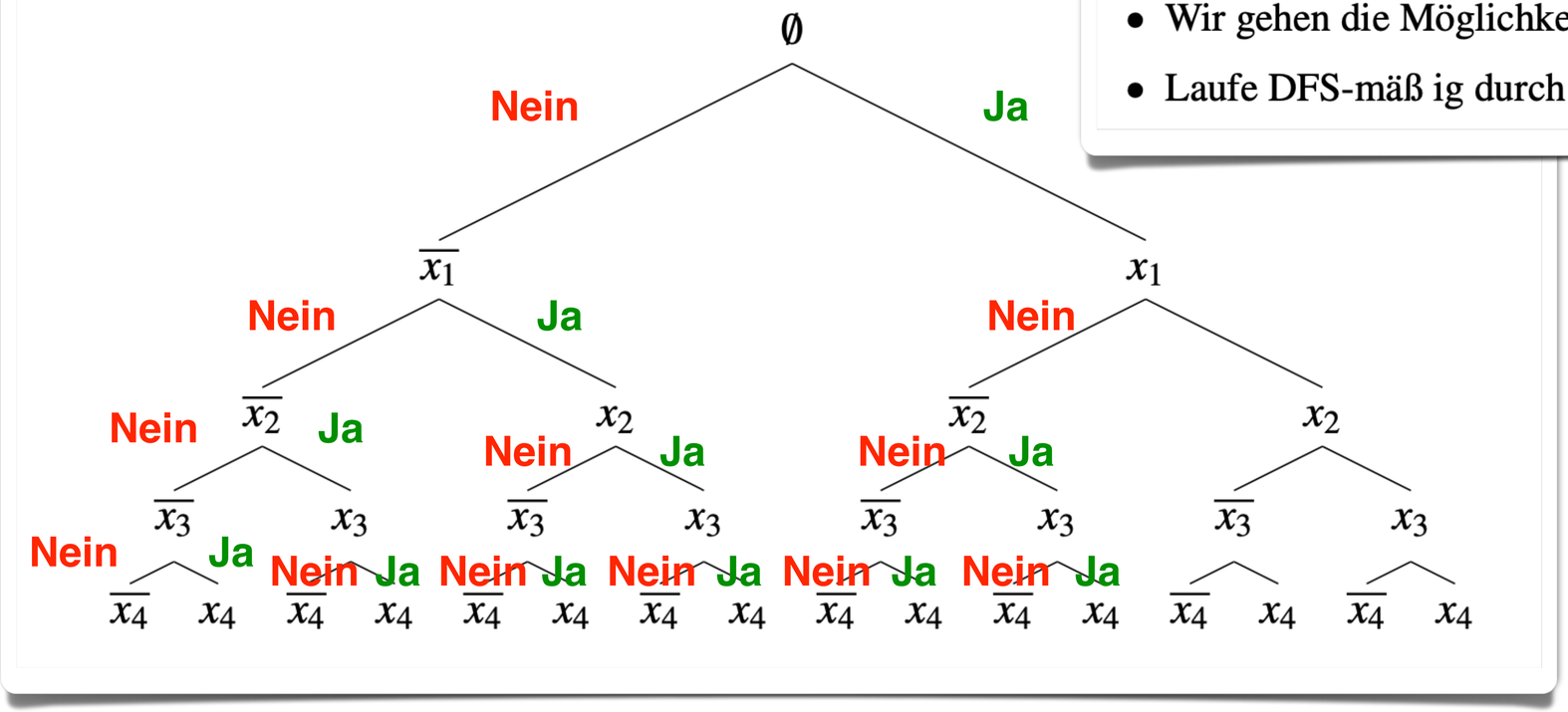

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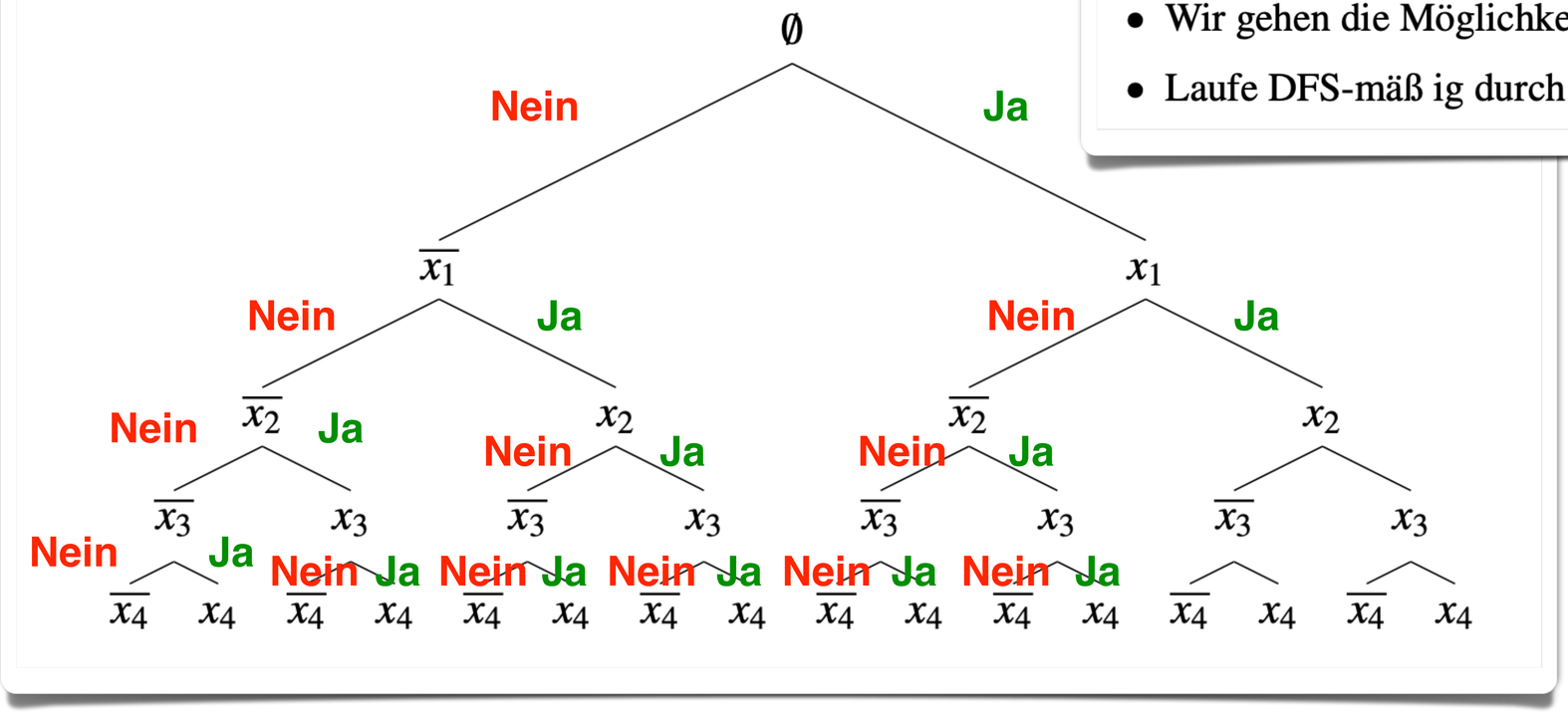

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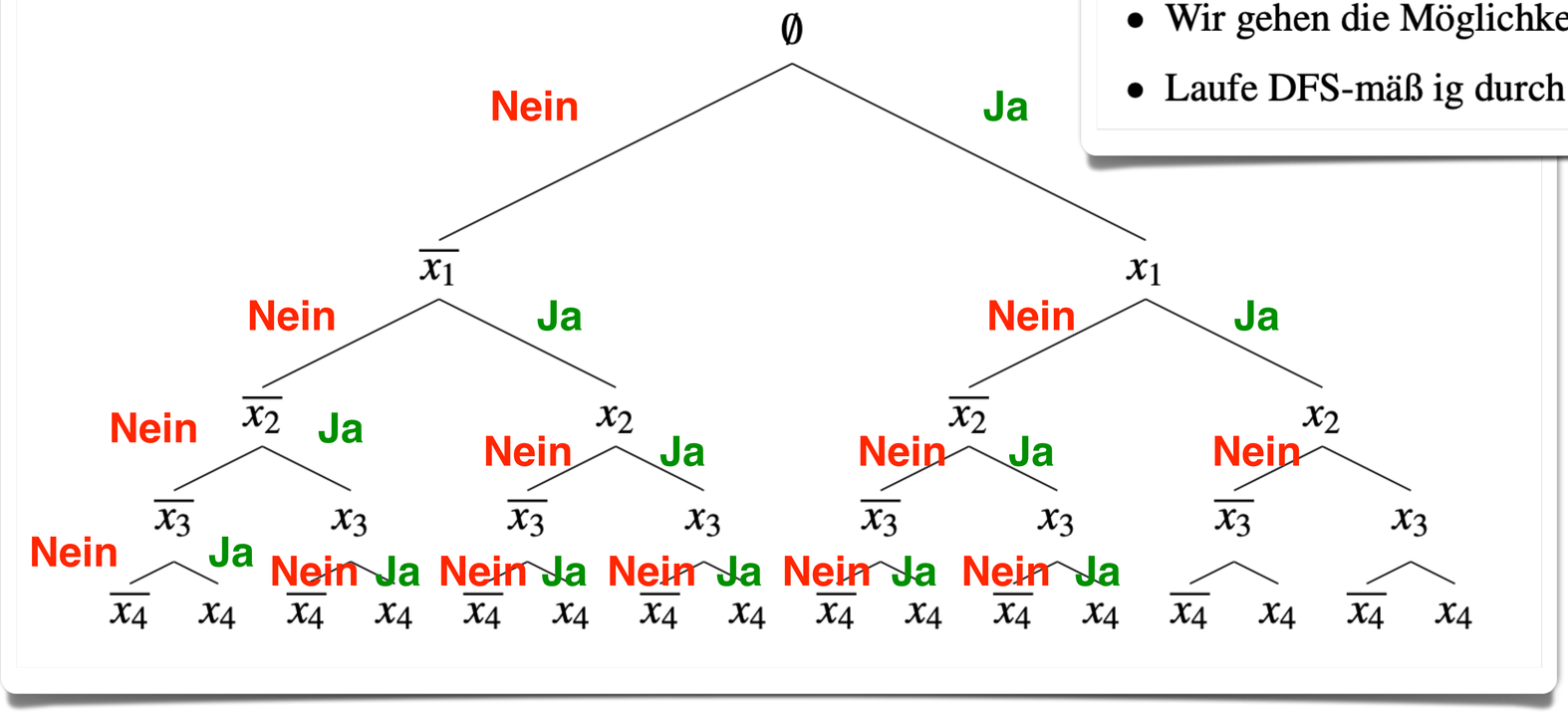

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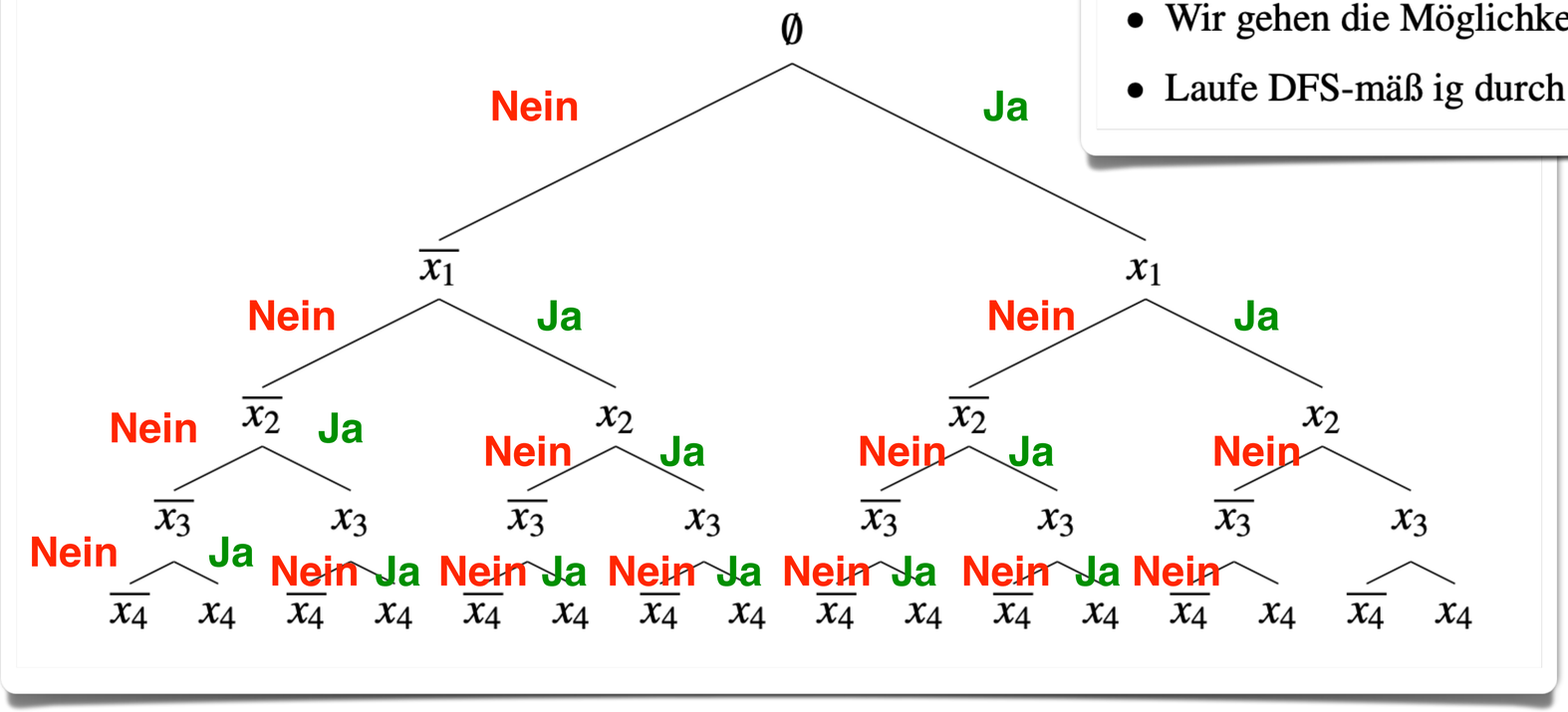

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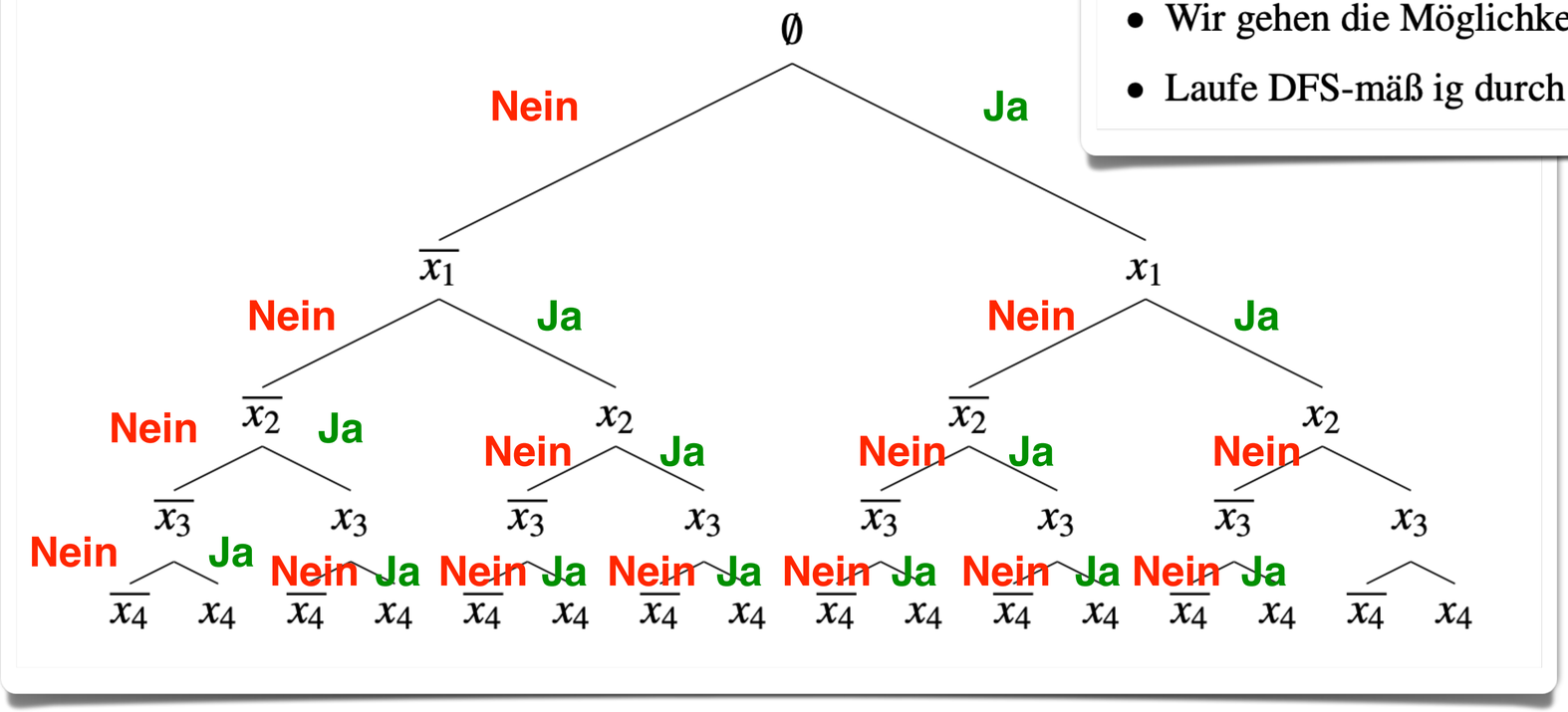

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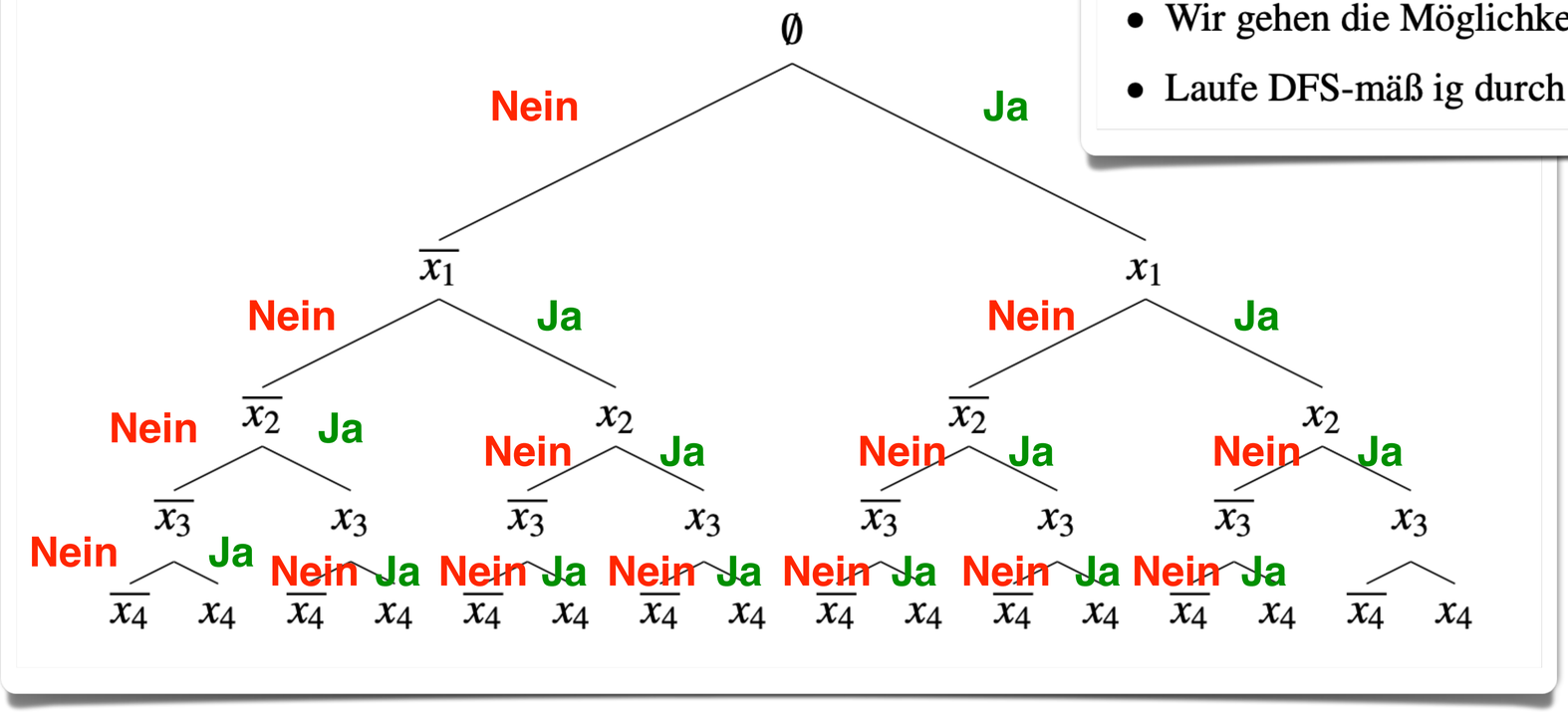

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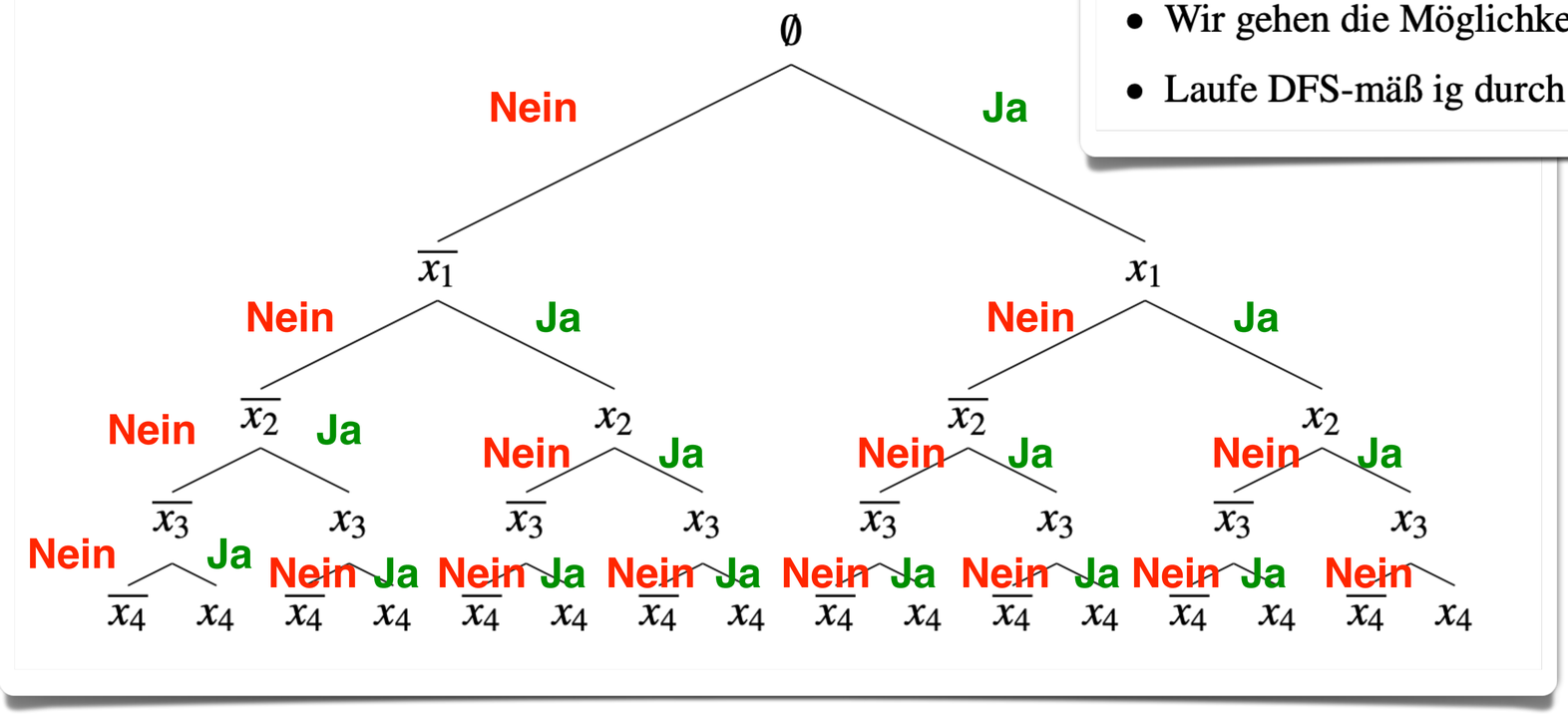

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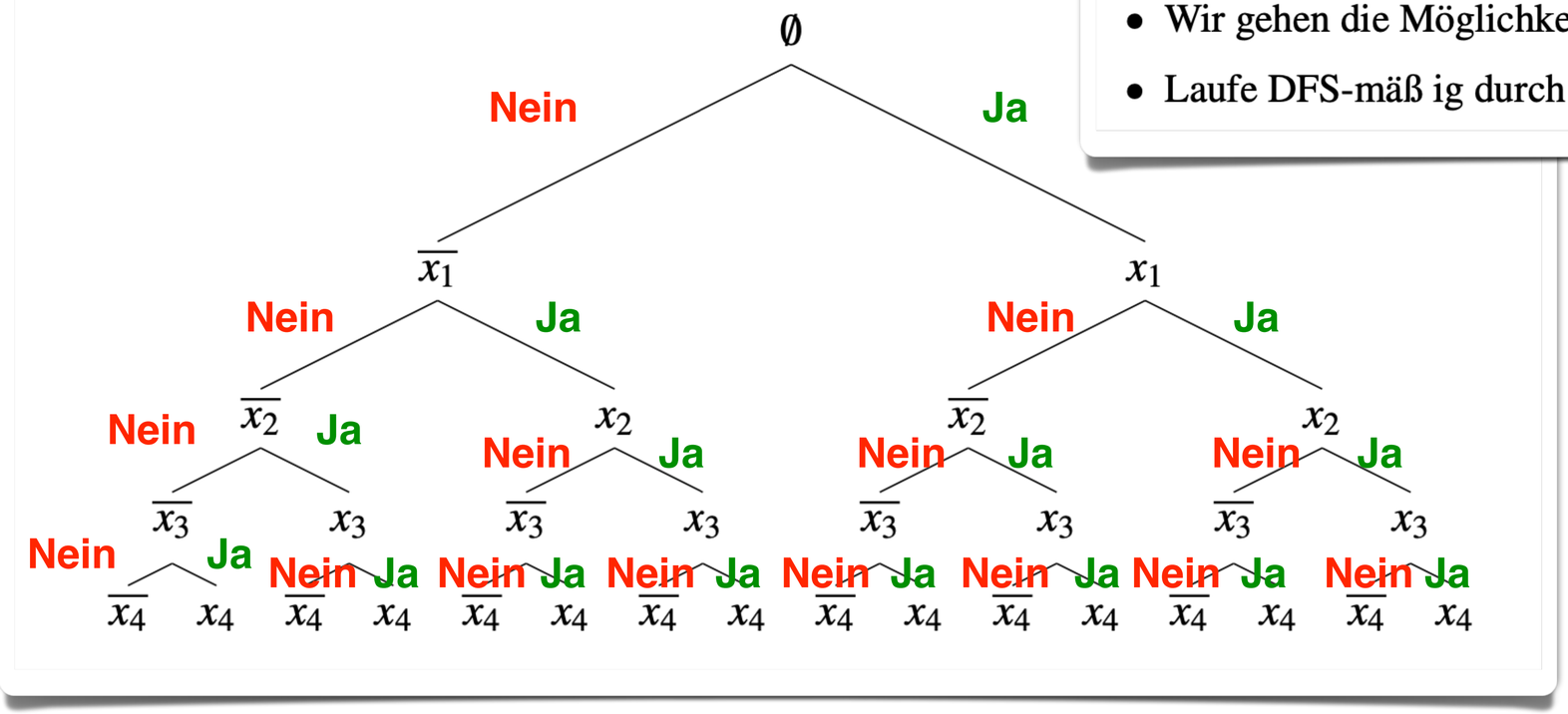

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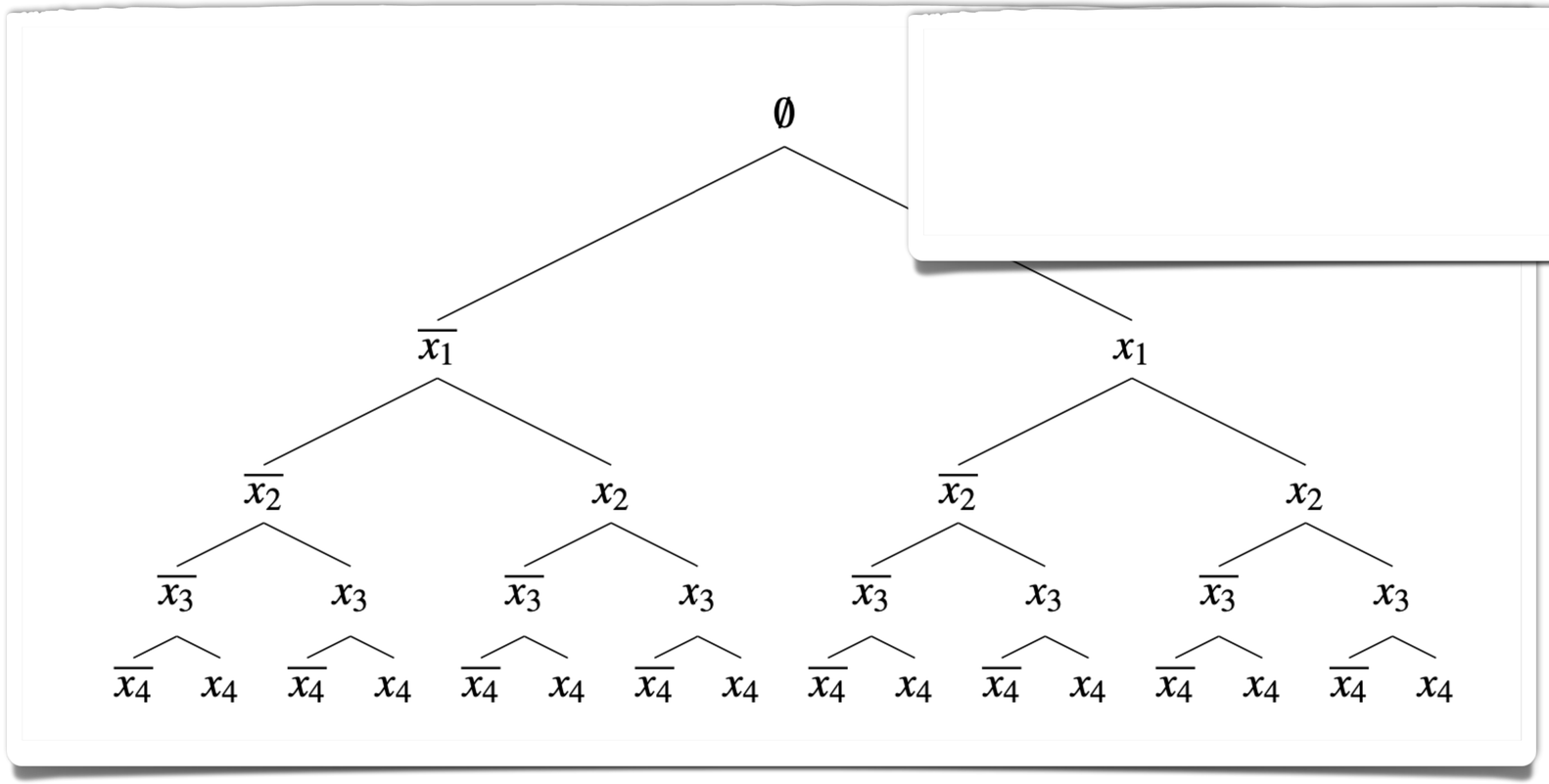

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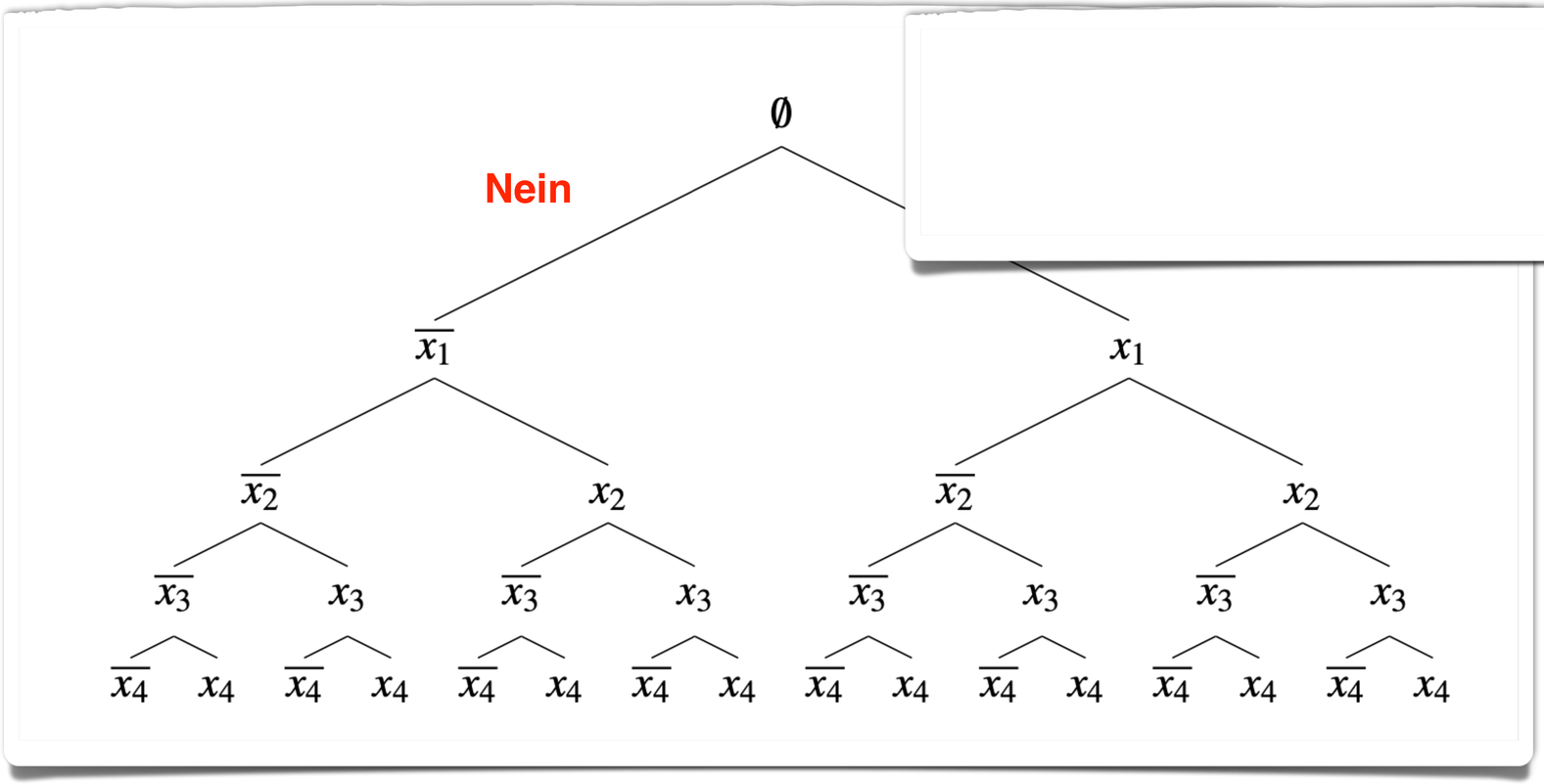

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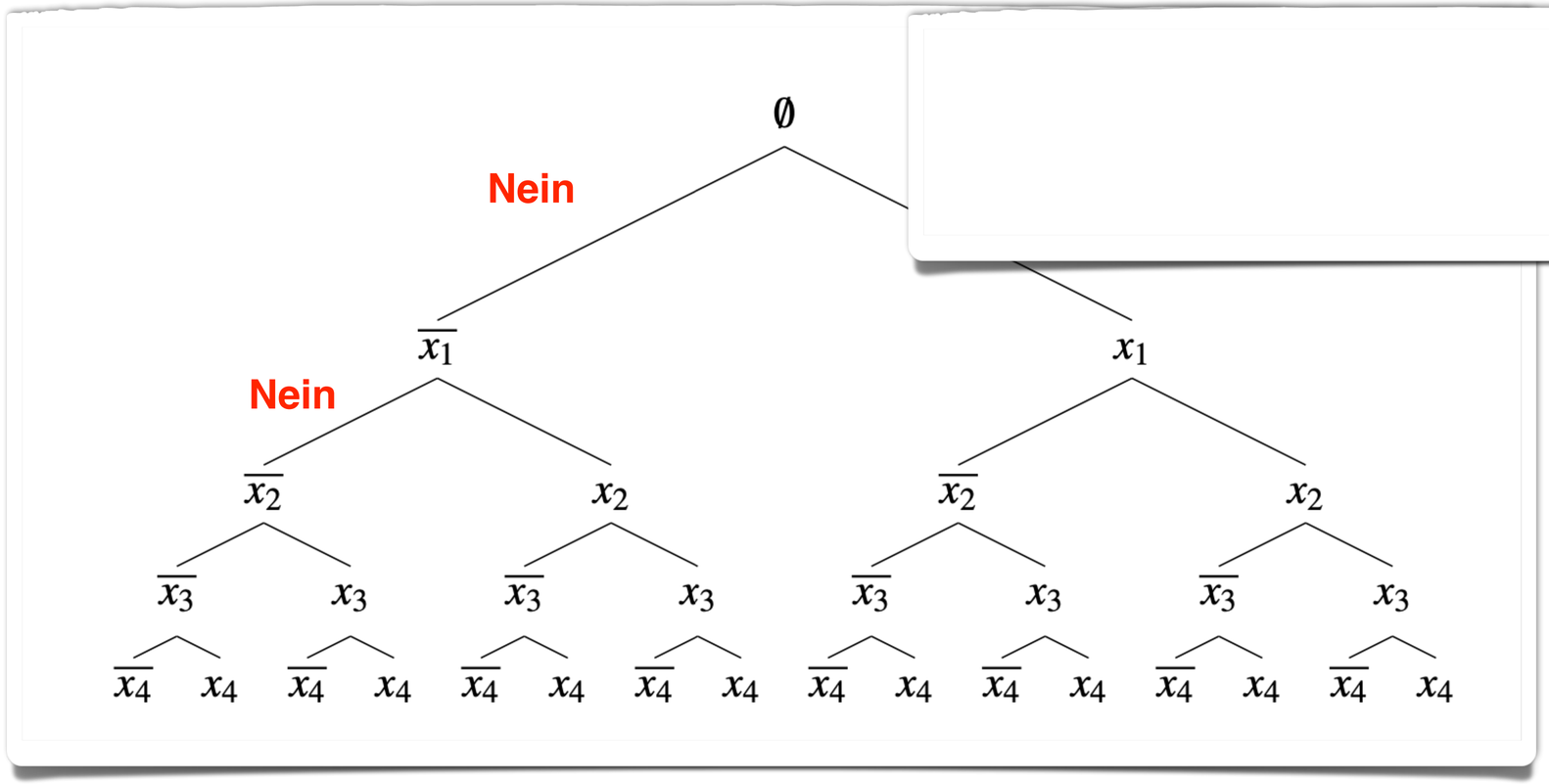

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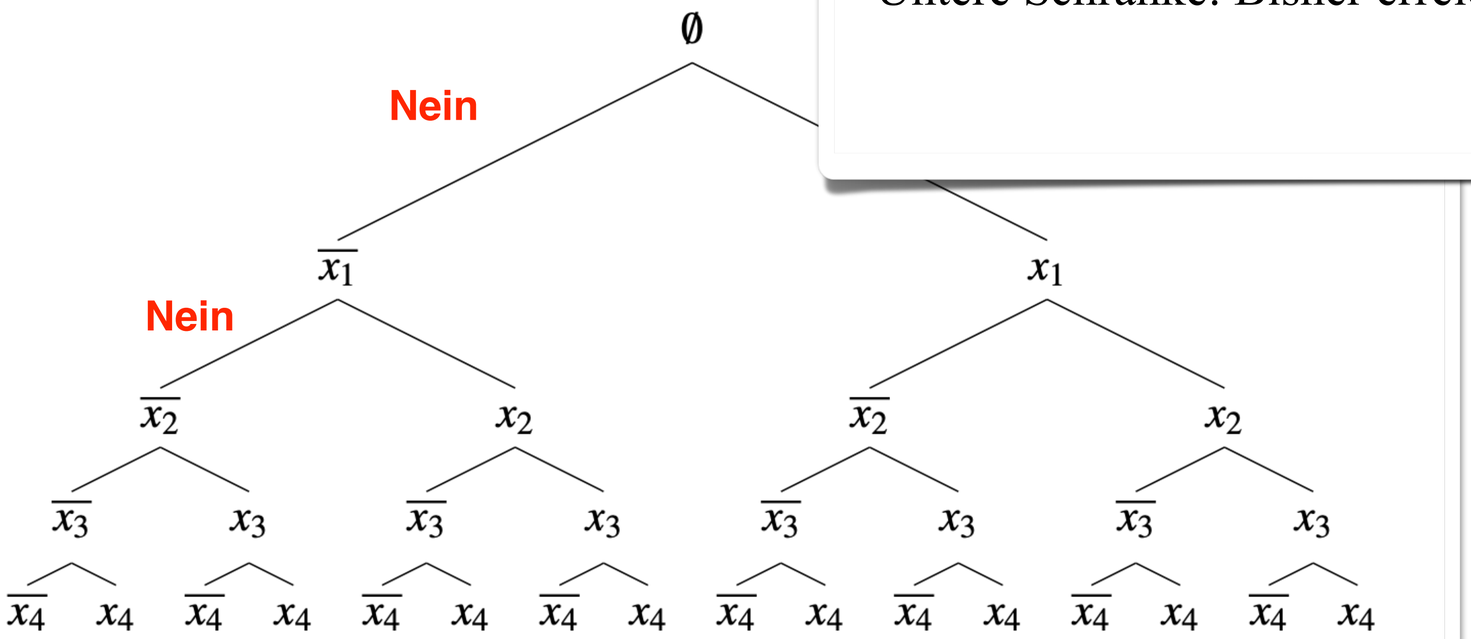

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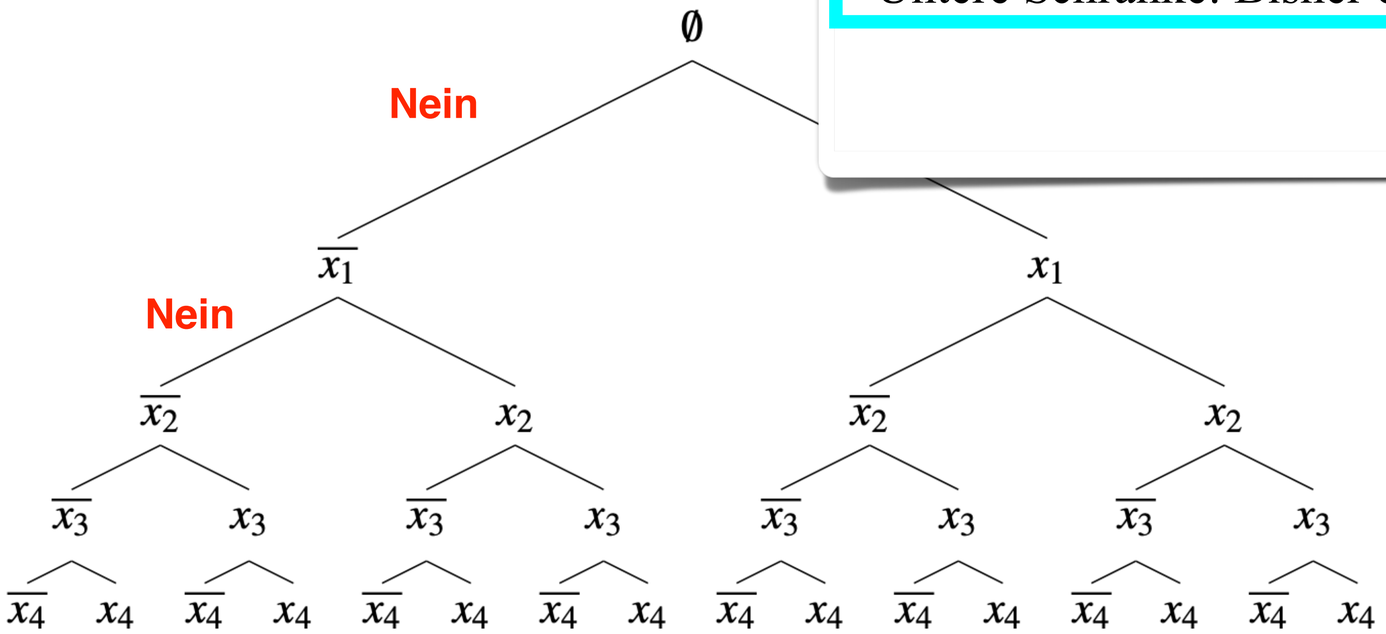

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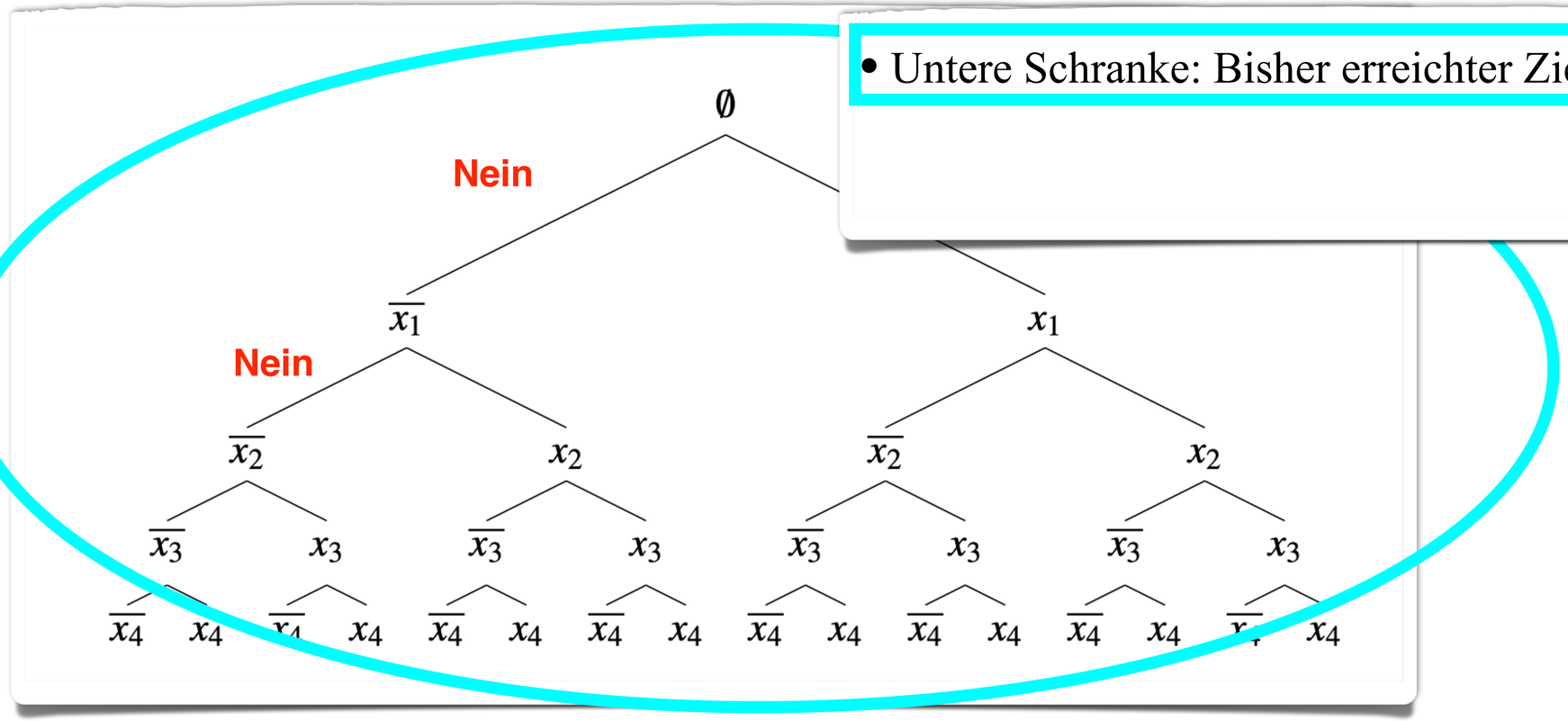

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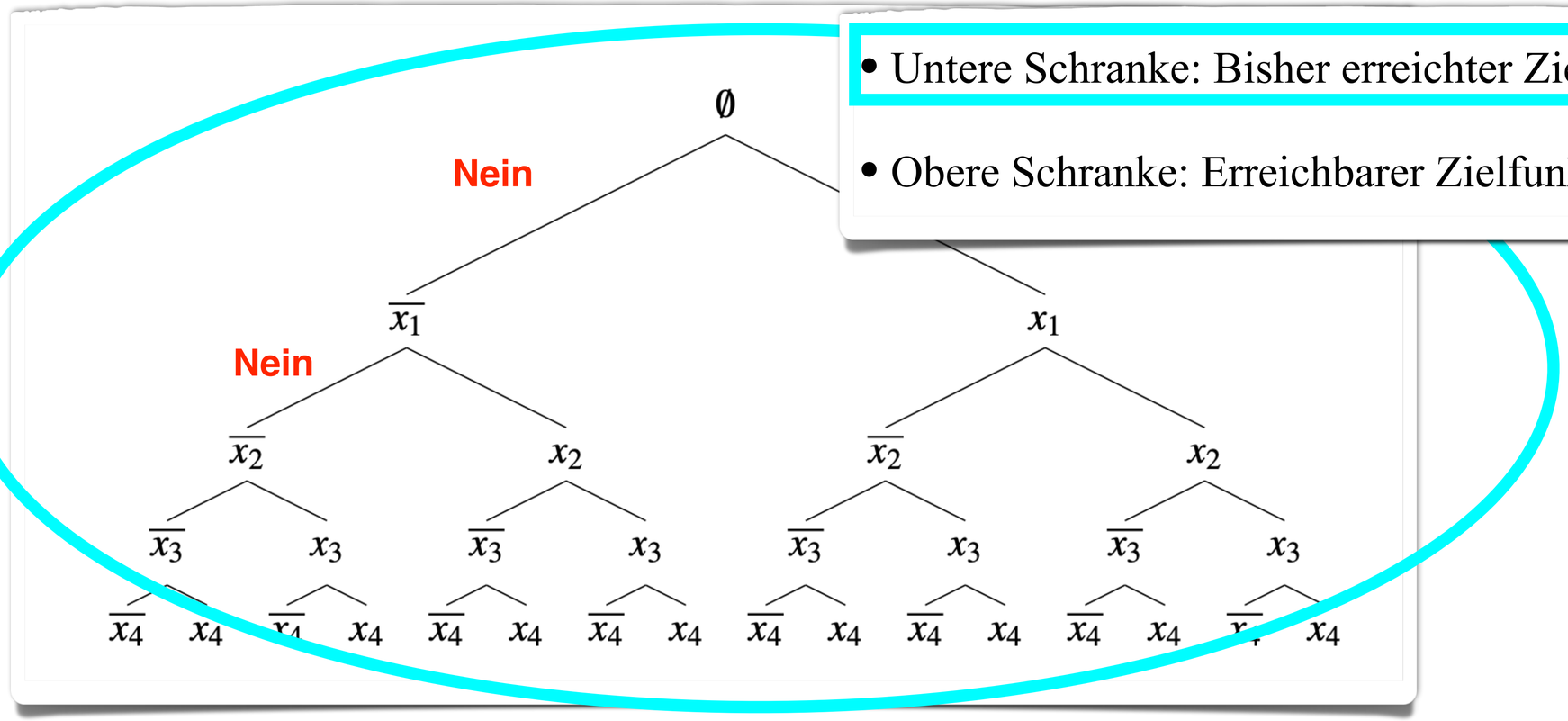

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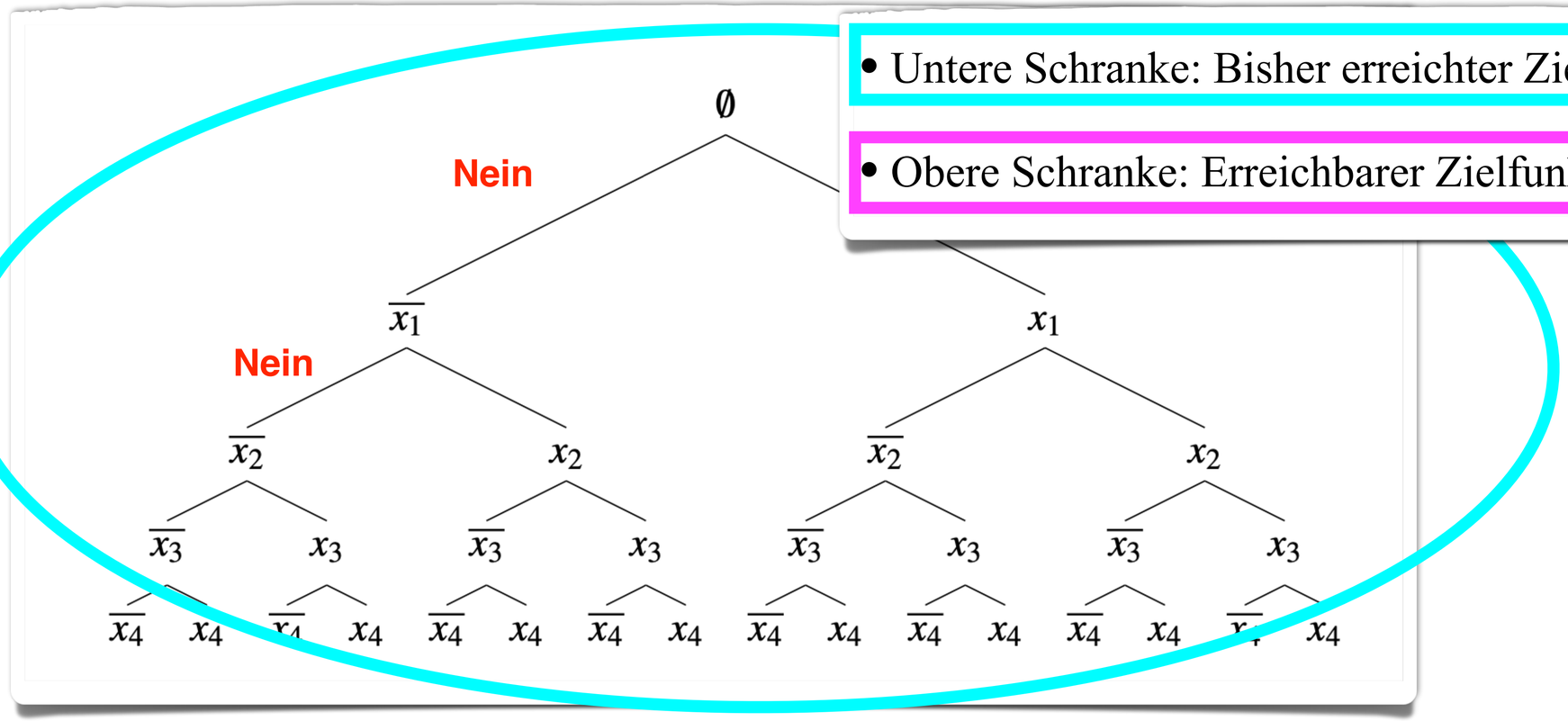

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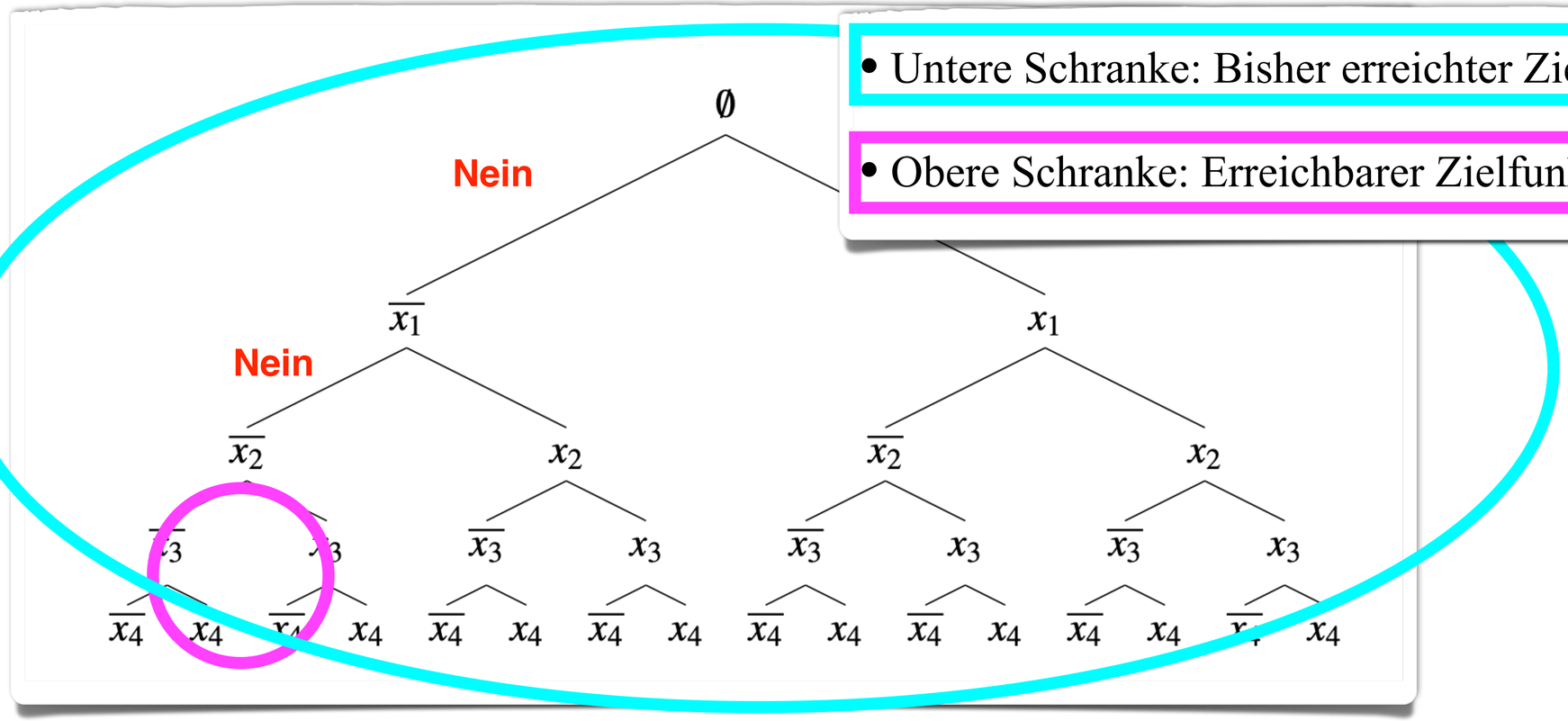

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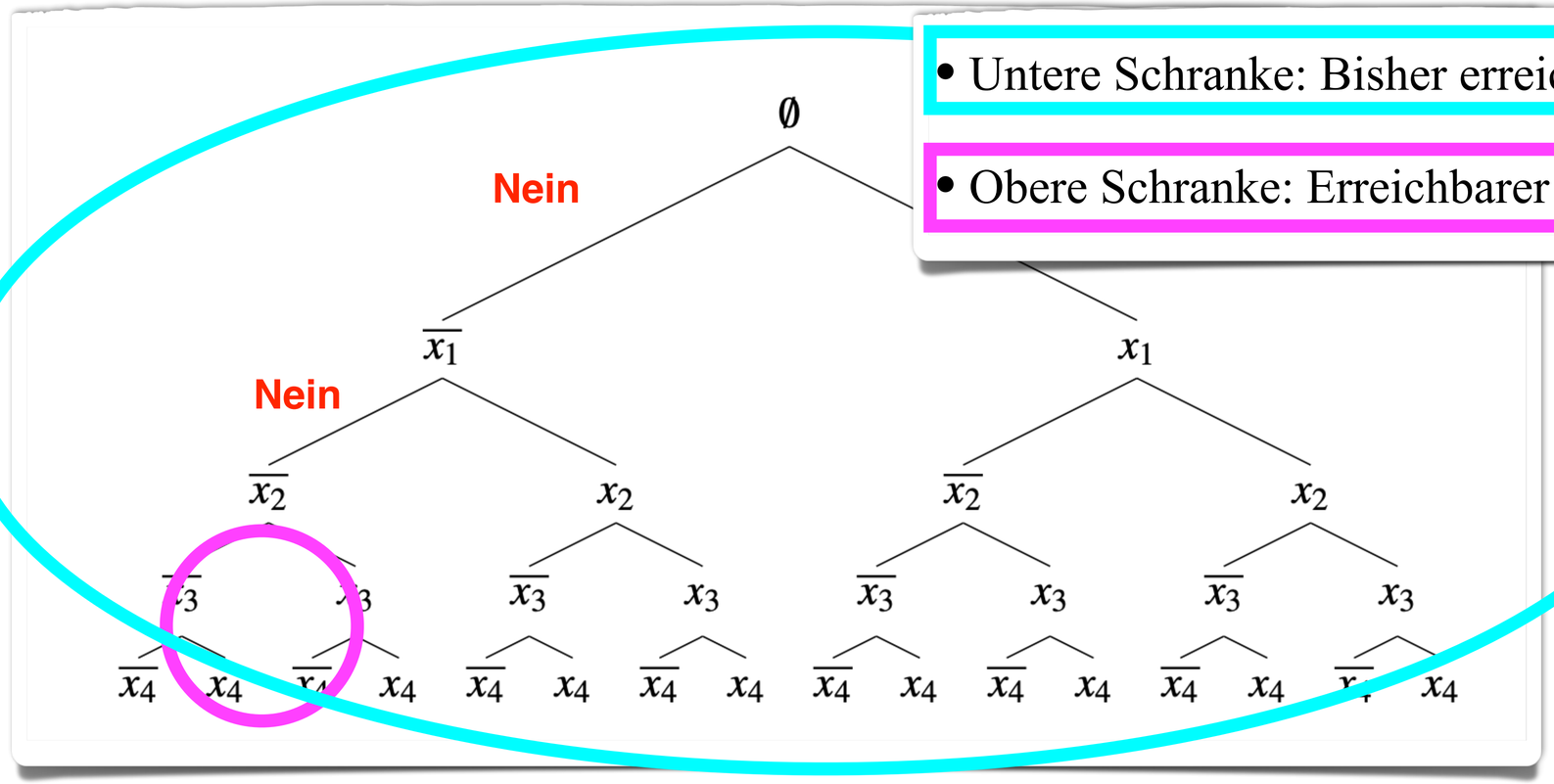

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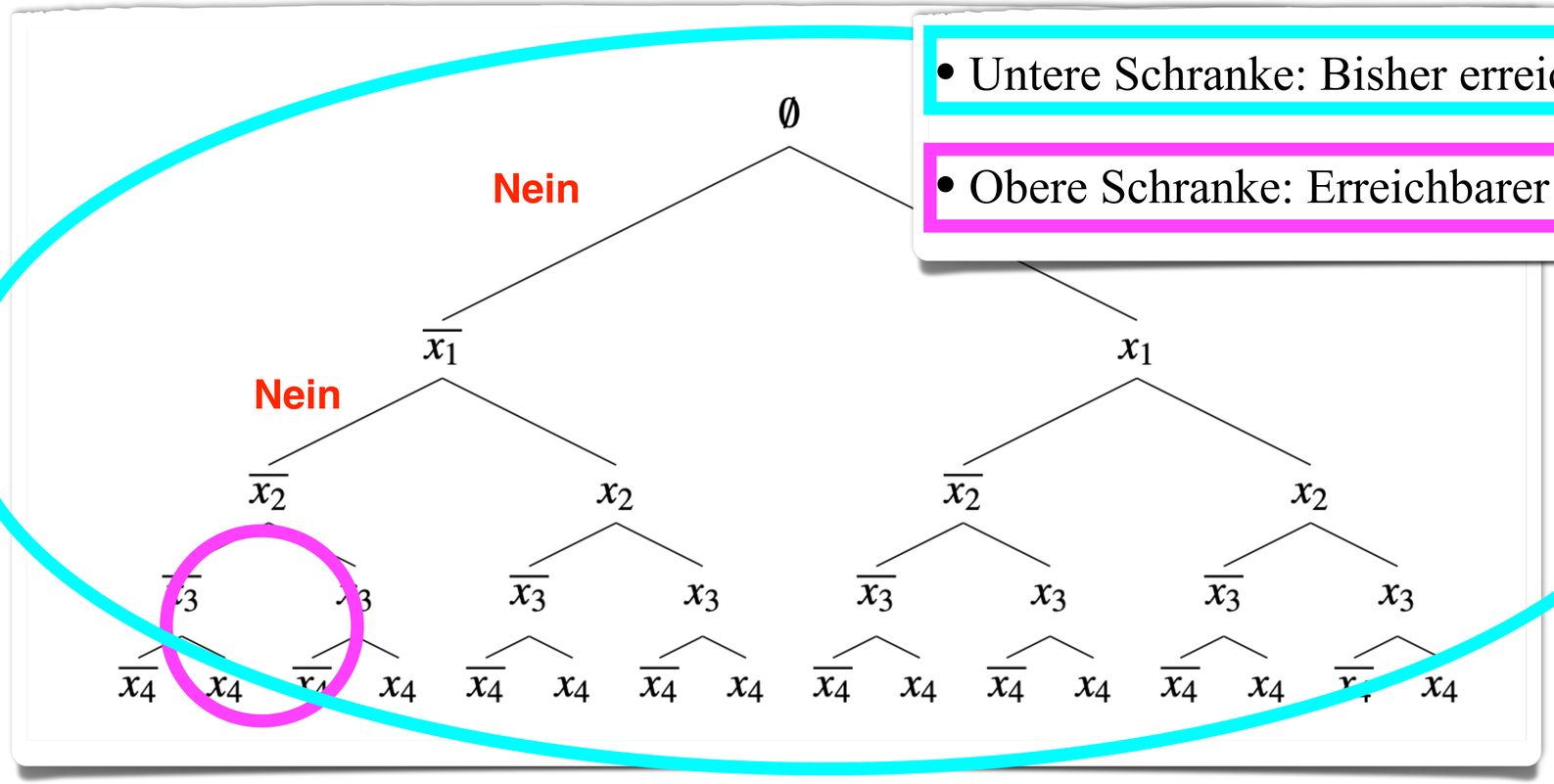

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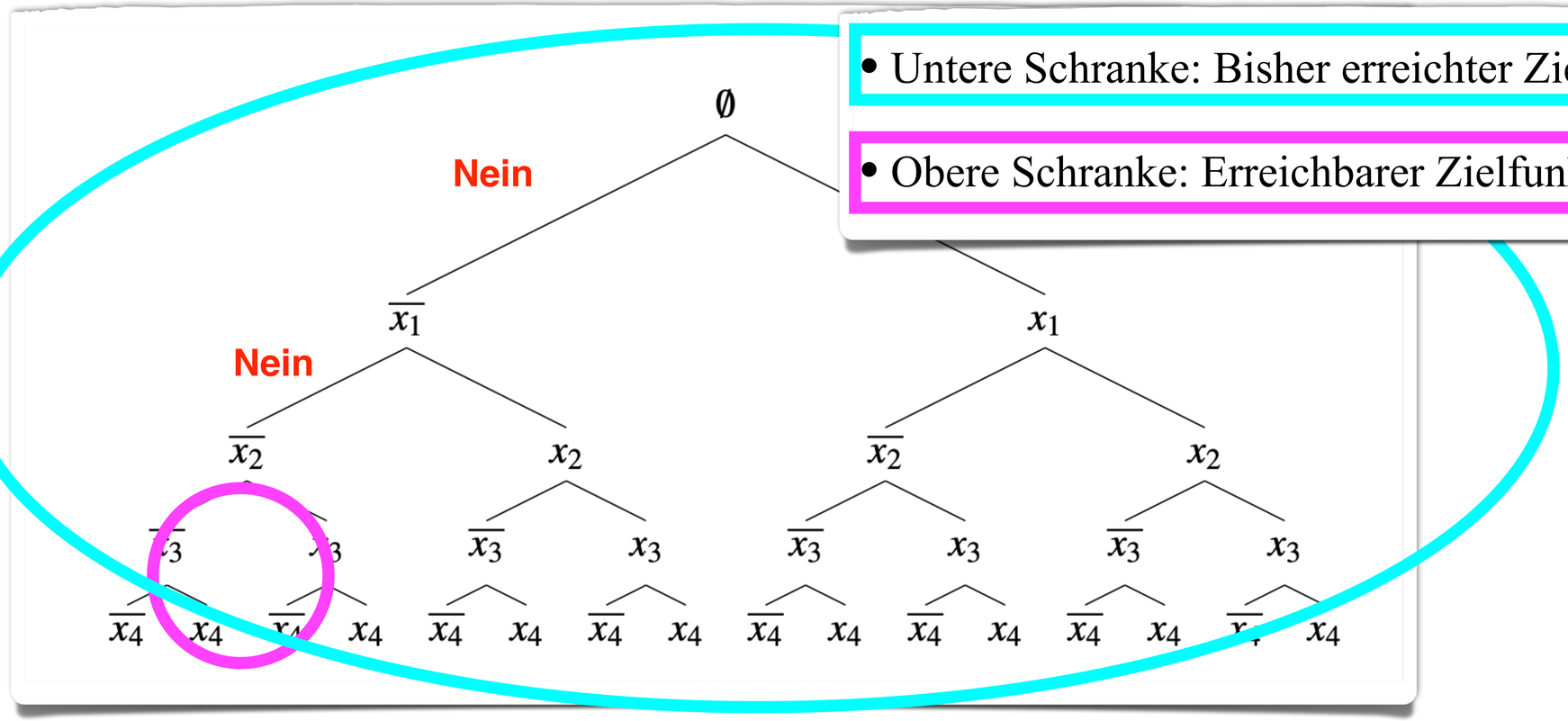

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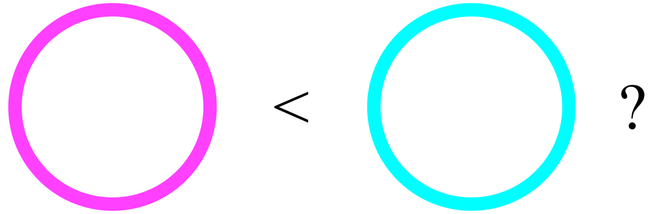

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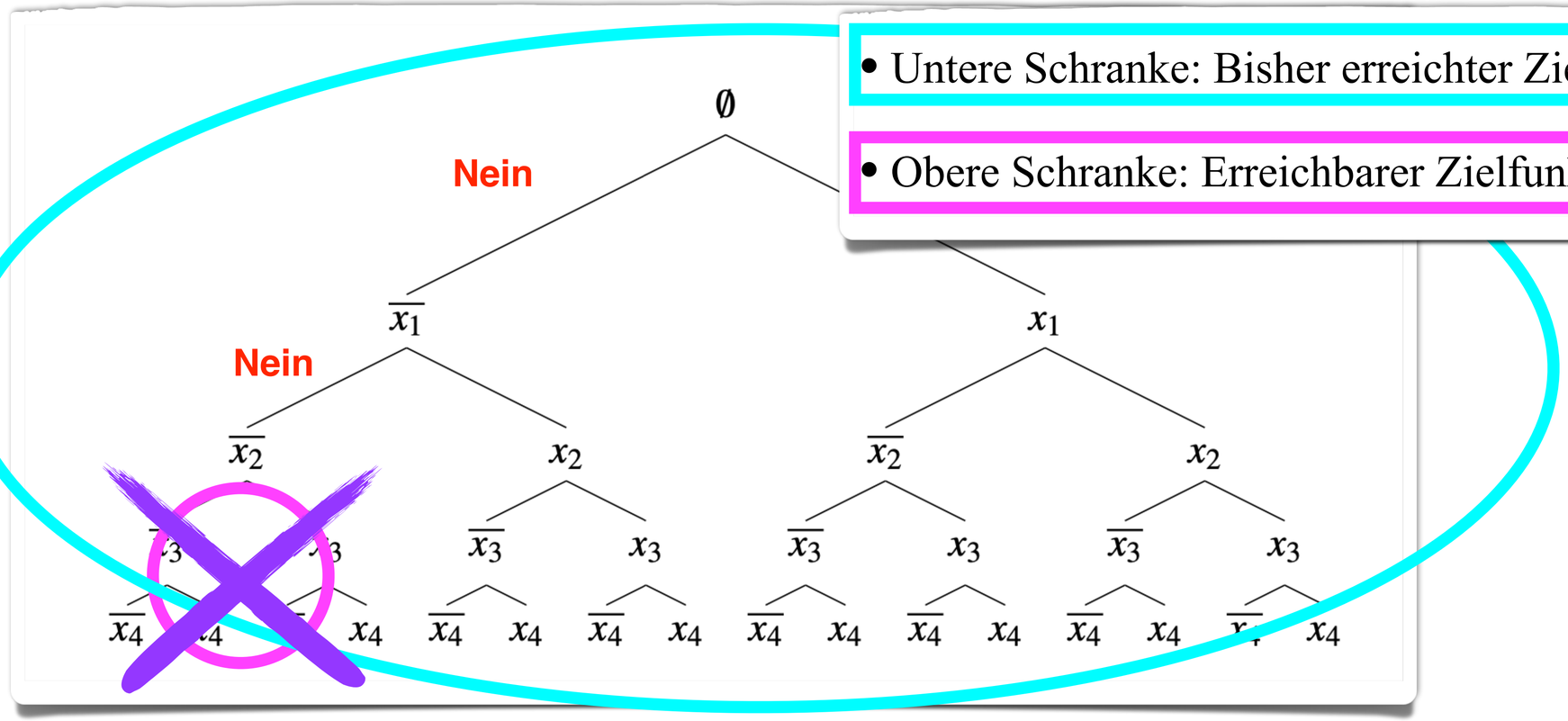

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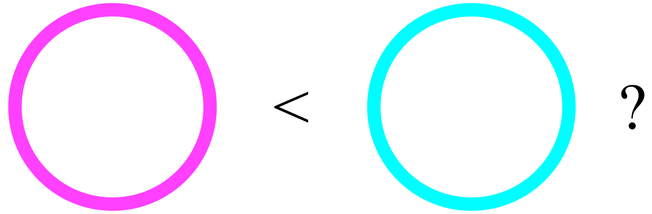

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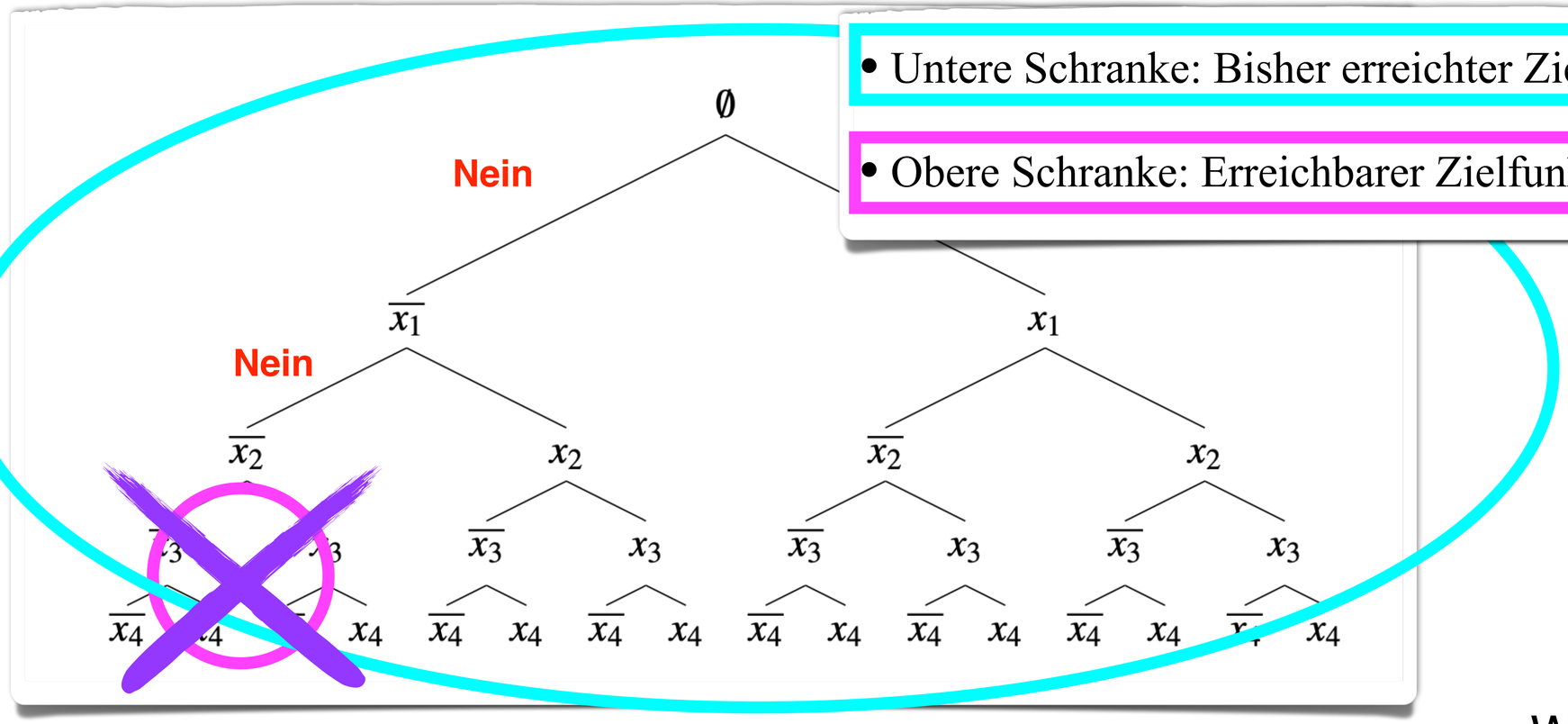

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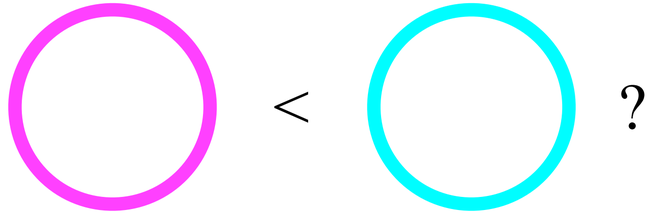

Was sind geeignete Schranken?

2. Geeignete Schranken - Knapsack

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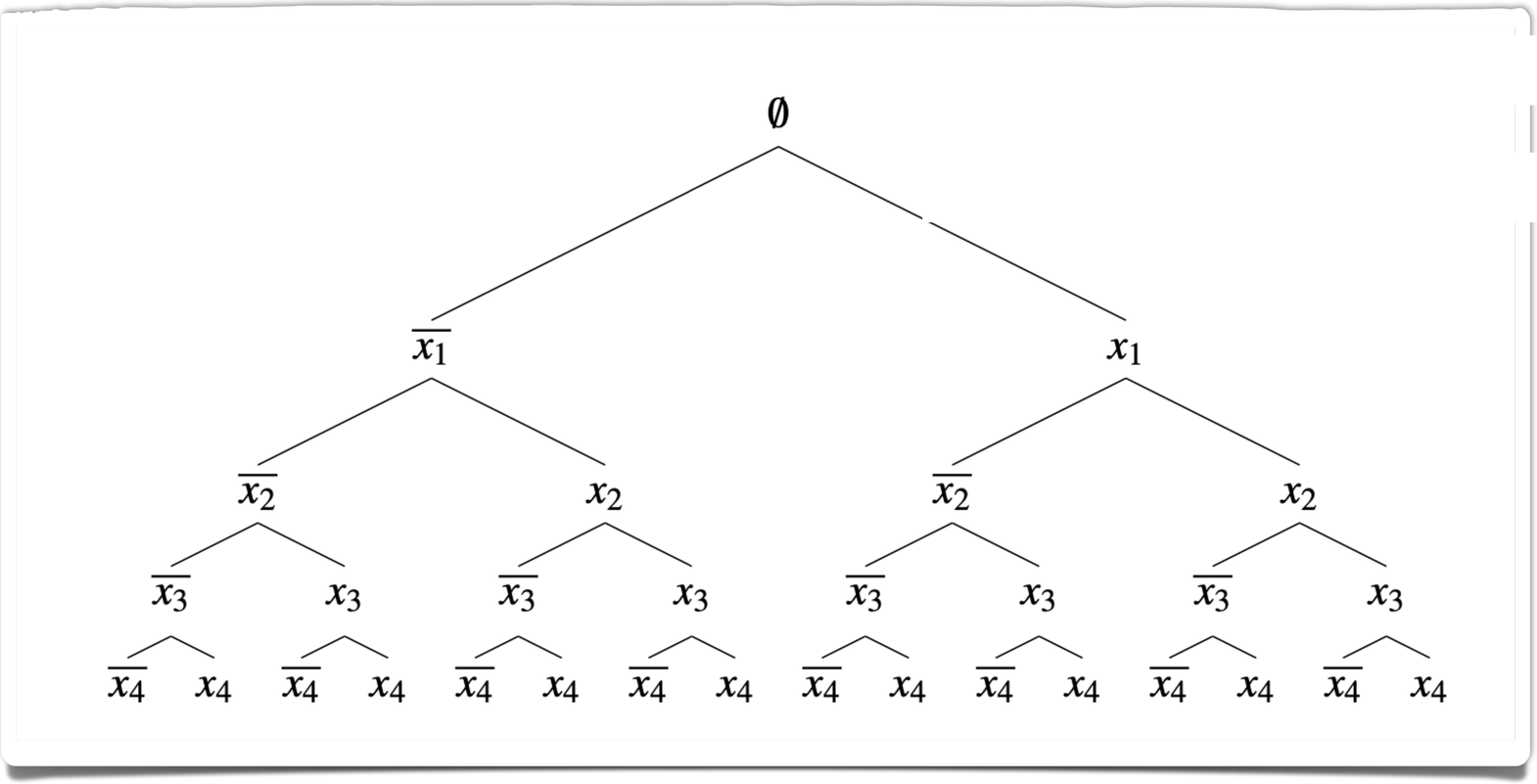

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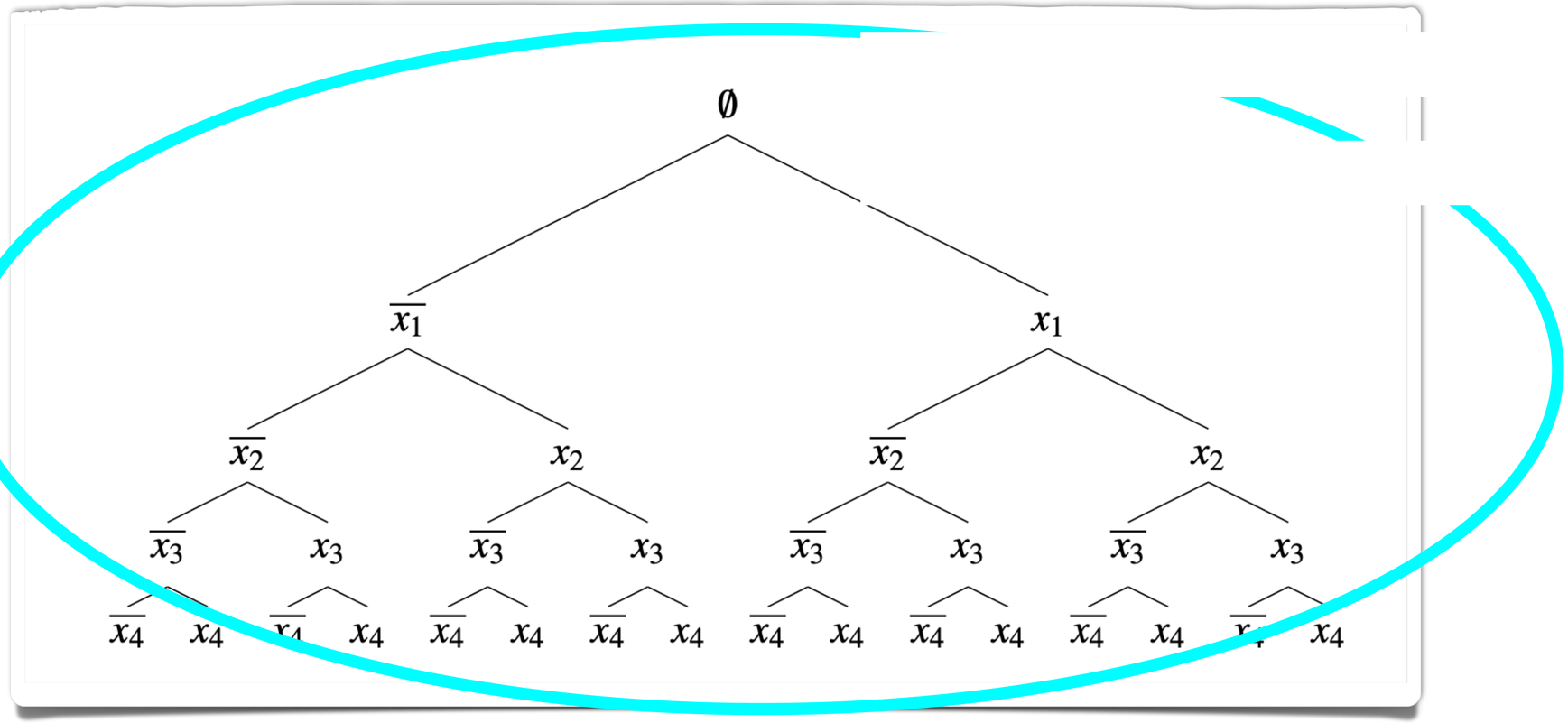

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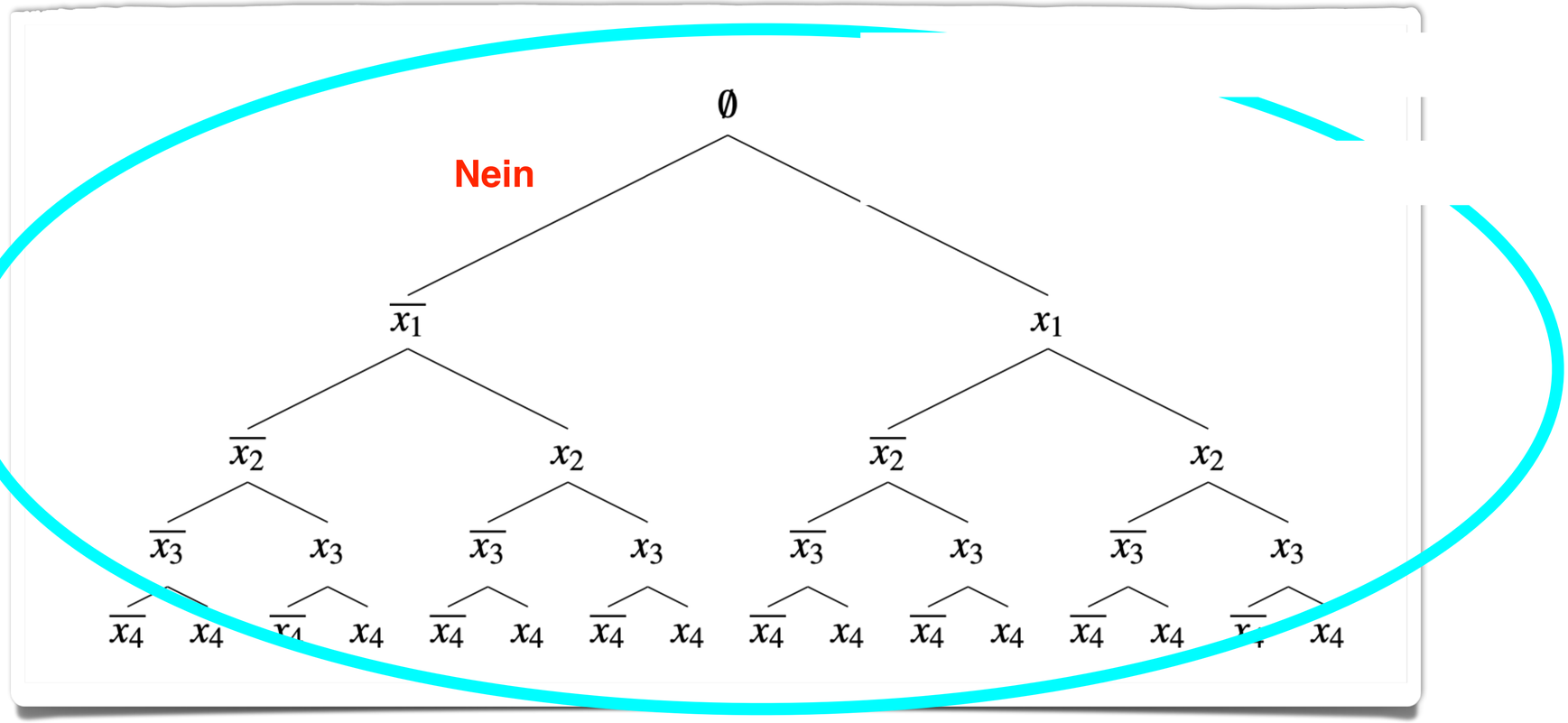

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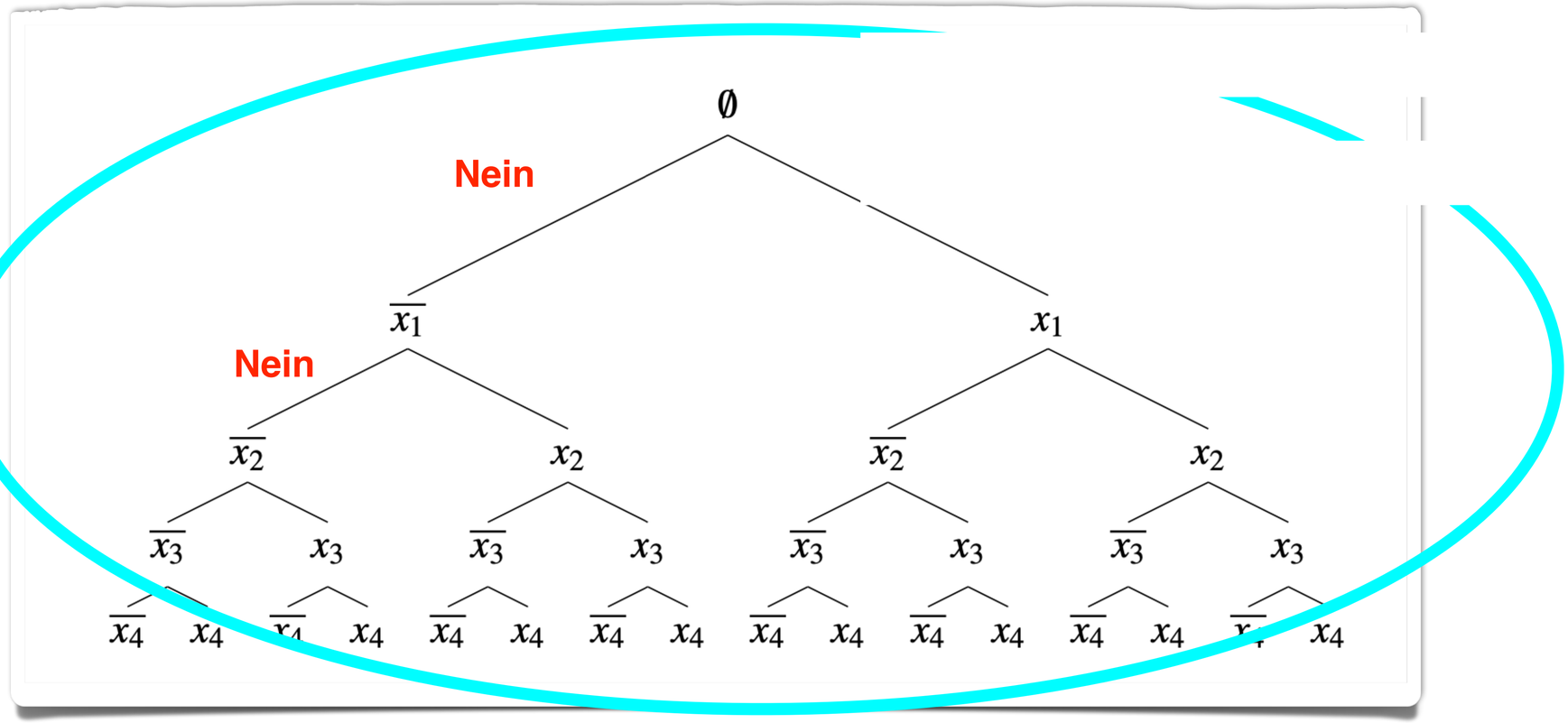

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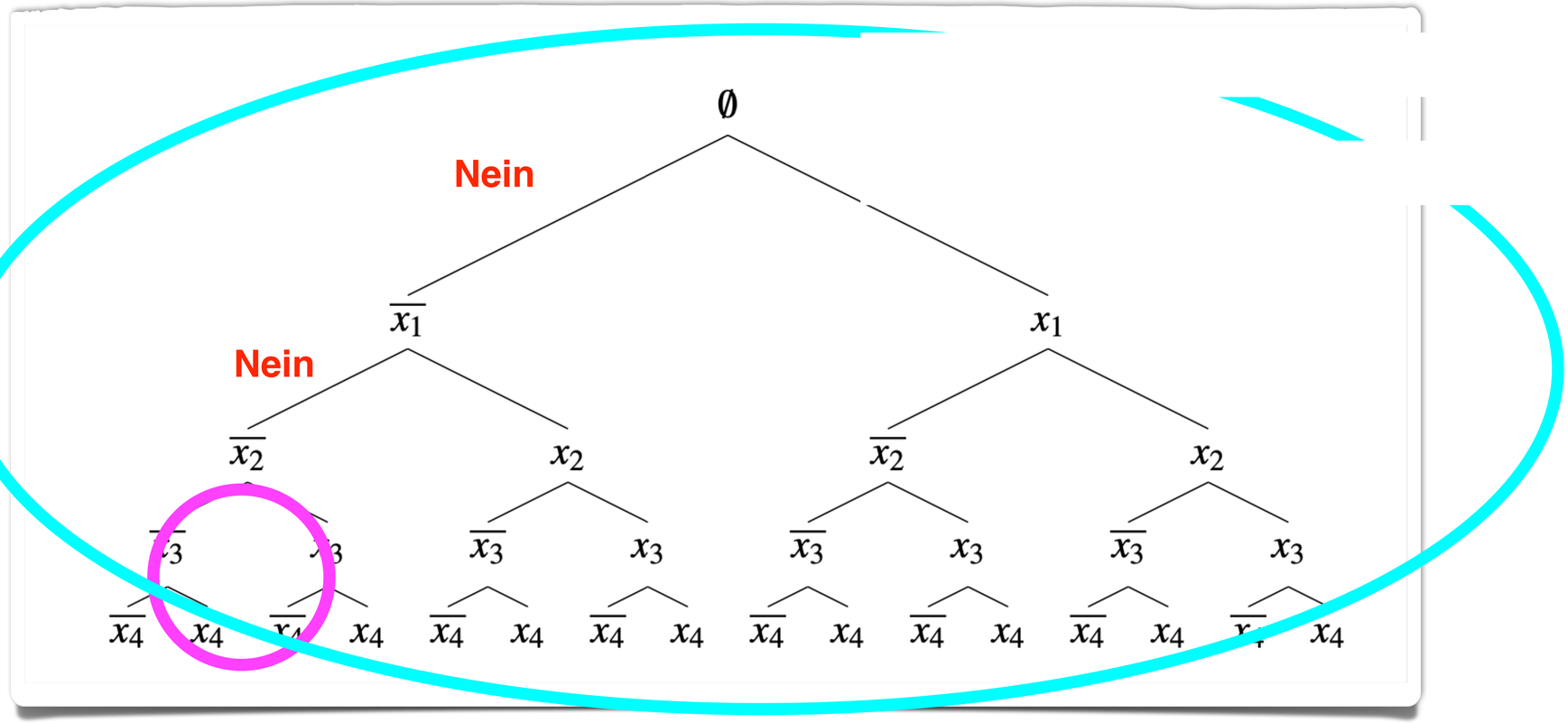

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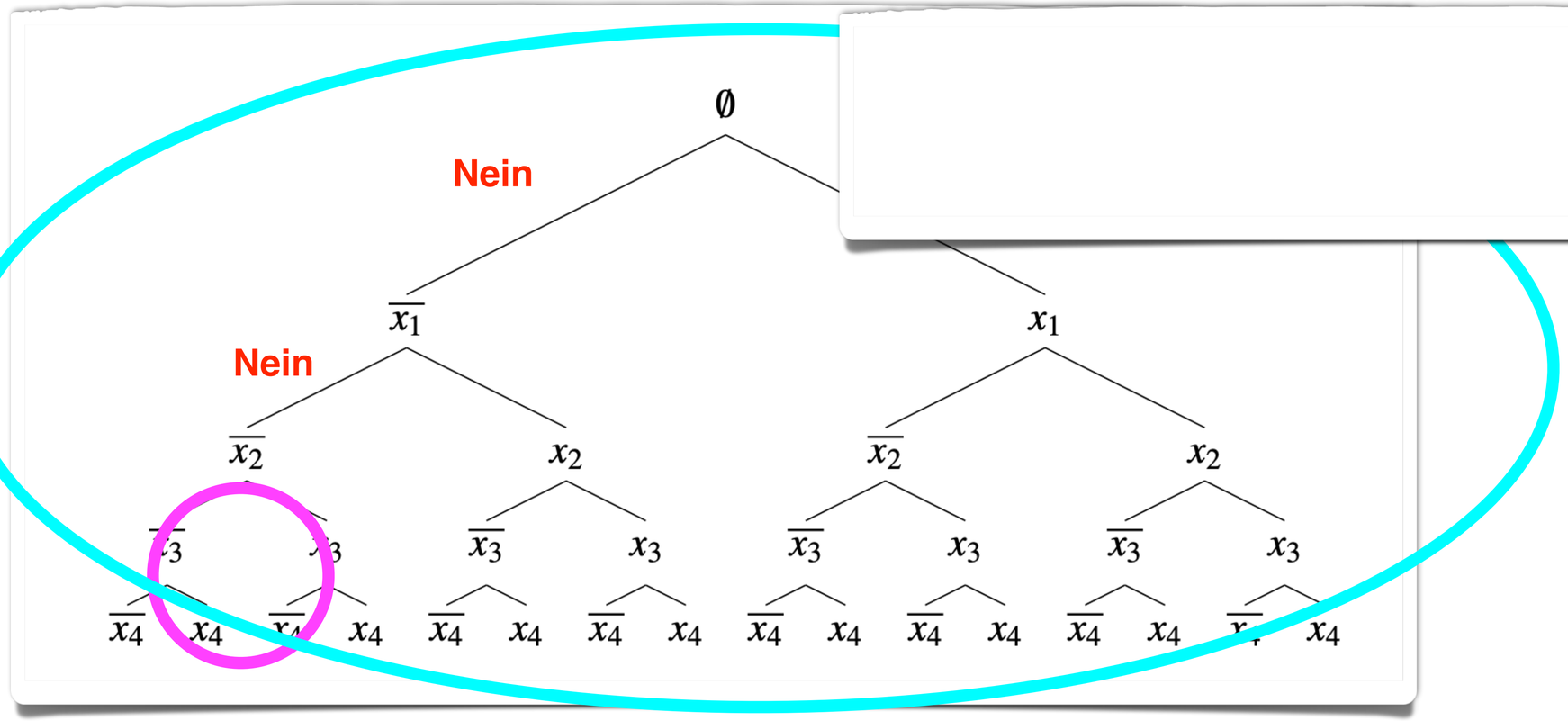

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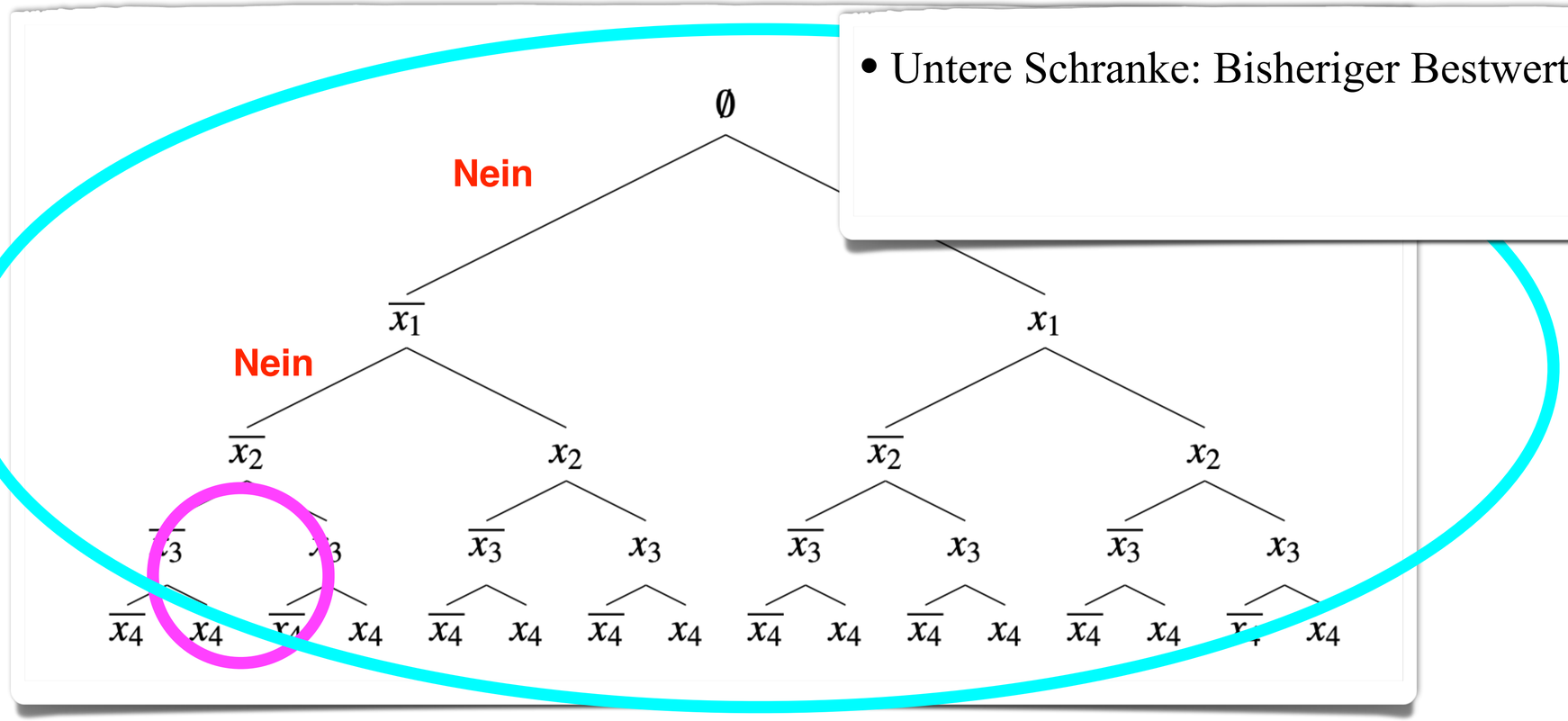

- Untere Schranke: Bisheriger Bestwert für Greedy (ganzzahlig)

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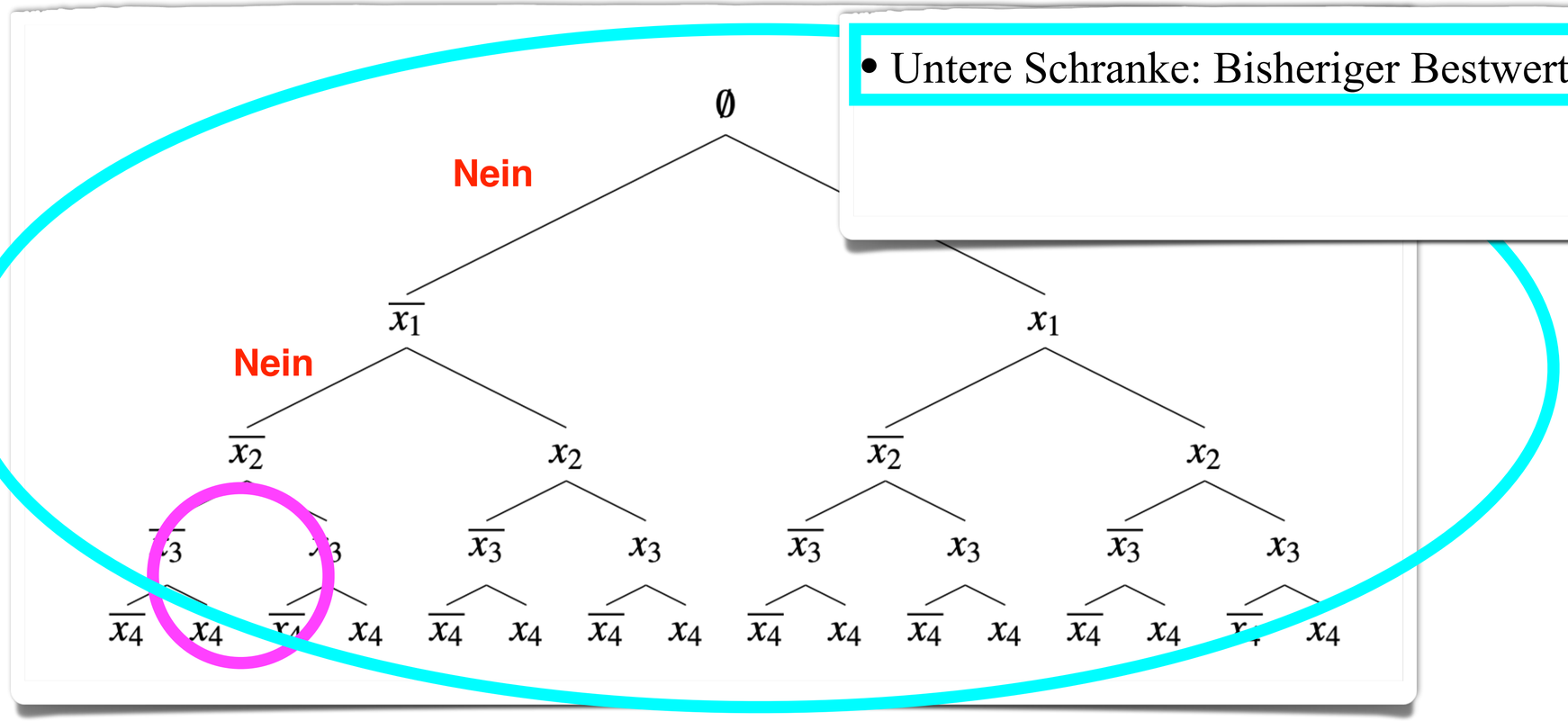

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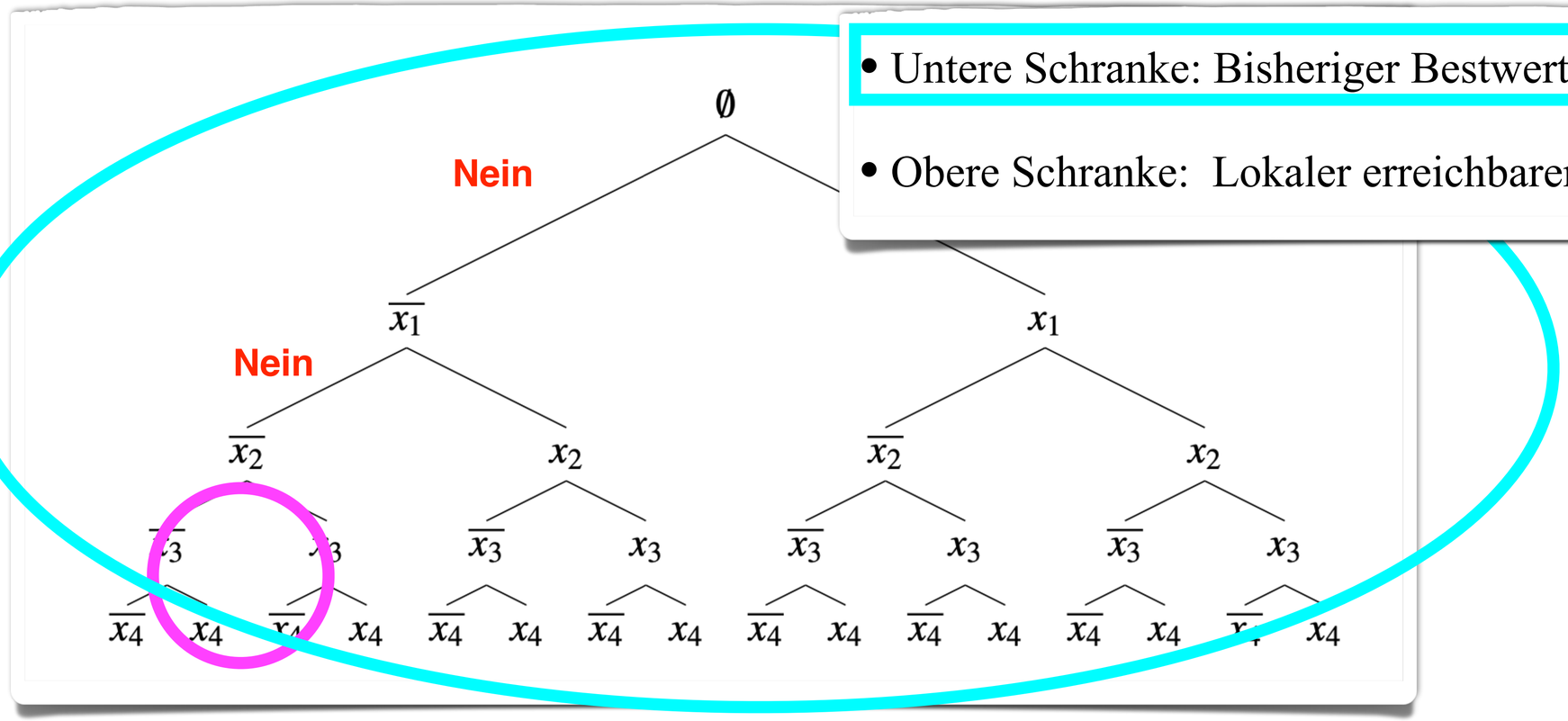

- Untere Schranke: Bisheriger Bestwert für Greedy (ganzzahlig)
- Obere Schranke: Lokaler erreichbarer Bestwert für Greedy (fraktional)

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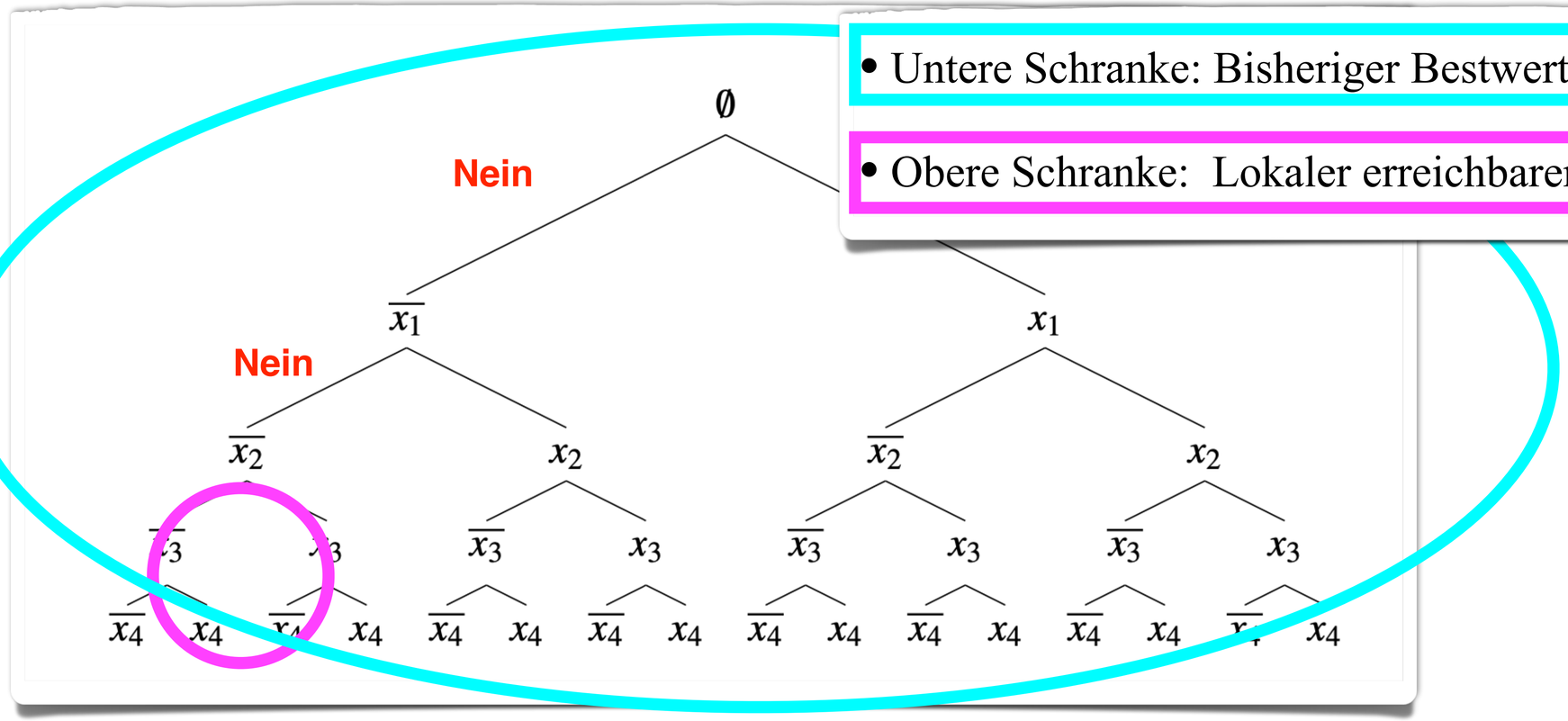

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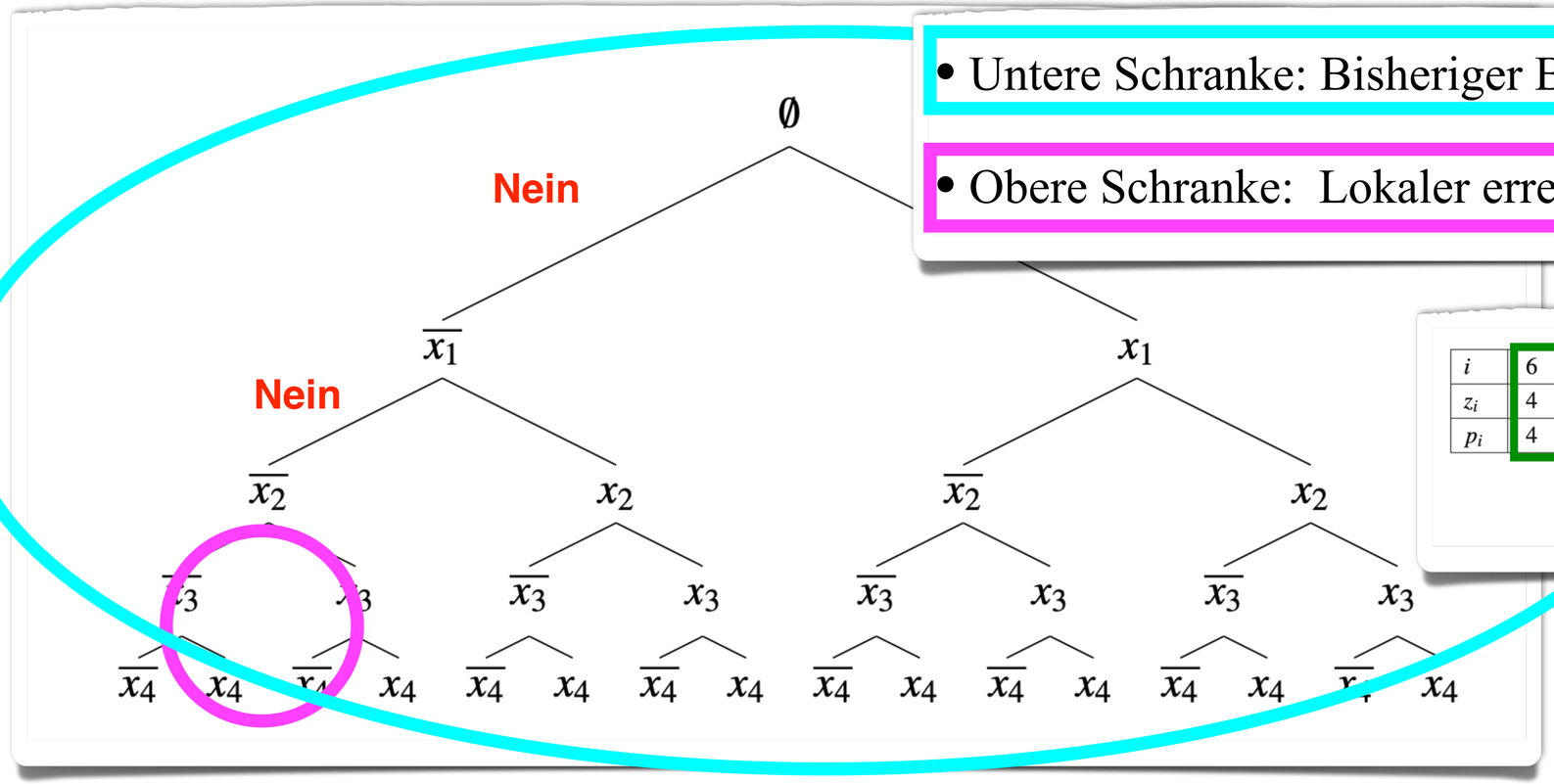

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z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} p_i x_i = 44$$

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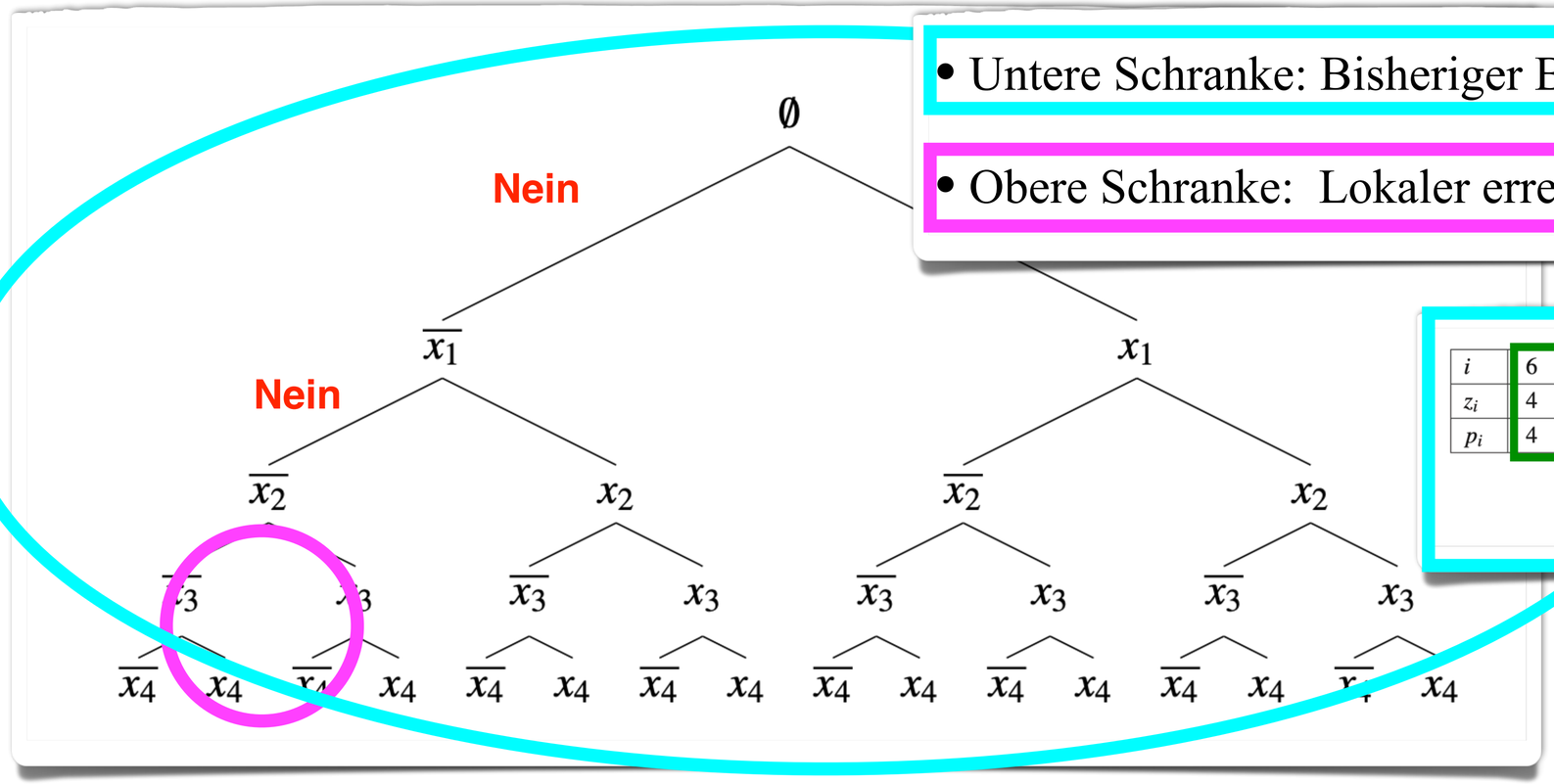

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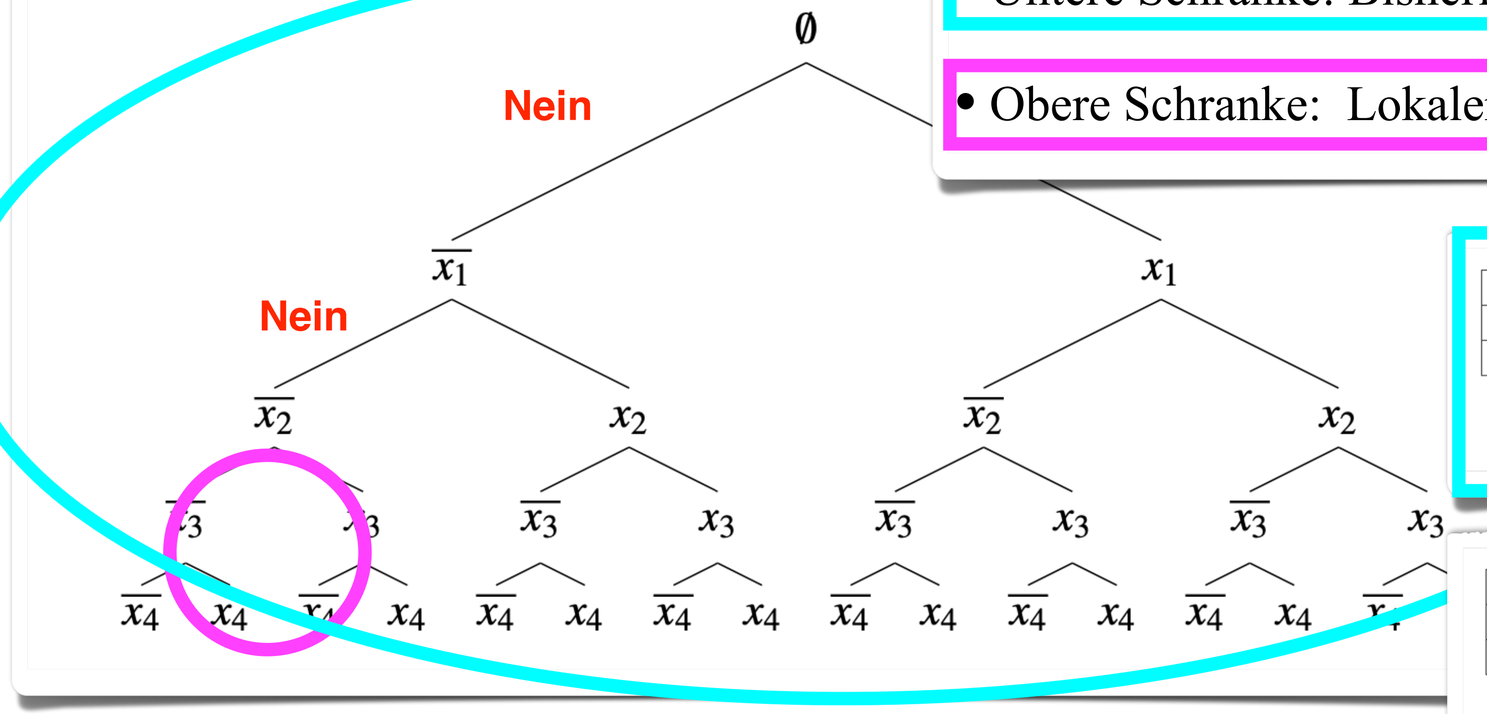

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$\sum_{i=1}^{n} p_i x_i$ = 46 x_{15} = 0,6

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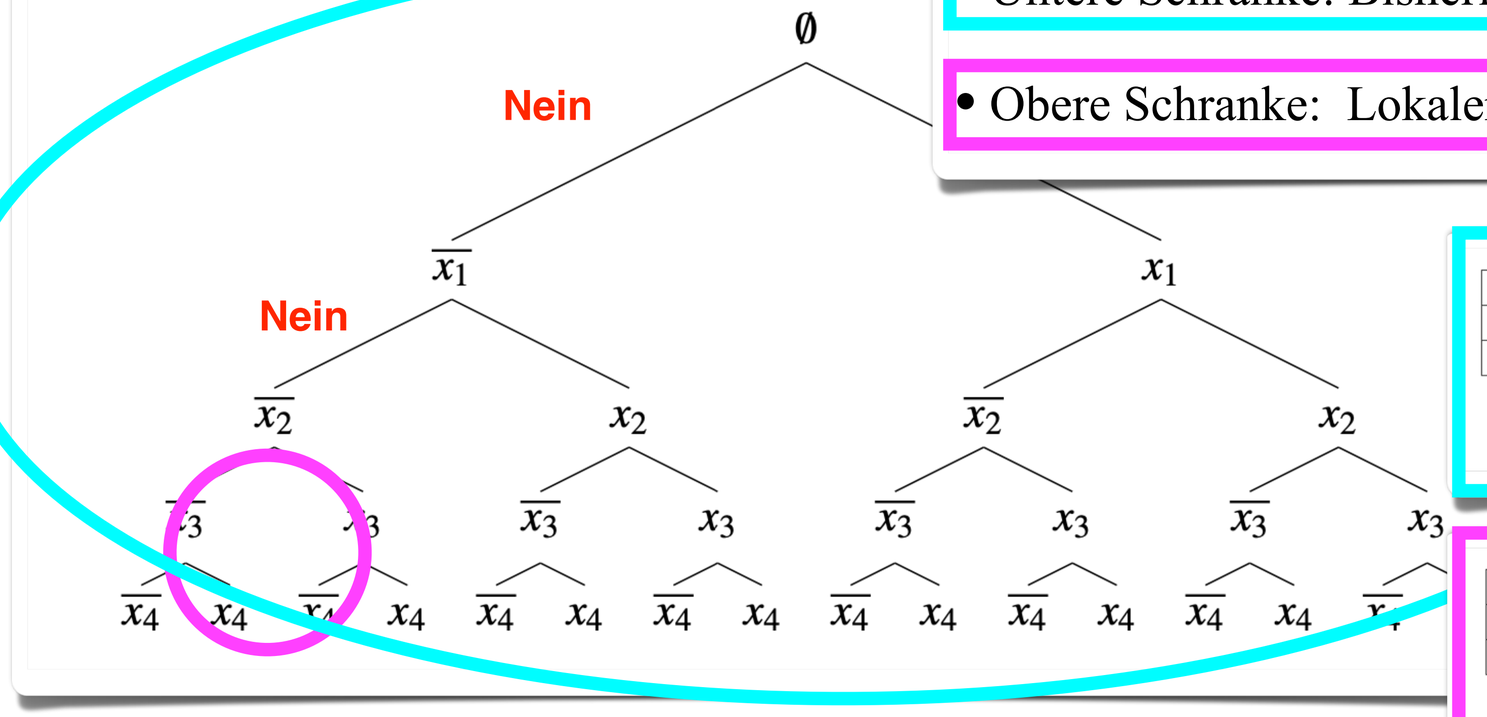

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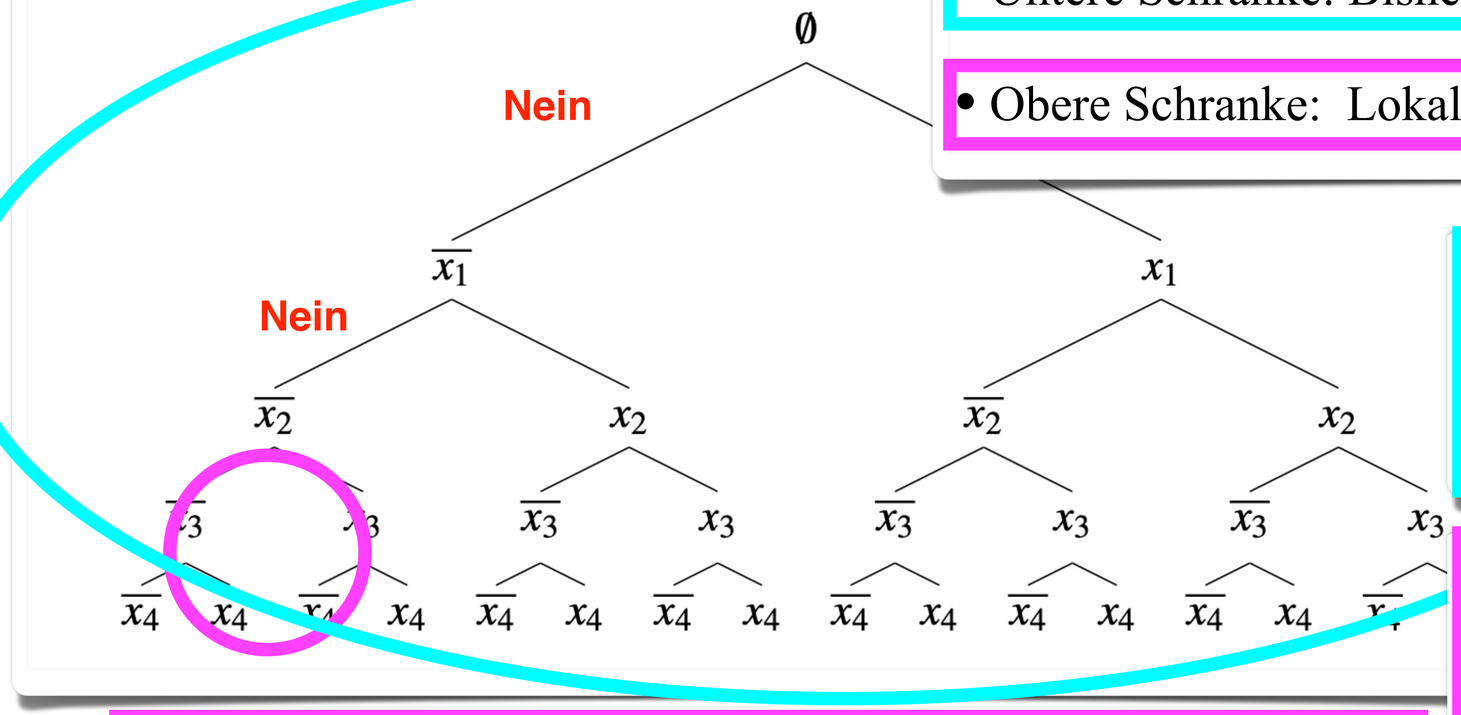

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$\sum_{i=1}^{n} p_i x_i$ = 46 x_{15} = 0,6

Relaxierung: Einfacher durch größere Lösungsmenge

3. Berechnung von Schranken

3. Berechnung von Schranken

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- Untere Schranke: Bisheriger Bestwert für Greedy (ganzzahlig)

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p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$\sum_{i=1}^{n} p_i x_i$ = 46 $\quad x_{15} = 0{,}6$

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Relaxierung: Schnell und optimal zu lösen durch größere Lösungsmenge

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$\sum_{i=1}^{n} p_i x_i$ = 46 $\quad x_{15} = 0{,}6$

Heuristik: Schnell zu lösen, aber nicht notwendig optimal

Relaxierung: Schnell und optimal zu lösen durch größere Lösungsmenge

3. Berechnung von Schranken II

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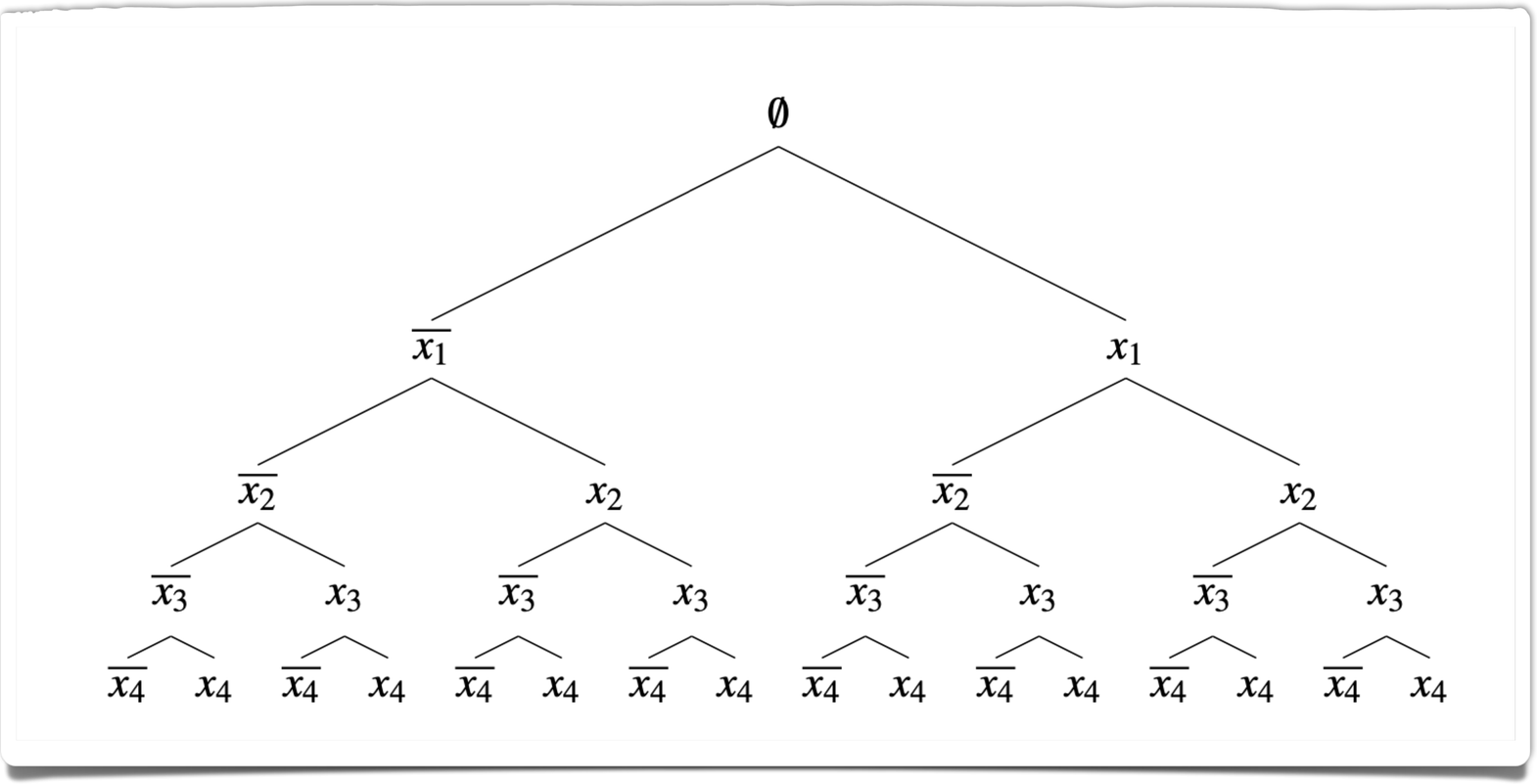
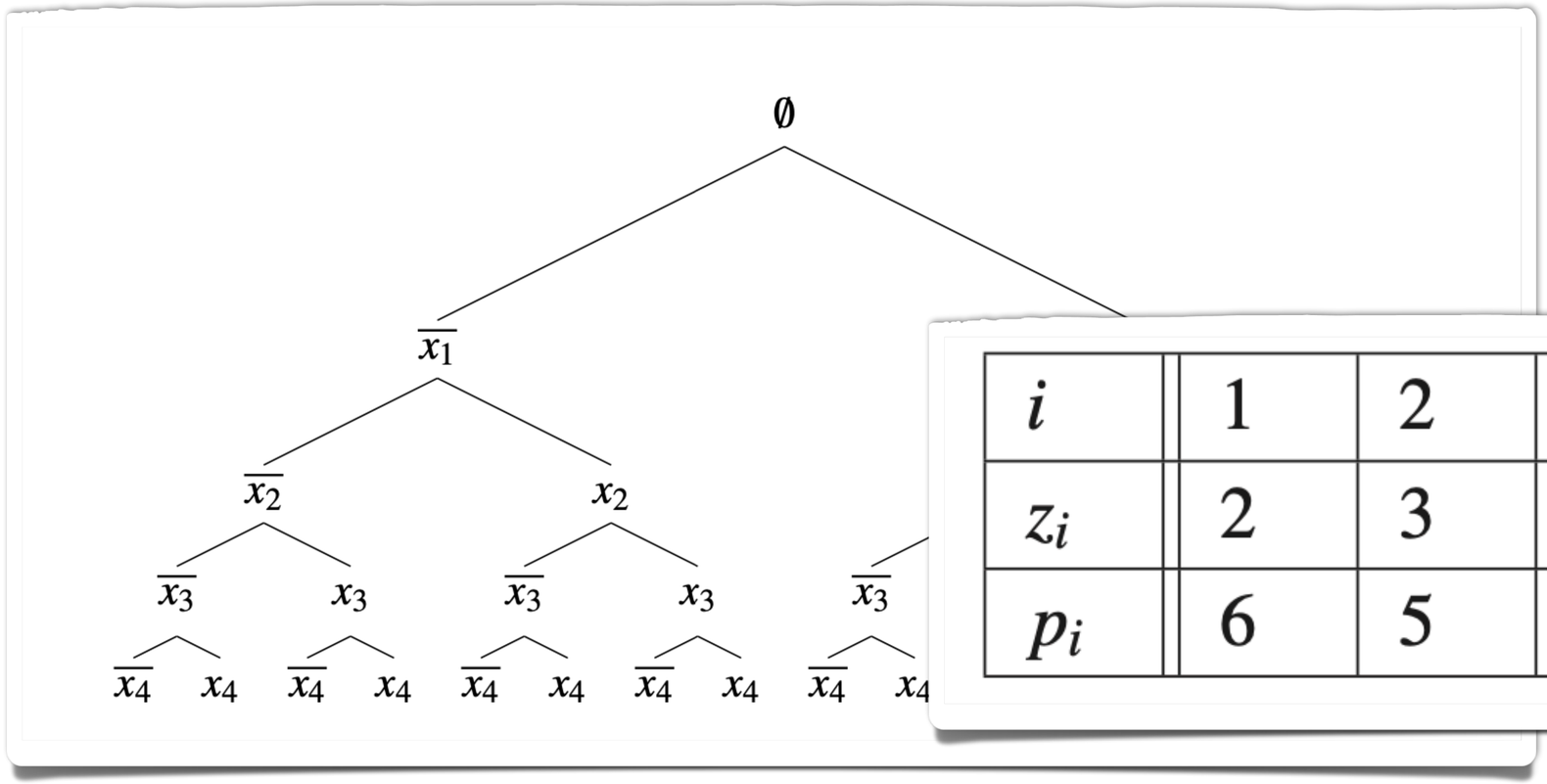
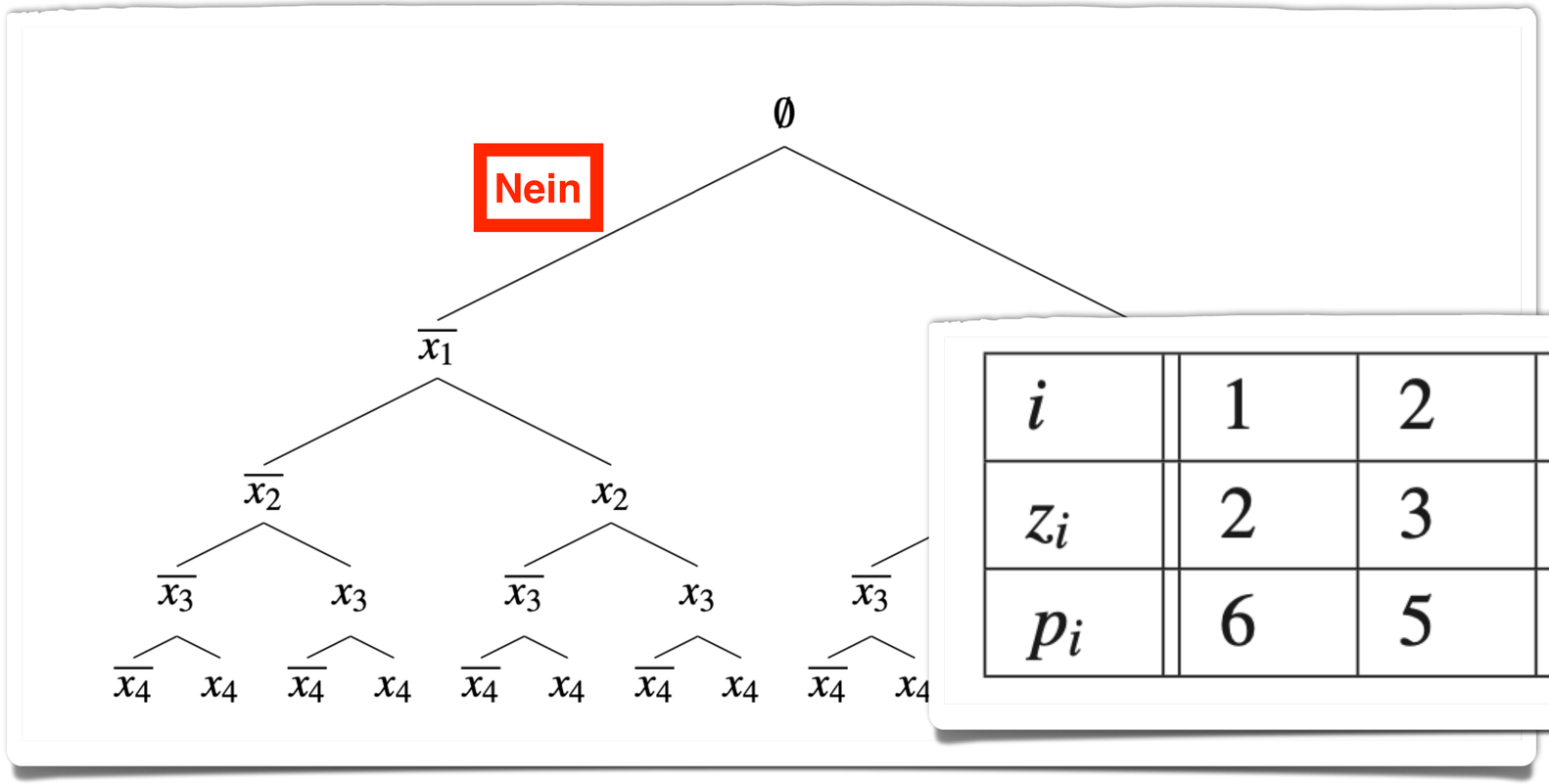


3. Berechnung von Schranken II



i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

3. Berechnung von Schranken II



i	1	2	3	3	5	6	7
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p_i	6	5	8	9	6	7	3

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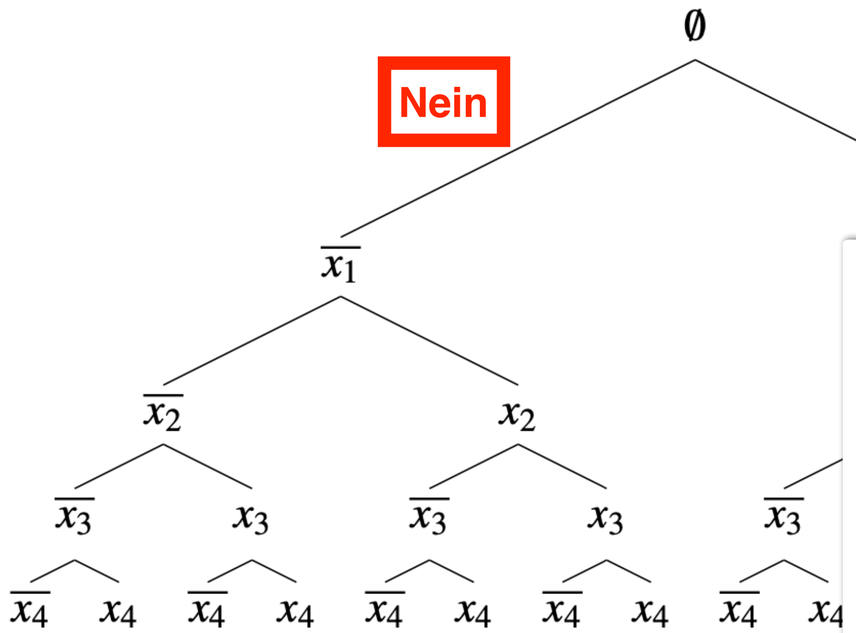

i	1	2	3	3	5	6	7
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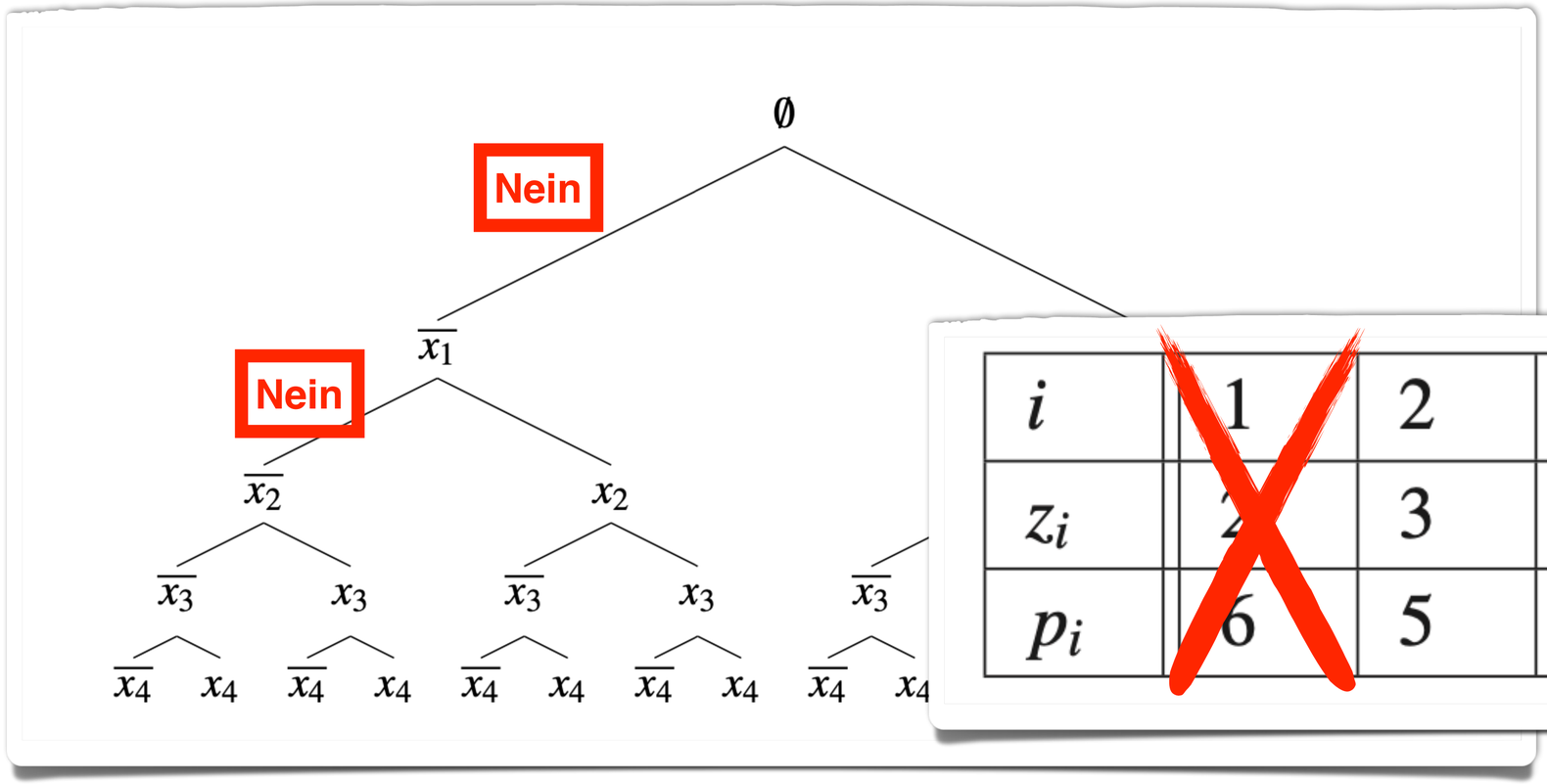
3. Berechnung von Schranken II



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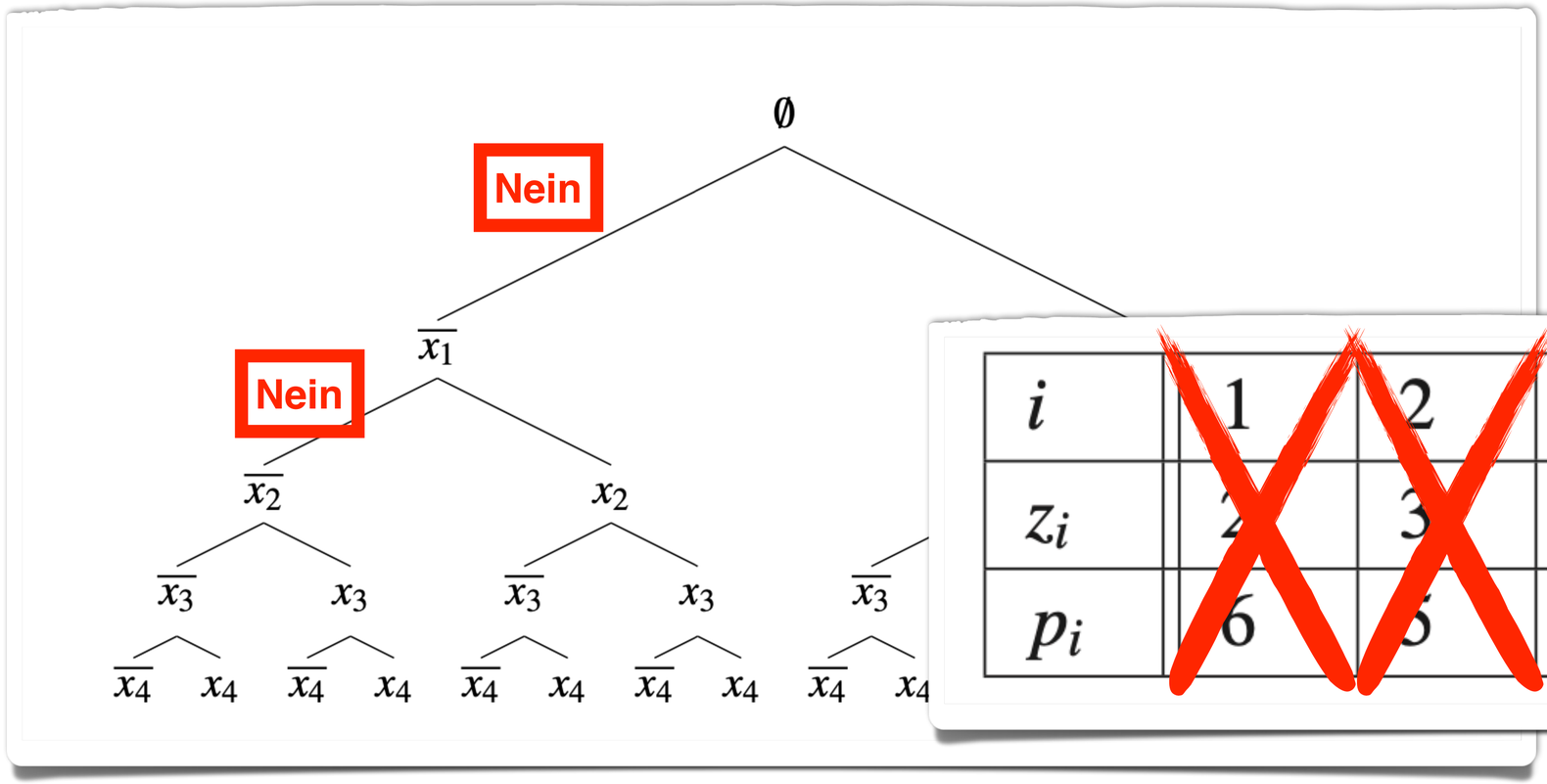

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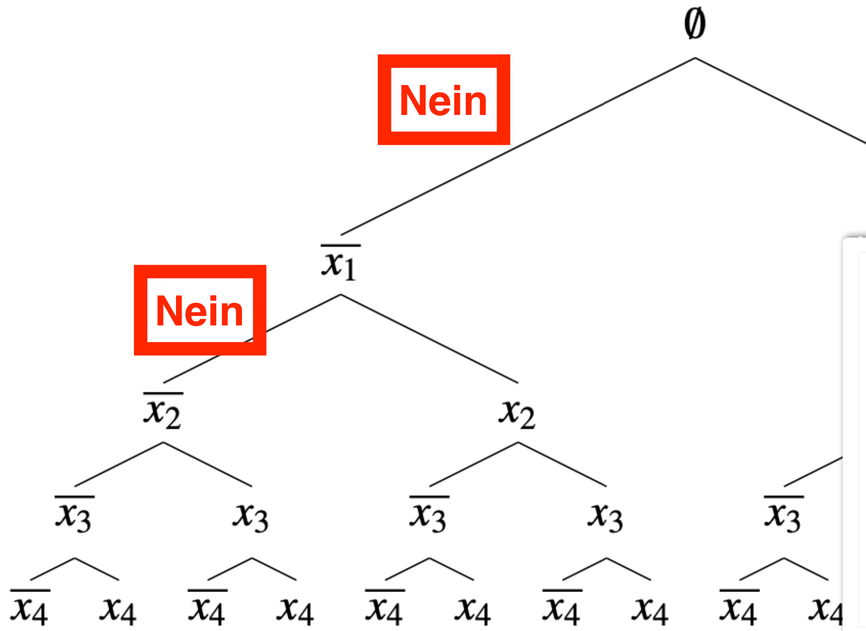
3. Berechnung von Schranken II

S: positiv fixiert ($x_i = 1$)
$\overline{S}$: negativ fixiert ($x_i = 0$)



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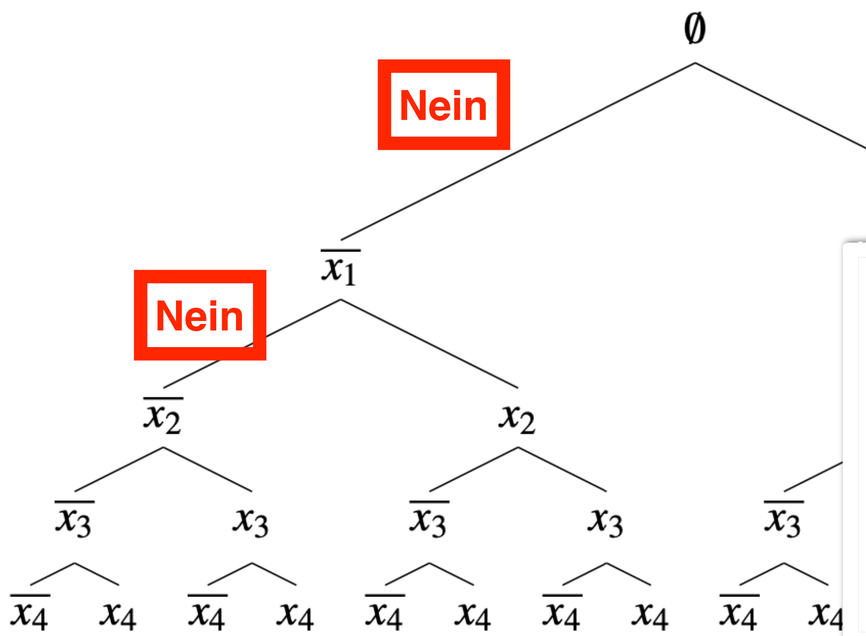

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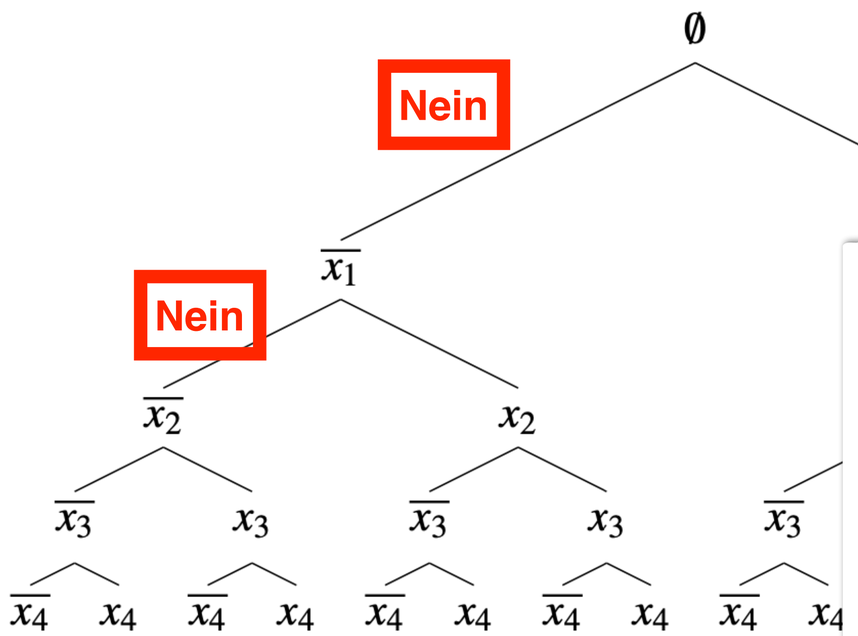

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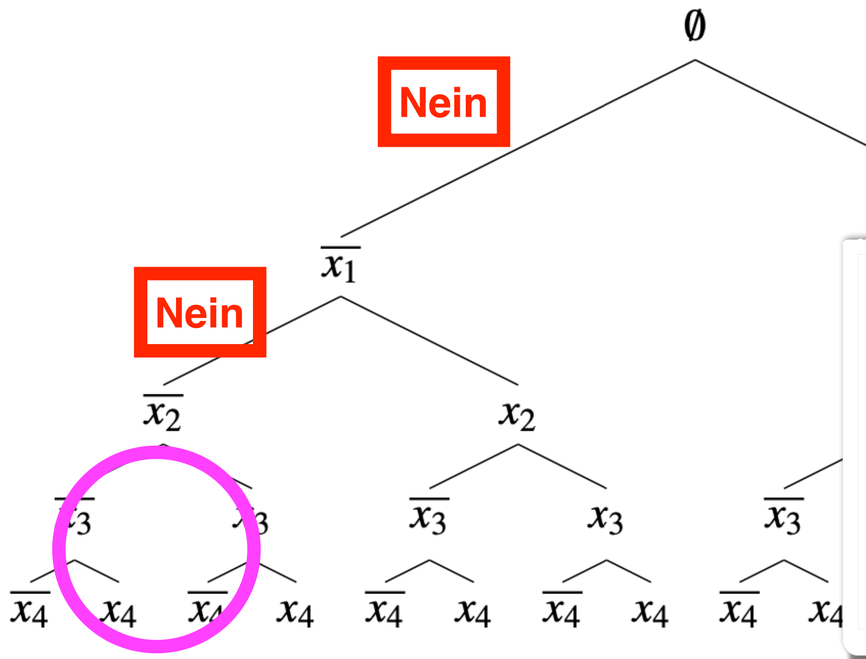

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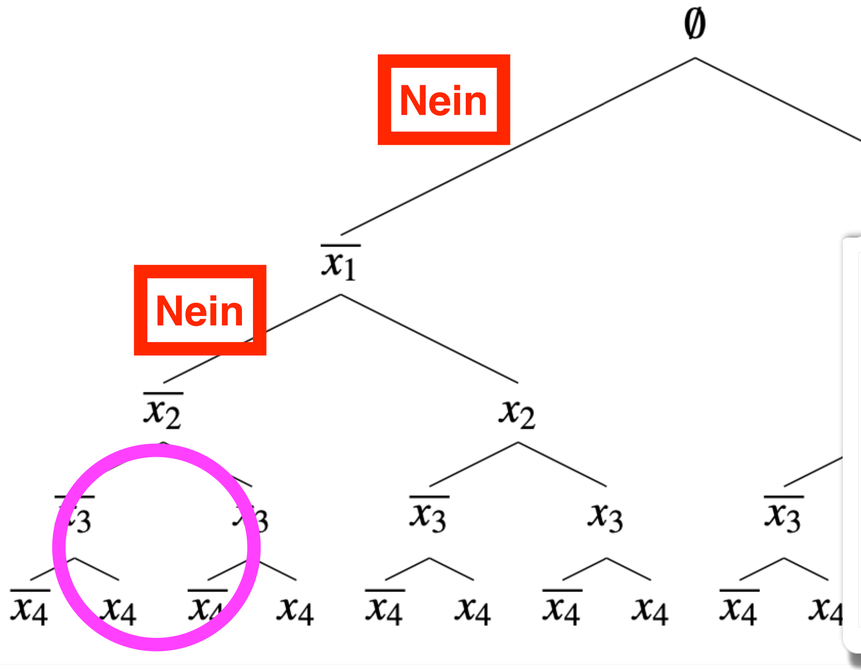

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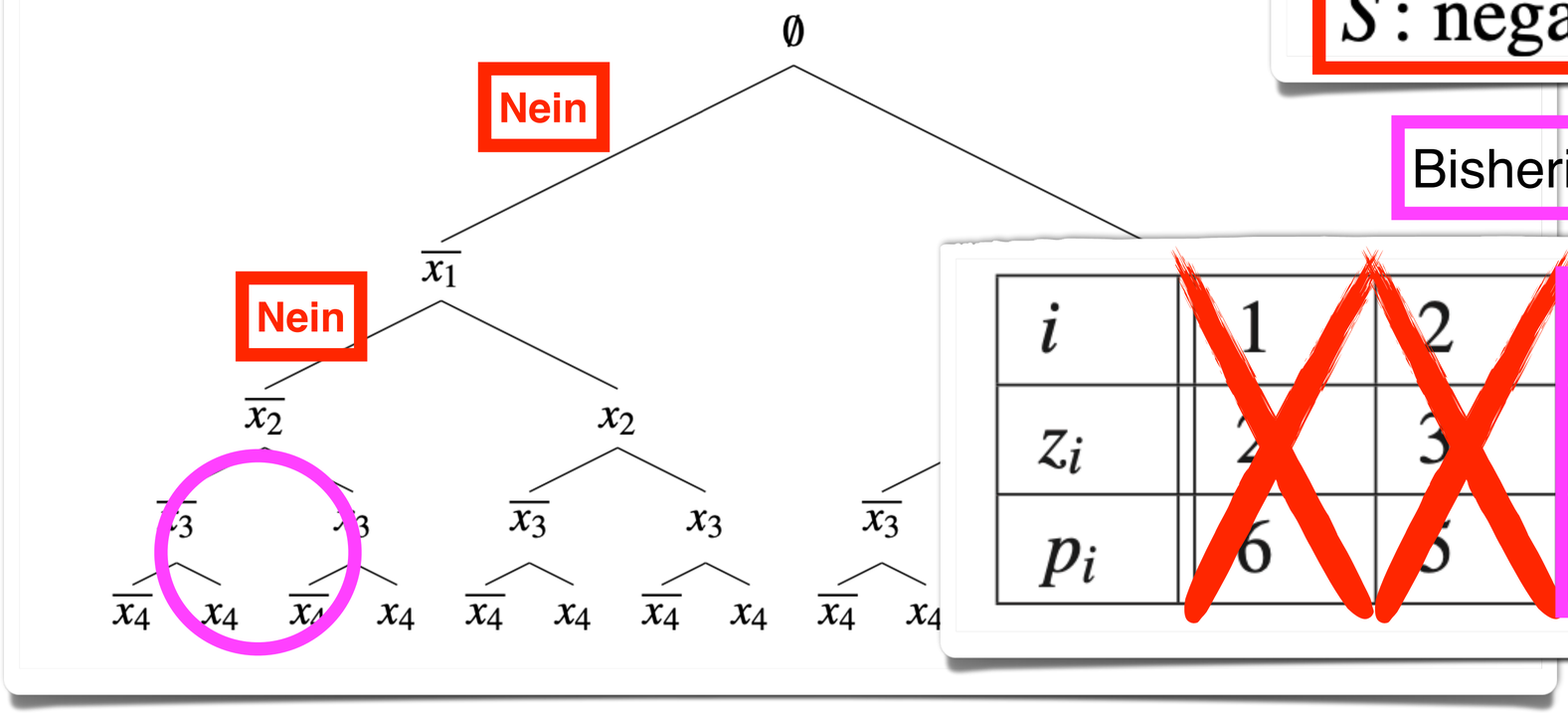

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Bisherige Entscheidungen fixieren Teilmenge!

i	1	2	3	3	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

3.3 Durchführung

Beispiel: Knapsack

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i						
z_i						
p_i						

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1
z_i	2
p_i	6

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2
z_i	2	3
p_i	6	5

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3
z_i	2	3	6
p_i	6	5	8

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4
z_i	2	3	6	7
p_i	6	5	8	9

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5
z_i	2	3	6	7	5
p_i	6	5	8	9	6

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5	6
z_i	2	3	6	7	5	9
p_i	6	5	8	9	6	7

Beispiel: Knapsack

Beispiel 2.4 (Knapsackproblem).

Sei $Z = 9$ und seien folgende sieben Objekte gegeben:

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Untere Schranke: Greedy (ganzzahlig)

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i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Untere Schranke: Greedy (ganzzahlig)

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

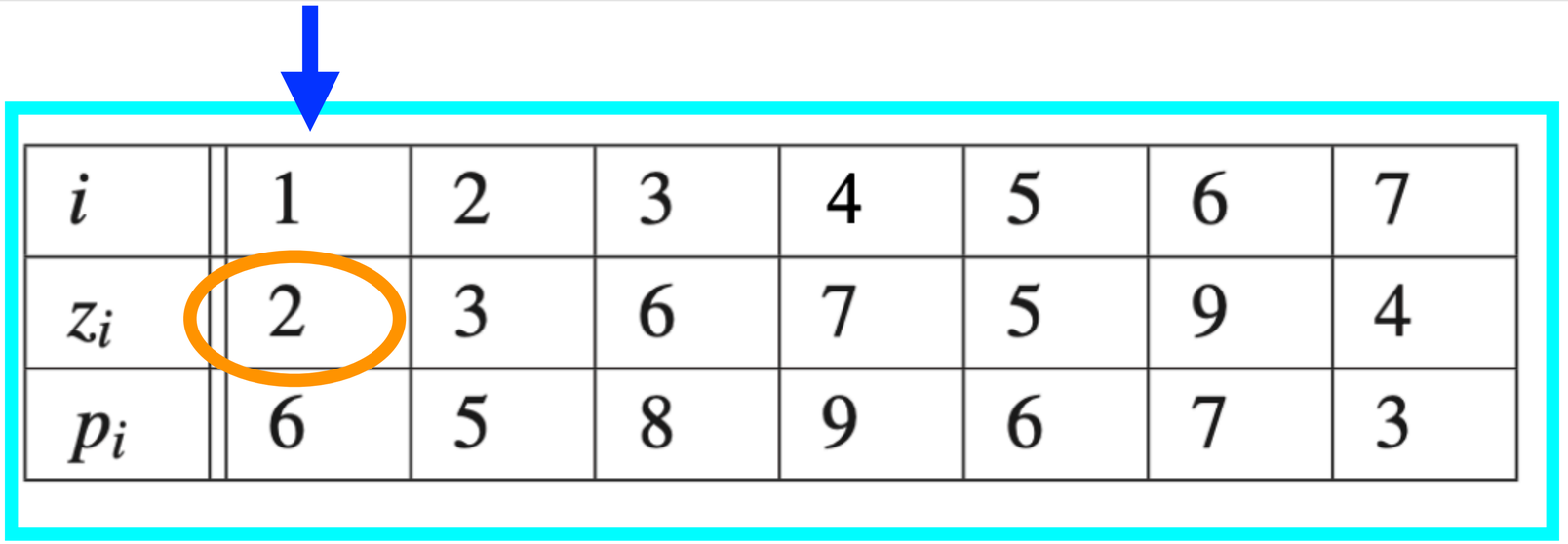
S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)



S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

$S=\emptyset$

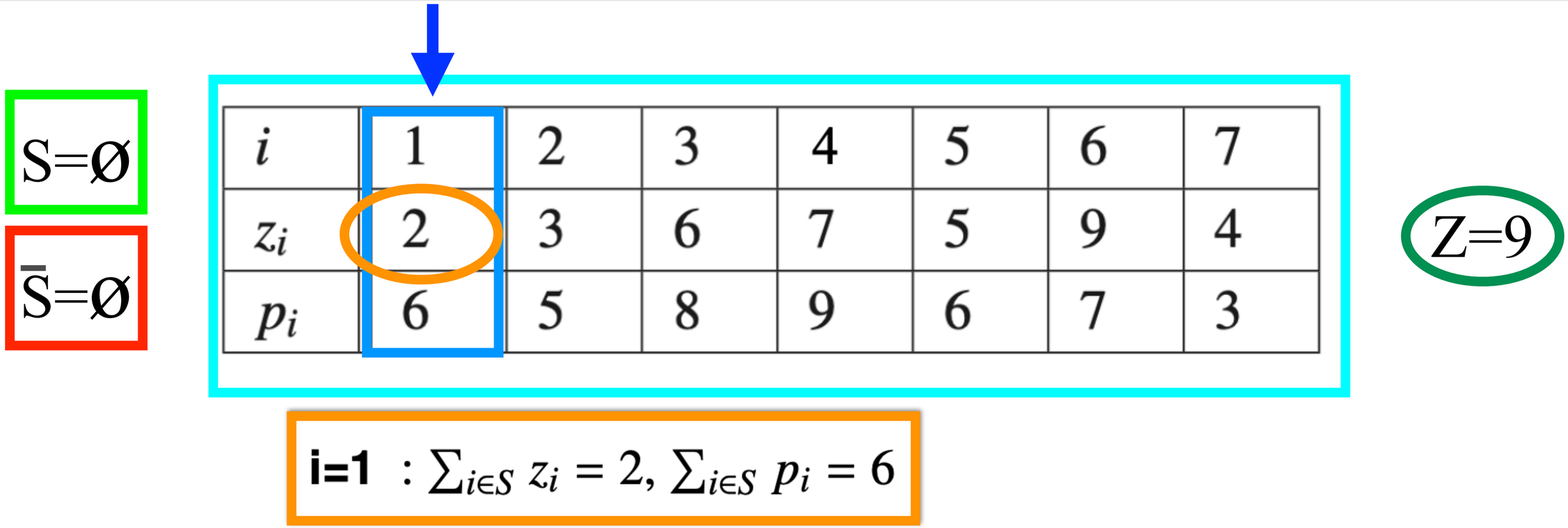
$\bar{S}=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=1 : $\sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$

Untere Schranke: Greedy (ganzzahlig)



S=Ø

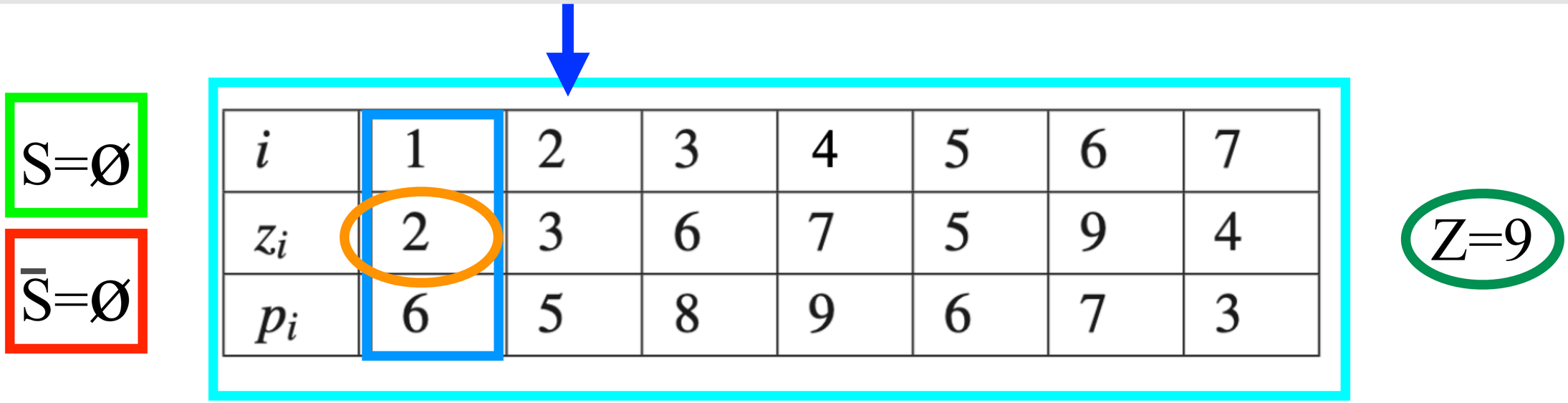
$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

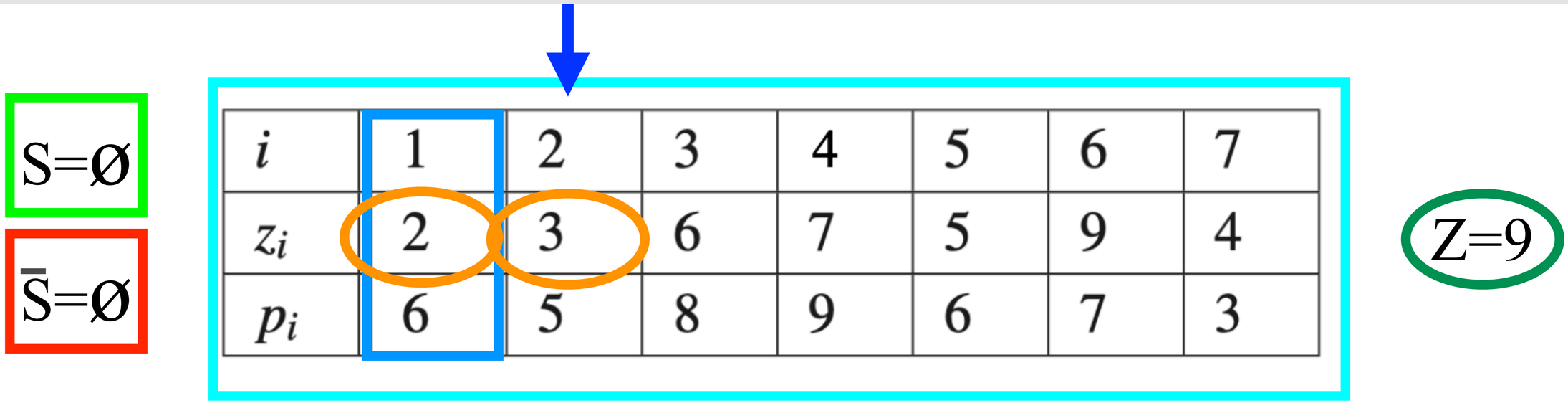
Z=9

i=1 : $\sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$

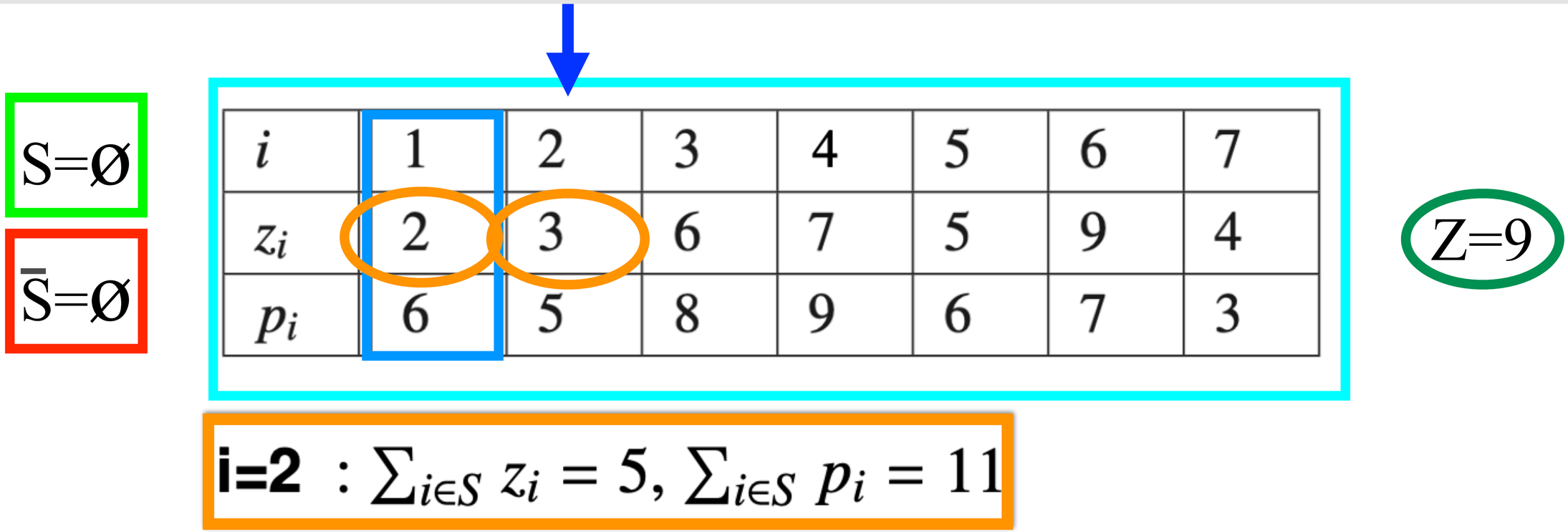
Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)



S=Ø

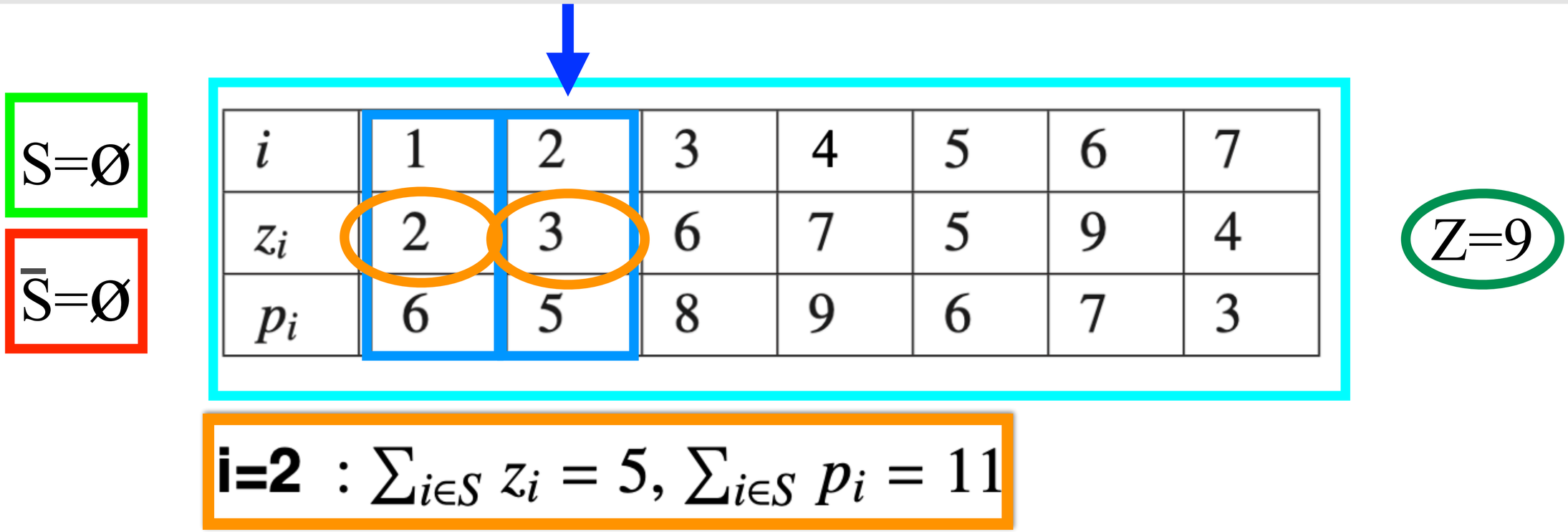
$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

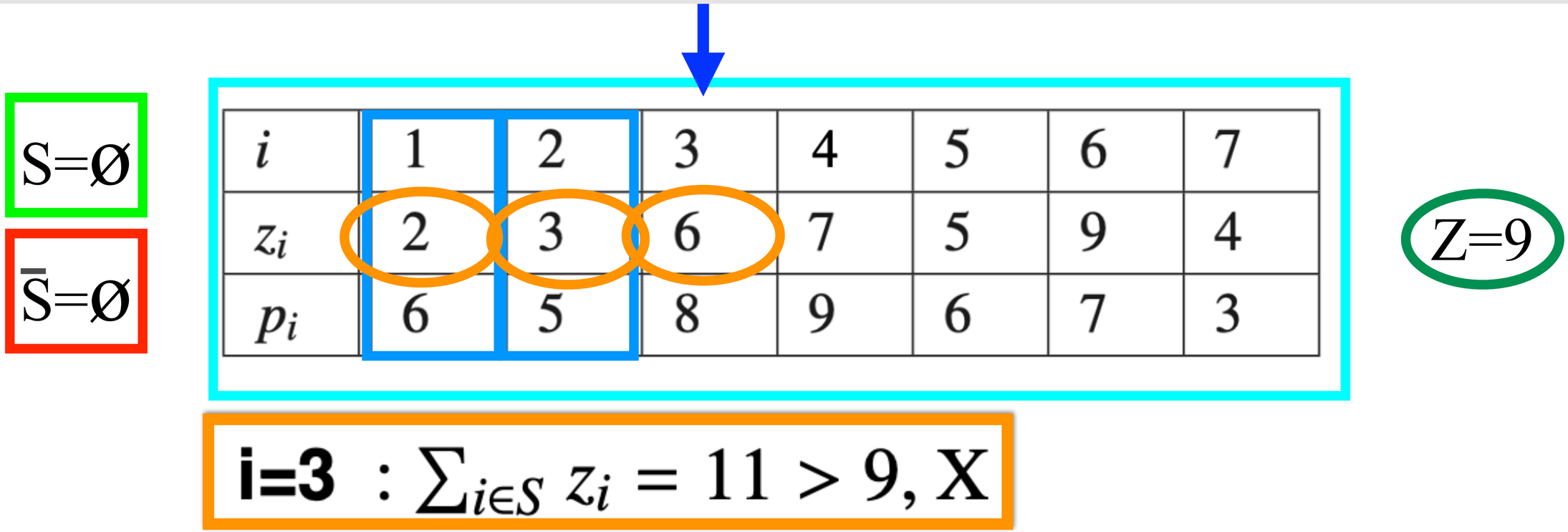
S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)

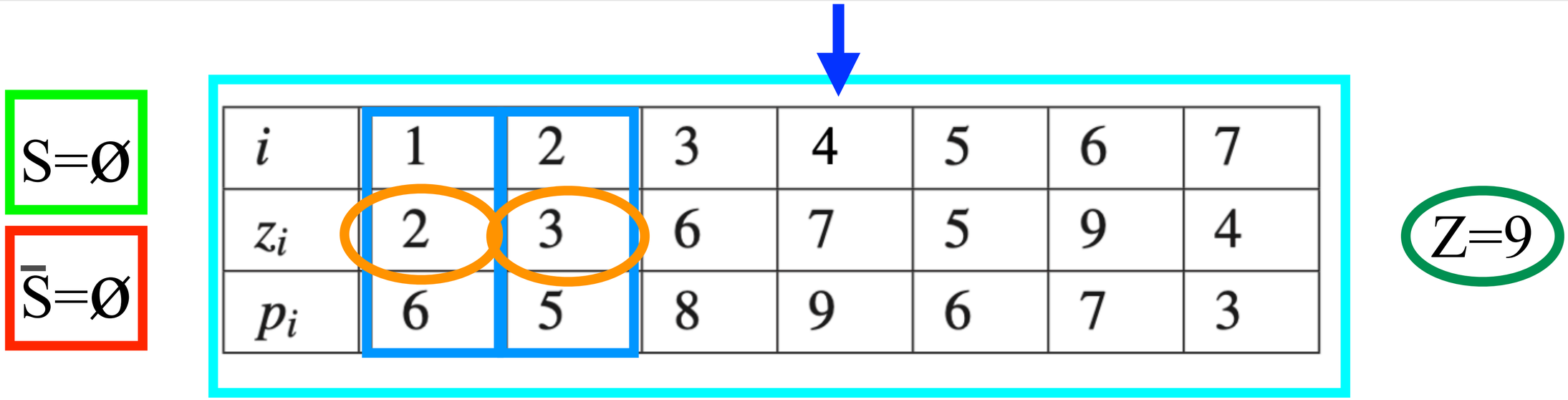
S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)

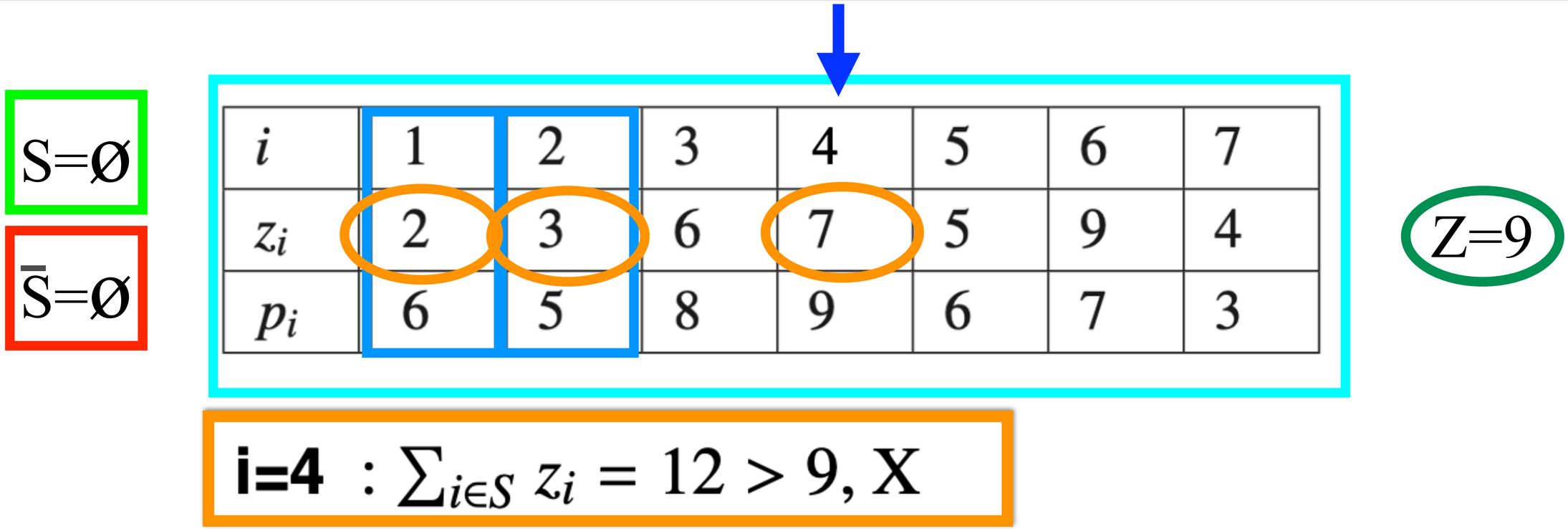
S=Ø

$\bar{S}$=Ø

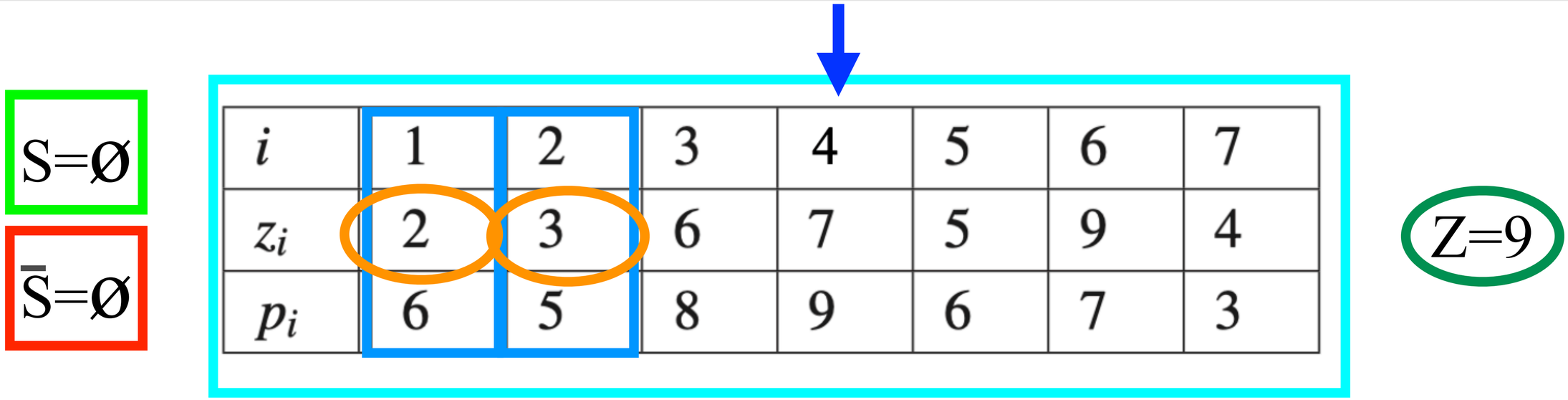
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

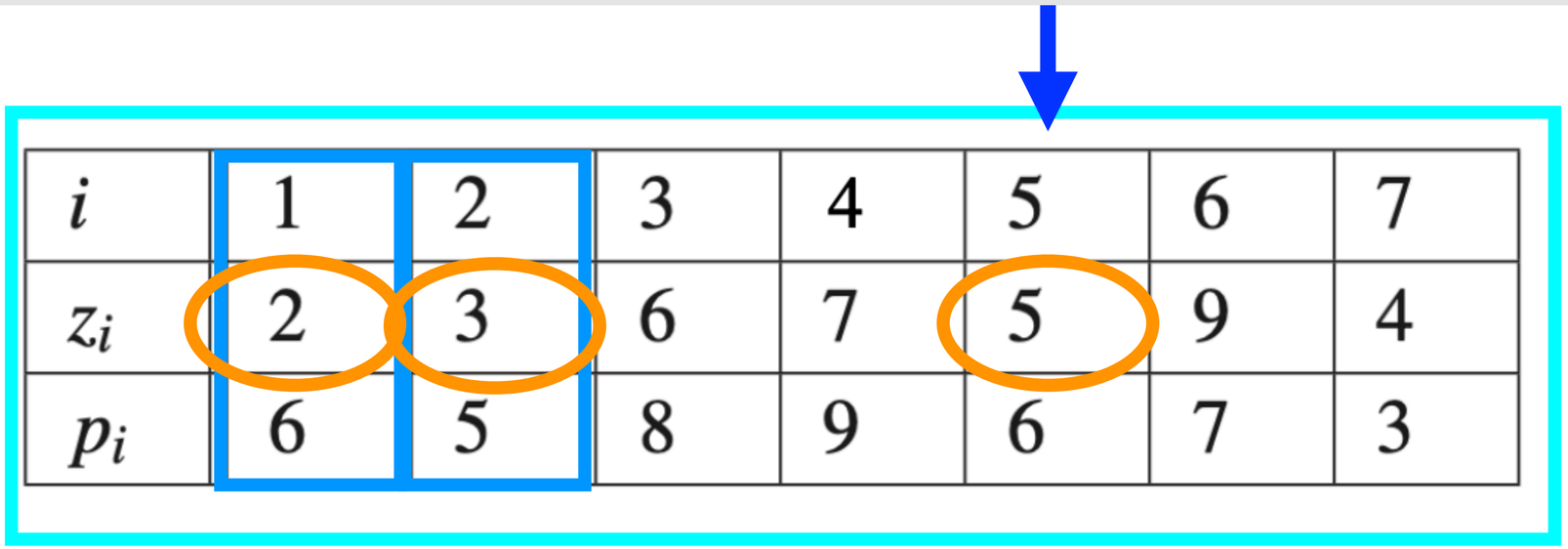
i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

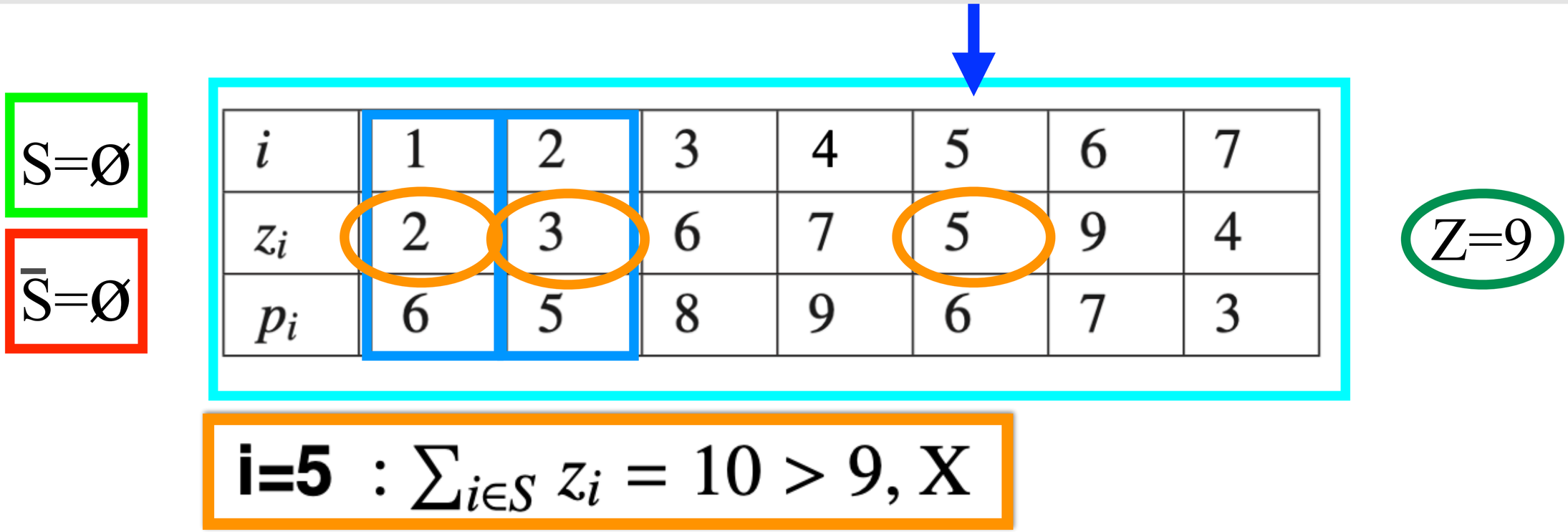
$\bar{S}$=Ø



i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)



Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=6 : $\sum_{i \in S} z_i = 14 > 9$, X

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=7 : $\sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=7 : $\sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=7 : $\sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=7 : $\sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$

Untere Schranke: Greedy (ganzzahlig)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=7 : $\sum_{i \in S} z_i = 9, \sum_{i \in S} p_i = 14$

LB=14

Obere Schranke: Greedy (fraktional)

Obere Schranke: Greedy (fraktional)

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Obere Schranke: Greedy (fraktional)

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Obere Schranke: Greedy (fraktional)

S=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

$\bar{S}=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=1 : $\sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=1 : $\sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=1 : $\sum_{i \in S} z_i = 2, \sum_{i \in S} p_i = 6$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
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Obere Schranke: Greedy (fraktional)

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i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: Z-$\sum_{i \in S} z_i = 4$, also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: $Z - \sum_{i \in S} z_i = 4$, also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: $Z - \sum_{i \in S} z_i = 4$, also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

$\bar{S}=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=2 : $\sum_{i\in S} z_i = 5, \sum_{i\in S} p_i = 11$

Bleibt: $Z-\sum_{i\in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z-\sum_{i\in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

$\bar{S}=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=2 : $\sum_{i\in S} z_i = 5, \sum_{i\in S} p_i = 11$

Bleibt: $Z-\sum_{i\in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z-\sum_{i\in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: $Z - \sum_{i \in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

$\bar{S}=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=2 : $\sum_{i\in S} z_i = 5, \sum_{i\in S} p_i = 11$

Bleibt: $Z-\sum_{i\in S} z_i = 4$; also $x_3 = \frac{2}{3}\left(=\frac{4}{6}=\frac{Z-\sum_{i\in S} z_i}{z_3}\right)$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: $Z - \sum_{i \in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

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i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

$Z=9$

i=2 : $\sum_{i\in S} z_i = 5, \sum_{i\in S} p_i = 11$

Bleibt: $Z-\sum_{i\in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z-\sum_{i\in S} z_i}{z_3})$

Damit: $\sum_{i\in S} x_i z_i = 9, \sum_{i\in S} x_i p_i = 11 + \frac{2}{3} * 8 = 16,\overline{3}$

Obere Schranke: Greedy (fraktional)

S=Ø

$\bar{S}$=Ø

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Z=9

i=2 : $\sum_{i \in S} z_i = 5, \sum_{i \in S} p_i = 11$

Bleibt: $Z - \sum_{i \in S} z_i = 4$; also $x_3 = \frac{2}{3} (= \frac{4}{6} = \frac{Z - \sum_{i \in S} z_i}{z_3})$

Damit: $\sum_{i \in S} x_i z_i = 9, \sum_{i \in S} x_i p_i = 11 + \frac{2}{3} * 8 = 16, \overline{3}$

Obere Schranke: Greedy (fraktional)

$S=\emptyset$

$\bar{S}=\emptyset$

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i=2 : $\sum_{i\in S} z_i = 5, \sum_{i\in S} p_i = 11$

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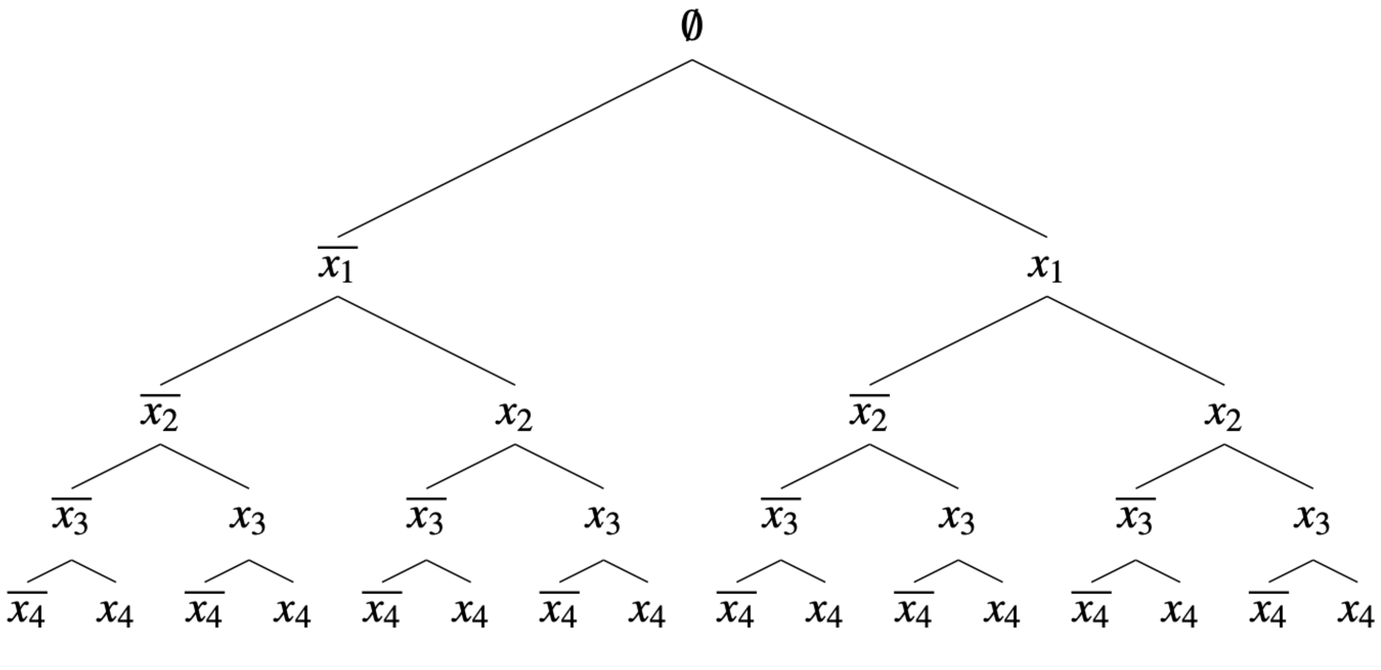
Damit: $\sum_{i\in S} x_i z_i = 9, \sum_{i\in S} x_i p_i = 11 + \frac{2}{3} * 8 = 16,\overline{3}$

UB=16

Gesamtbeispiel

Gesamtbeispiel



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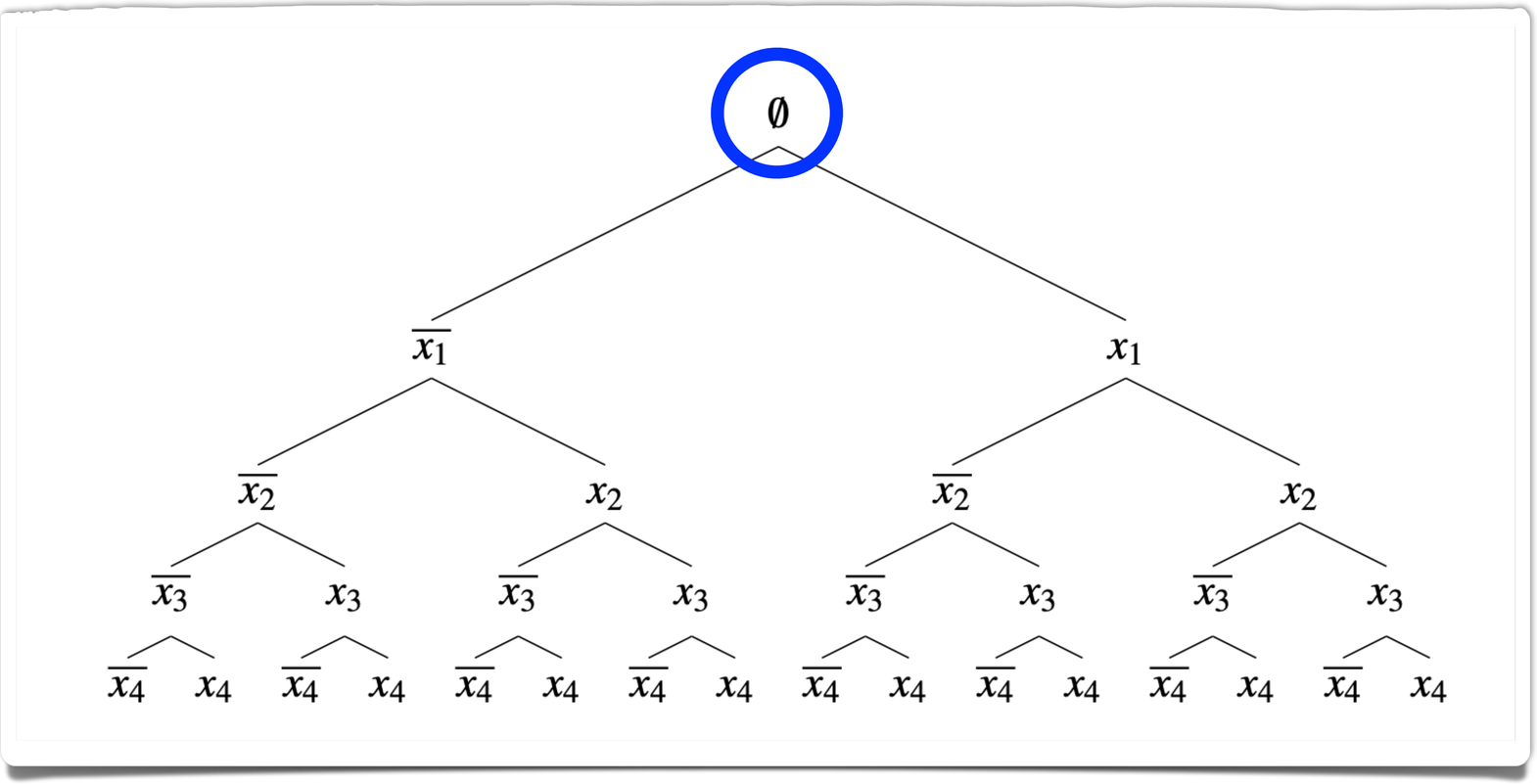

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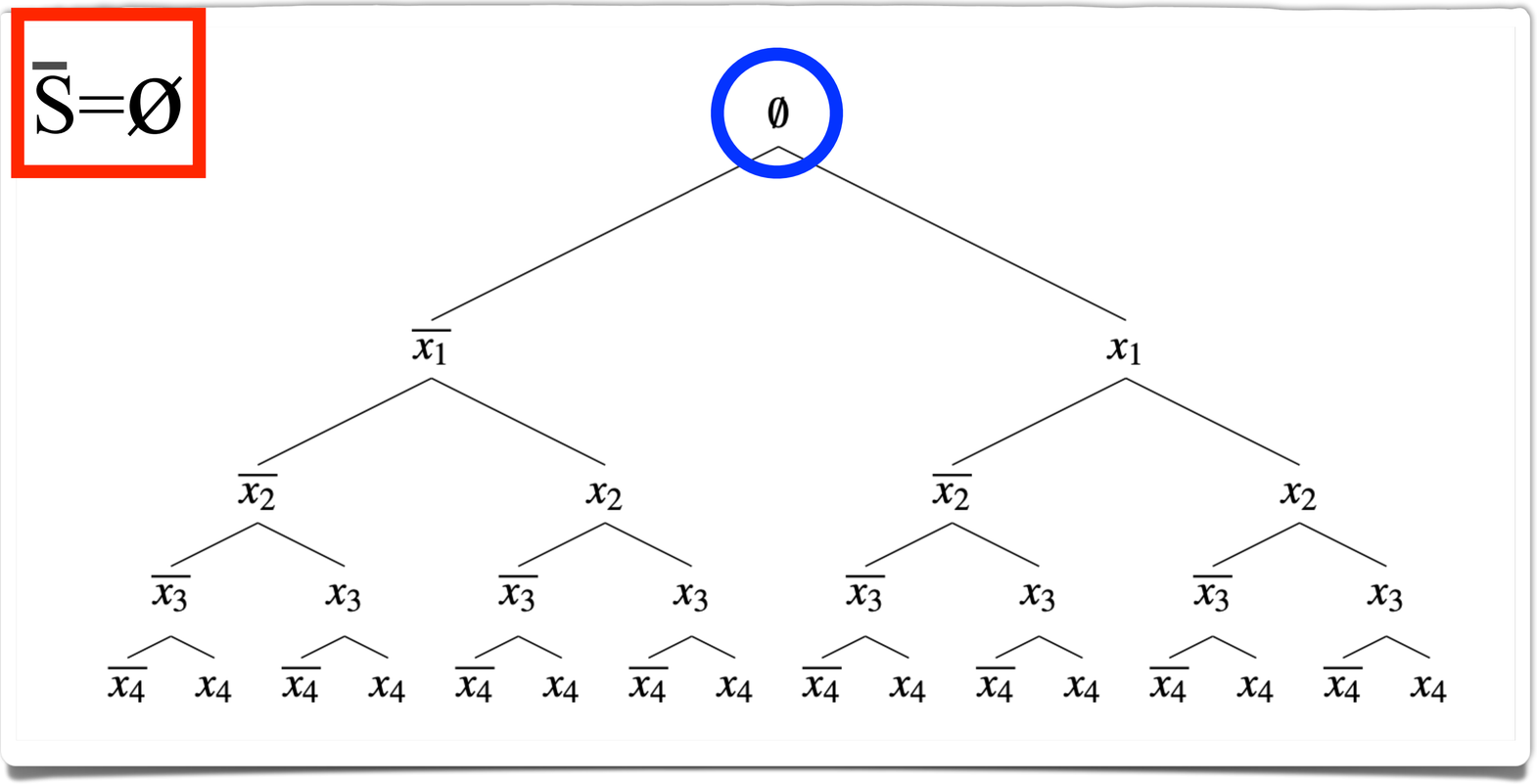

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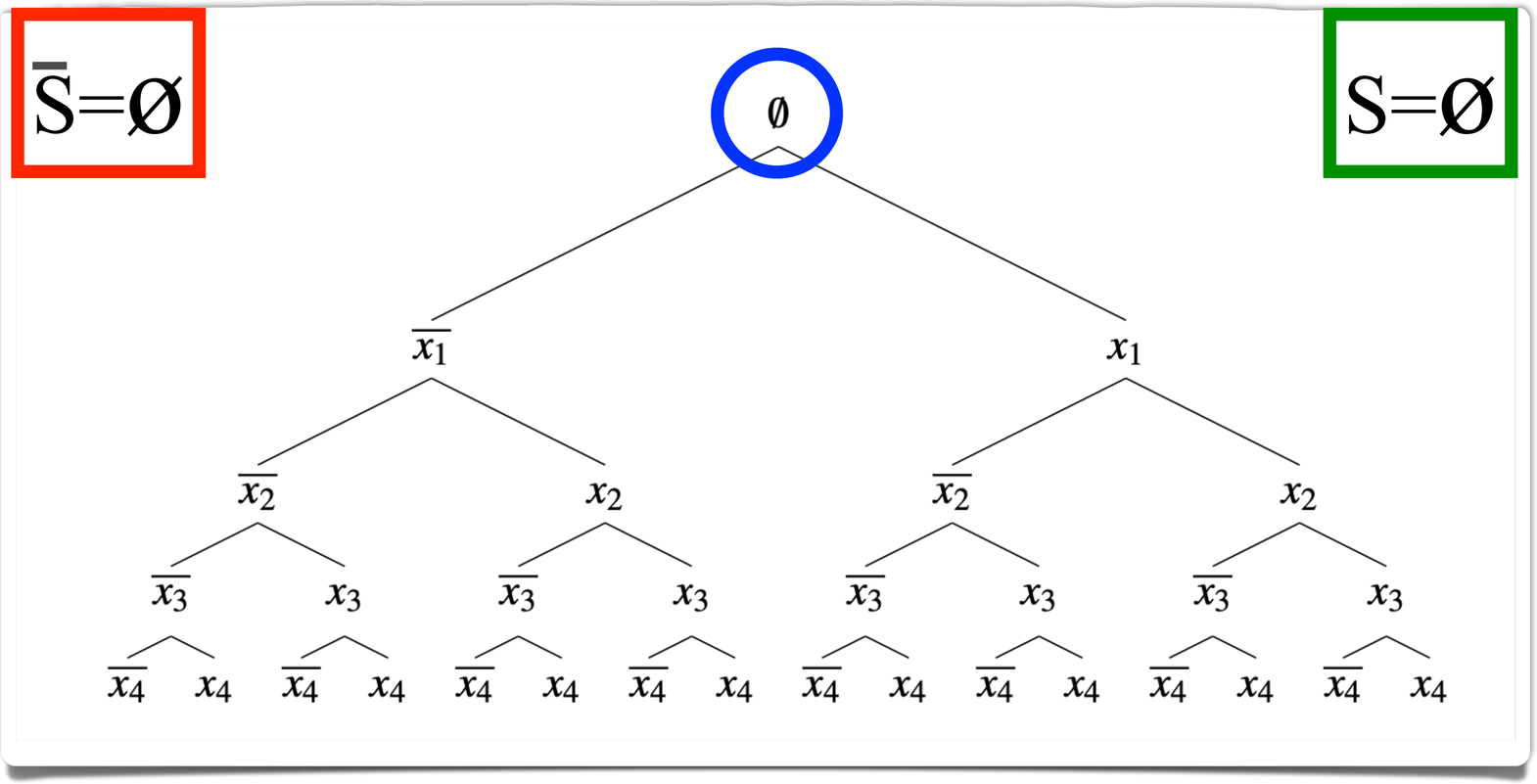
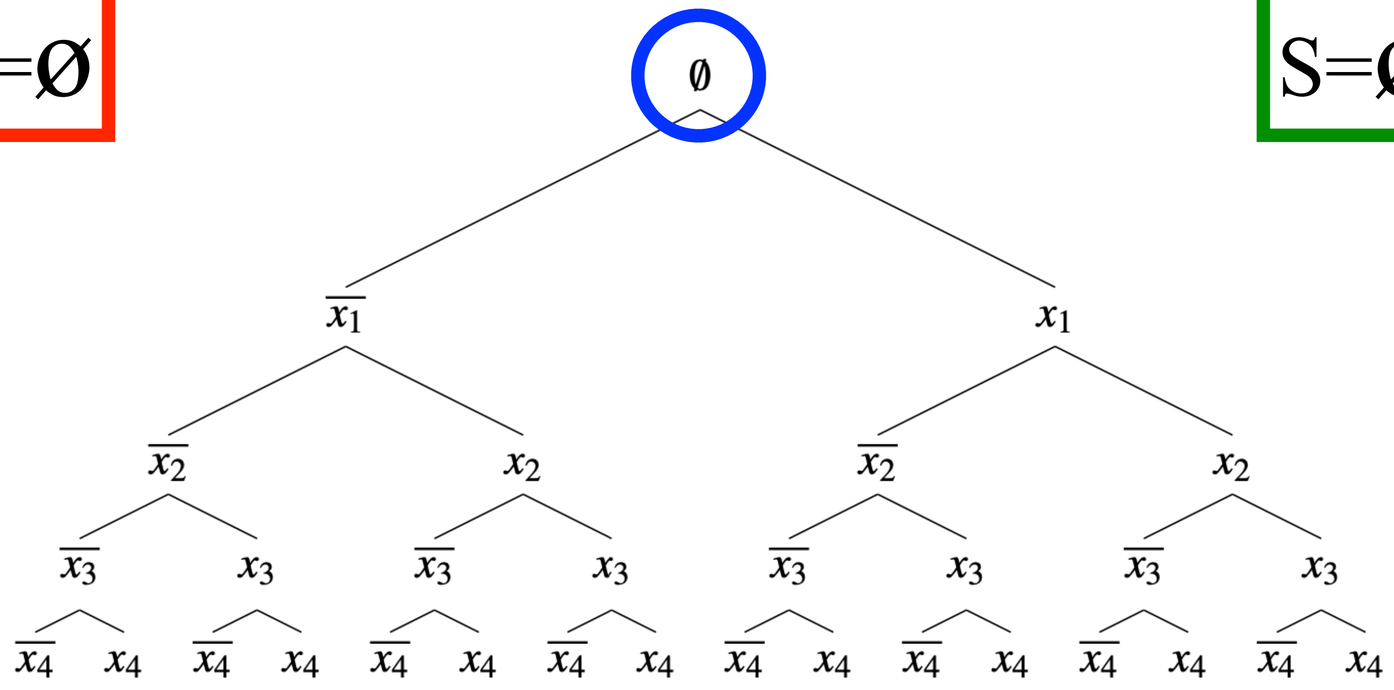


Gesamtbeispiel

$\bar{S}=\emptyset$



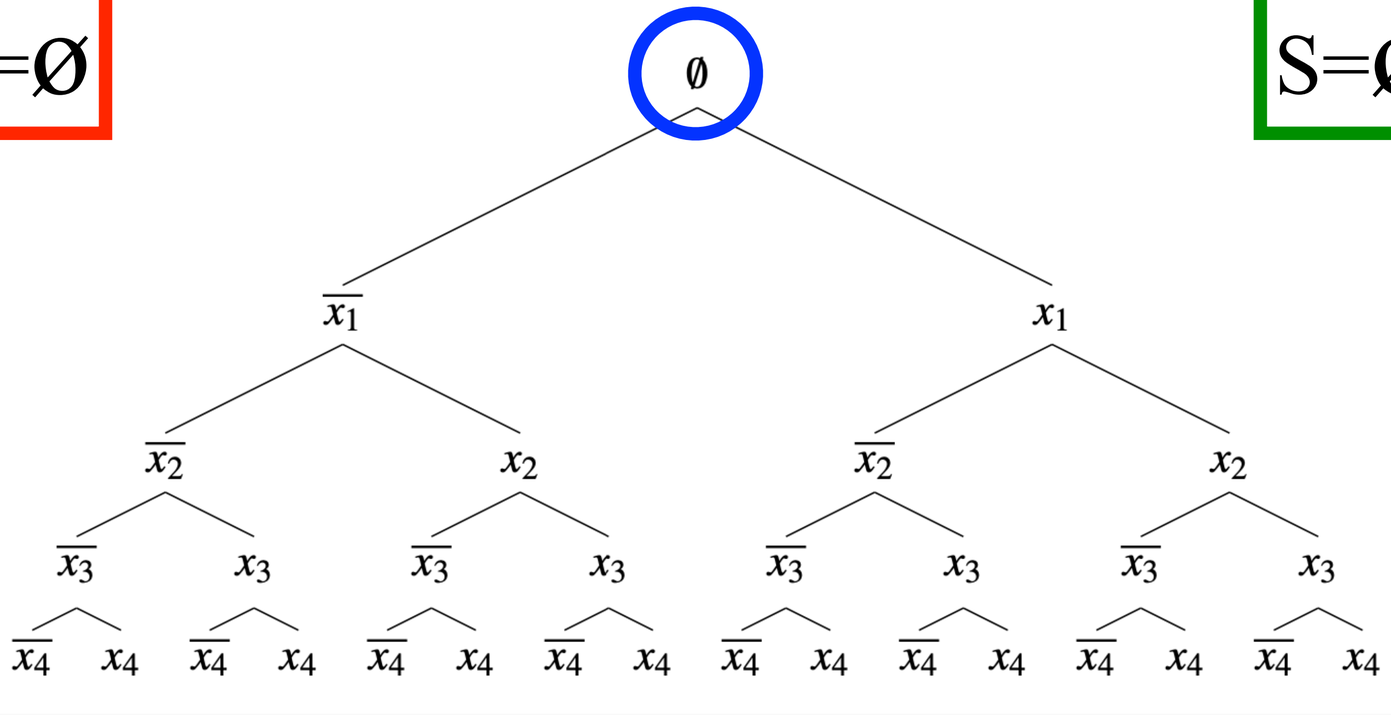
$S=\emptyset$

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

Gesamtbeispiel

$\bar{S}=\emptyset$

$S=\emptyset$



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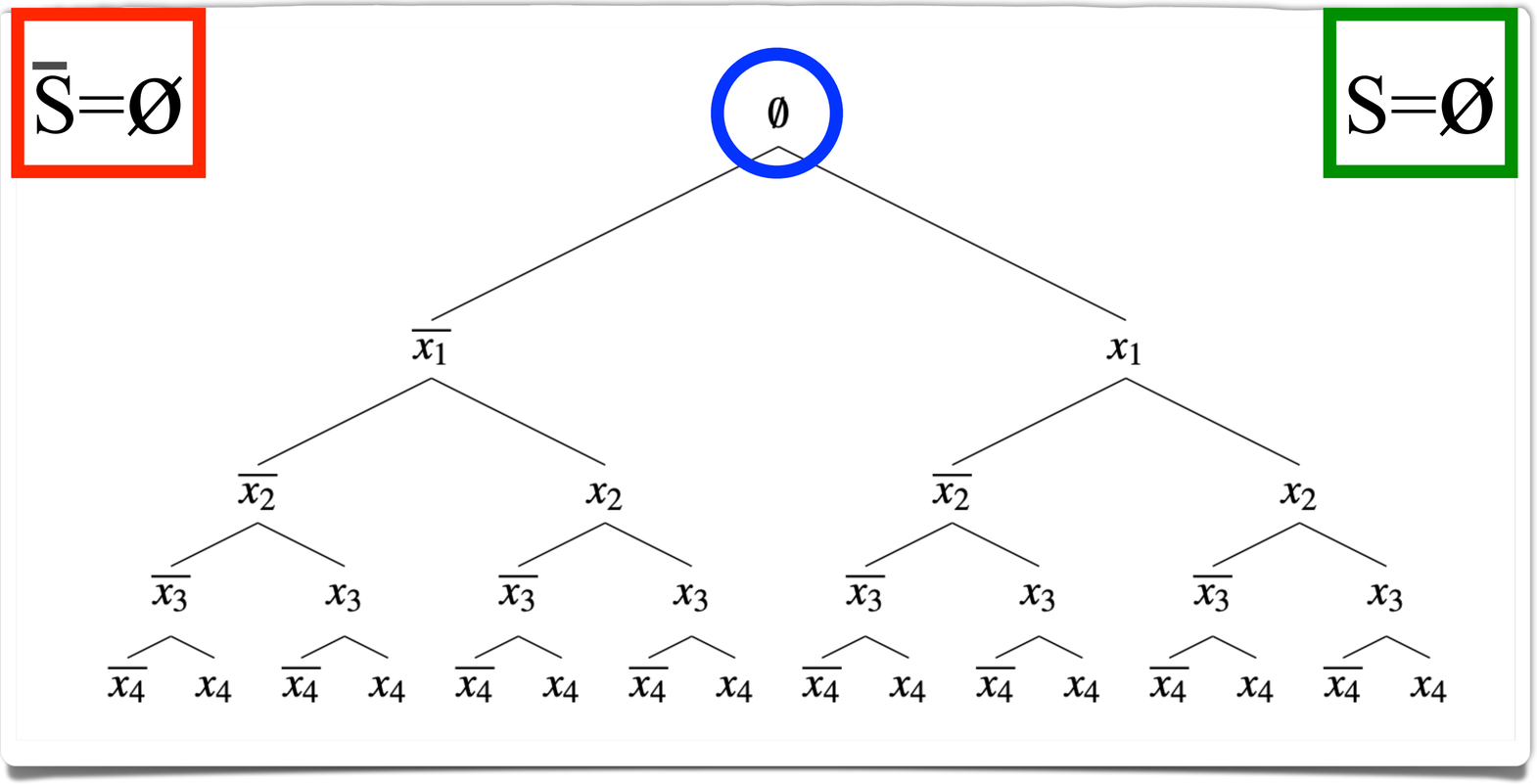
p_i	6	5	8	9	6	7	3

Gesamtbeispiel



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Gesamtbeispiel

$\bar{S}=\emptyset$

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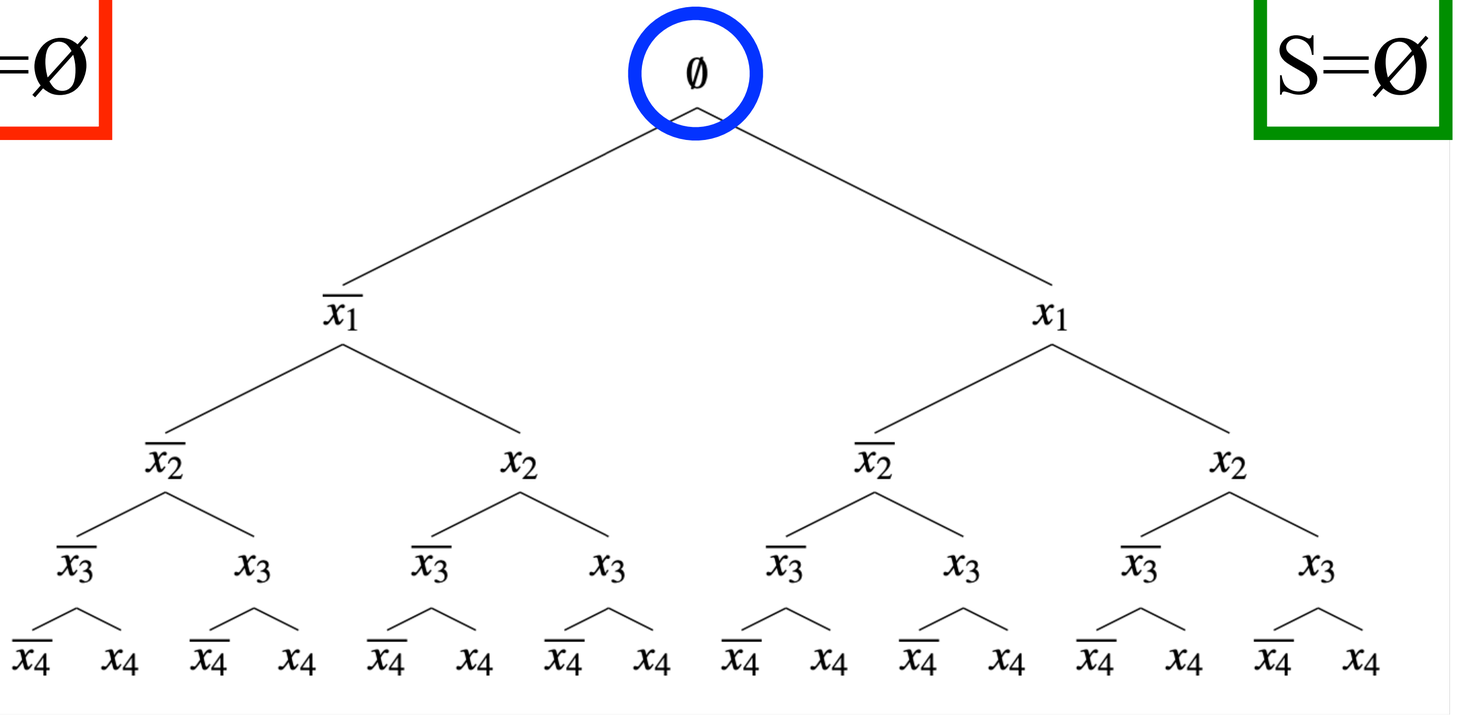

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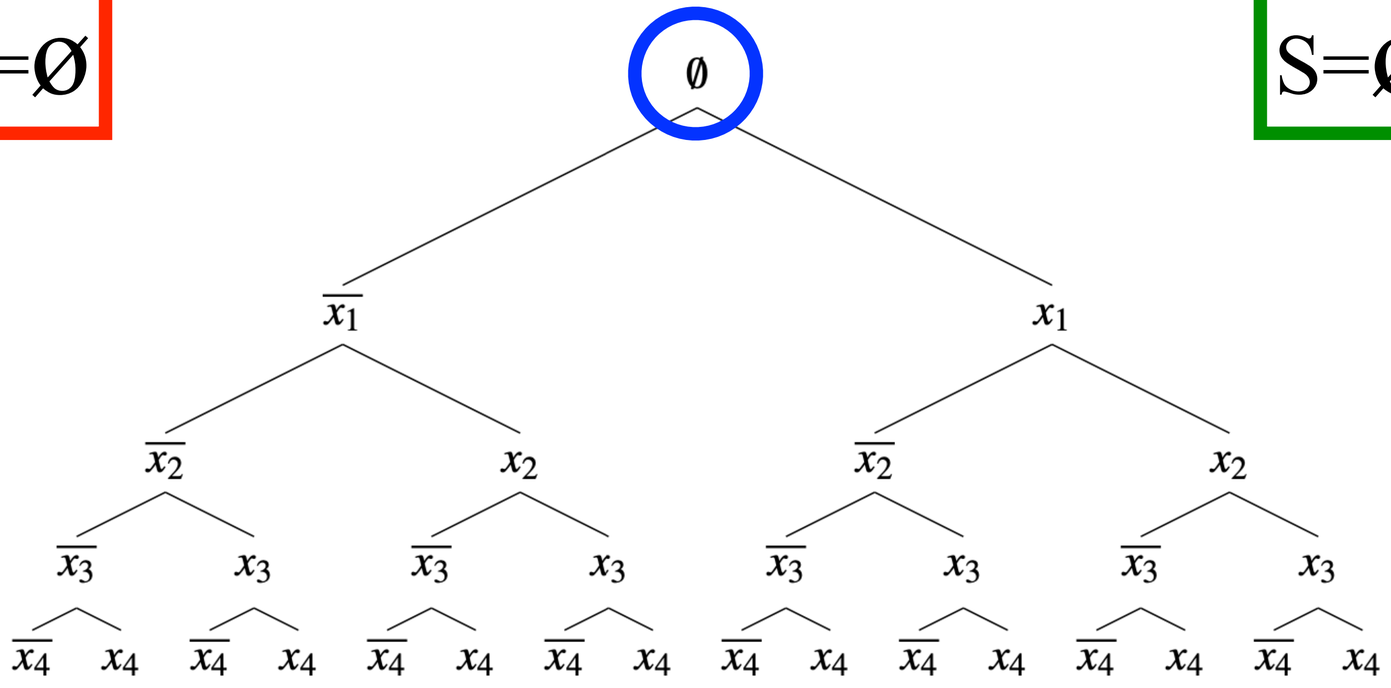
Gesamtbeispiel

$\bar{S}=\emptyset$



$S=\emptyset$

i	1	2	3	4	5	6	7
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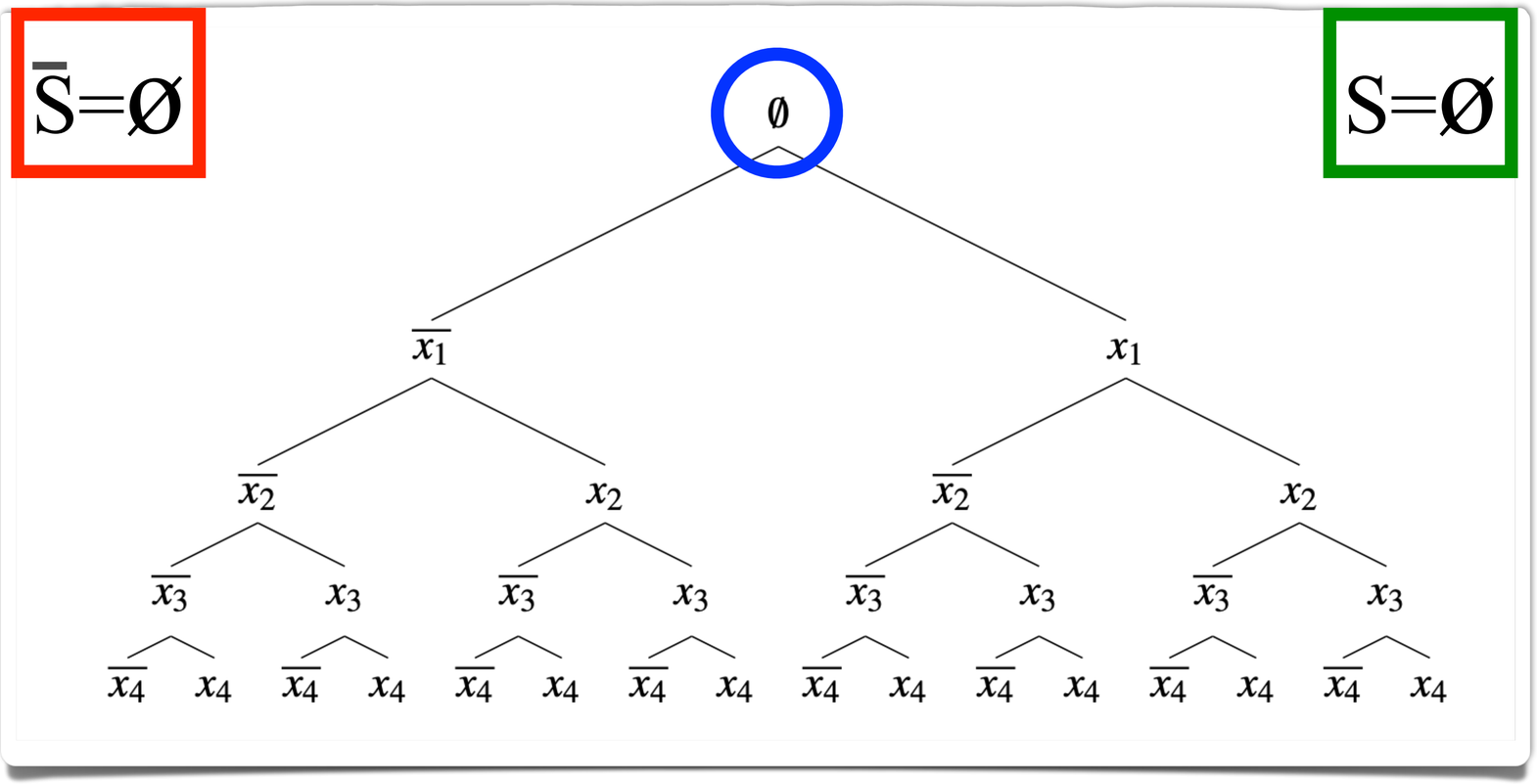
LB=14

Gesamtbeispiel

$\bar{S}=\emptyset$



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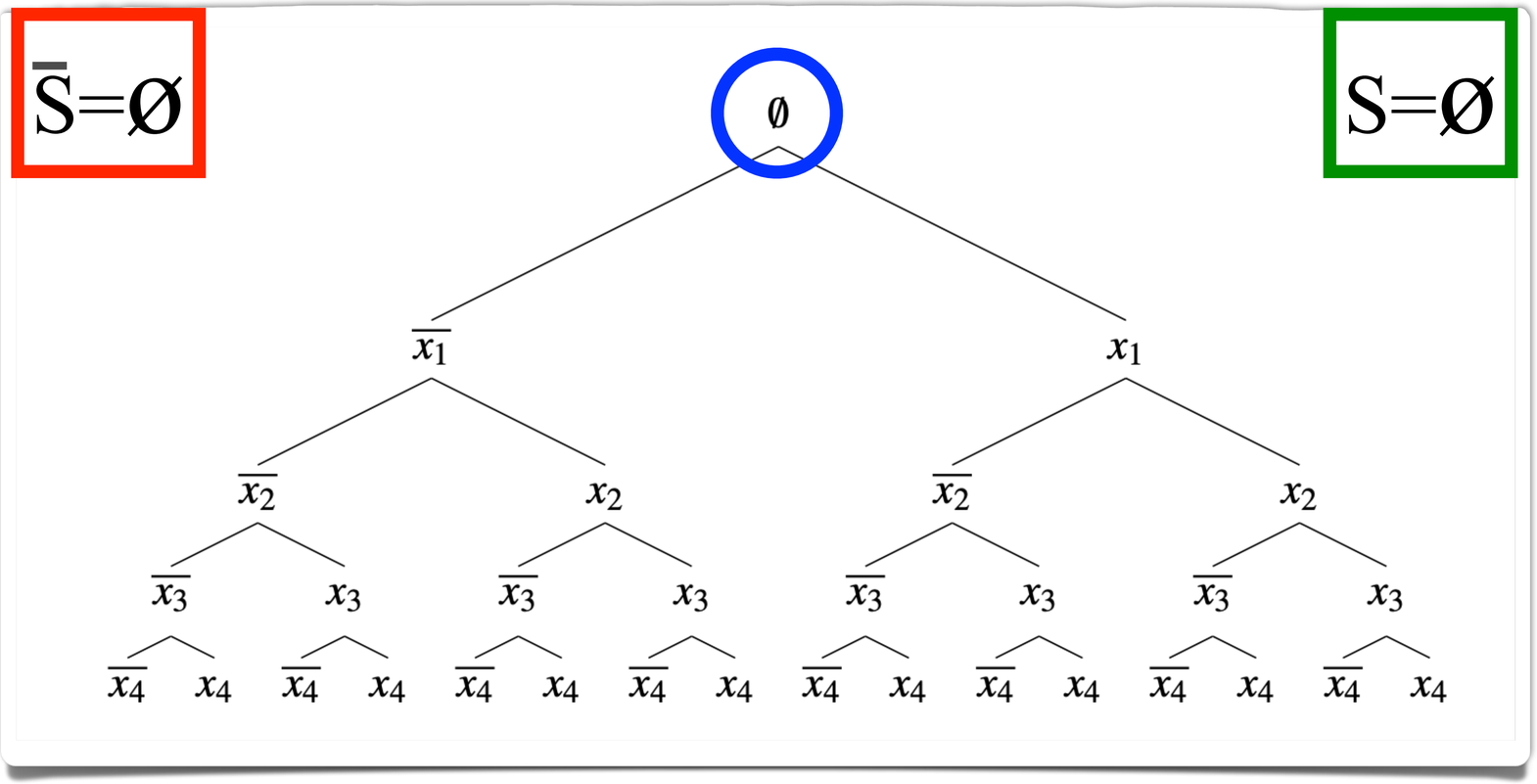
Gesamtbeispiel

$\bar{S}=\emptyset$



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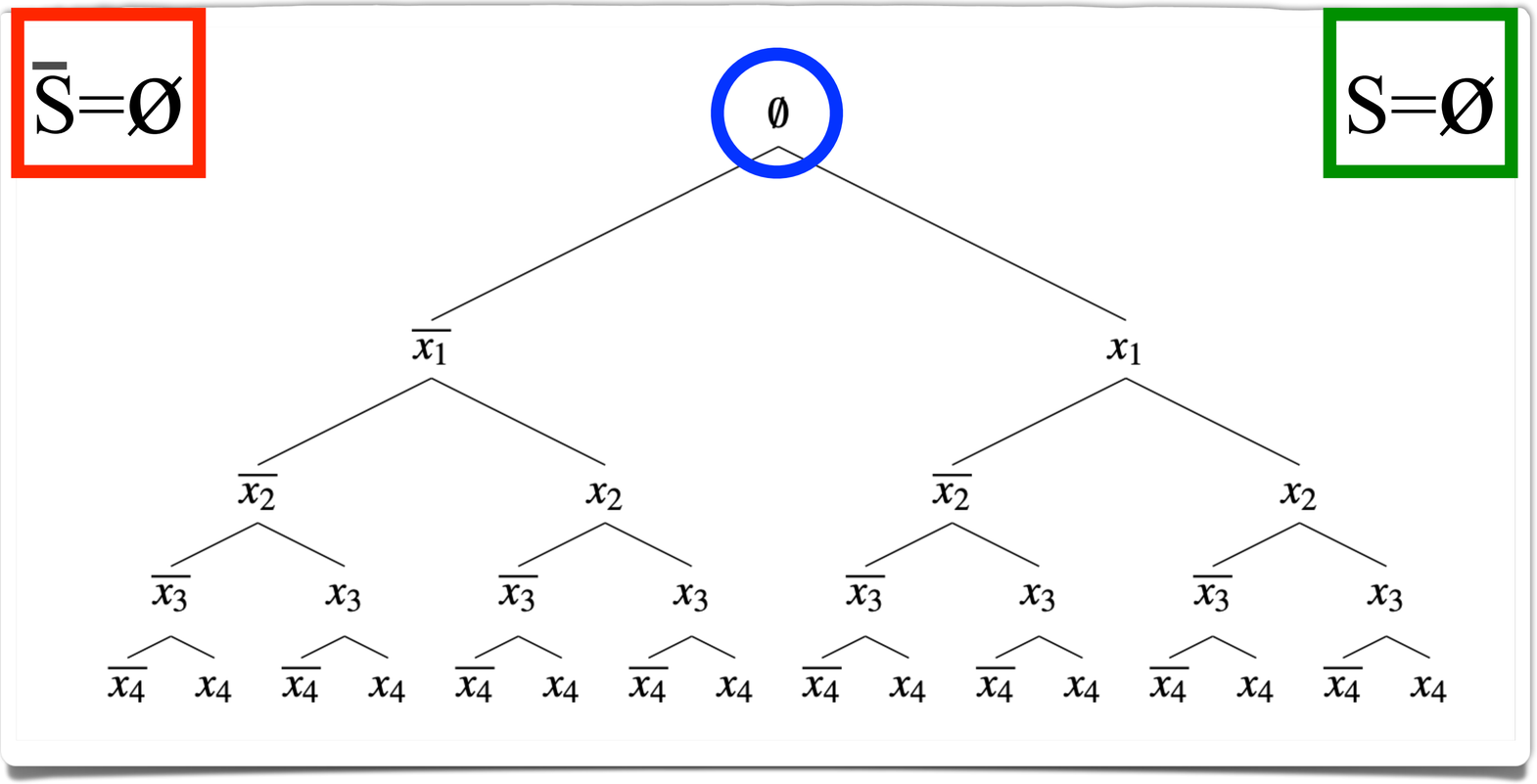
LB=14

Gesamtbeispiel

$\bar{S}=\emptyset$

$S=\emptyset$



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LB=14

Gesamtbeispiel

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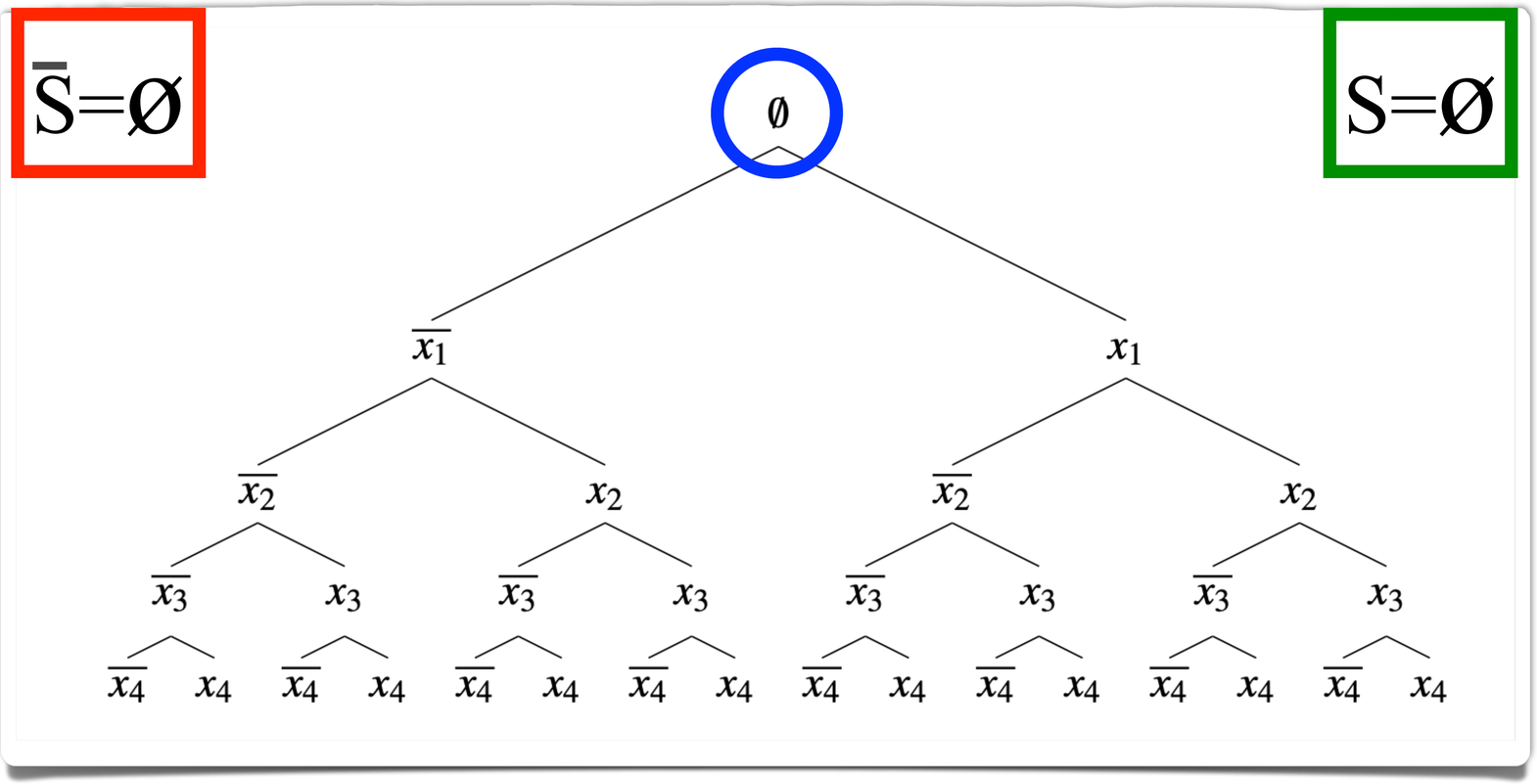

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LB=14

Gesamtbeispiel

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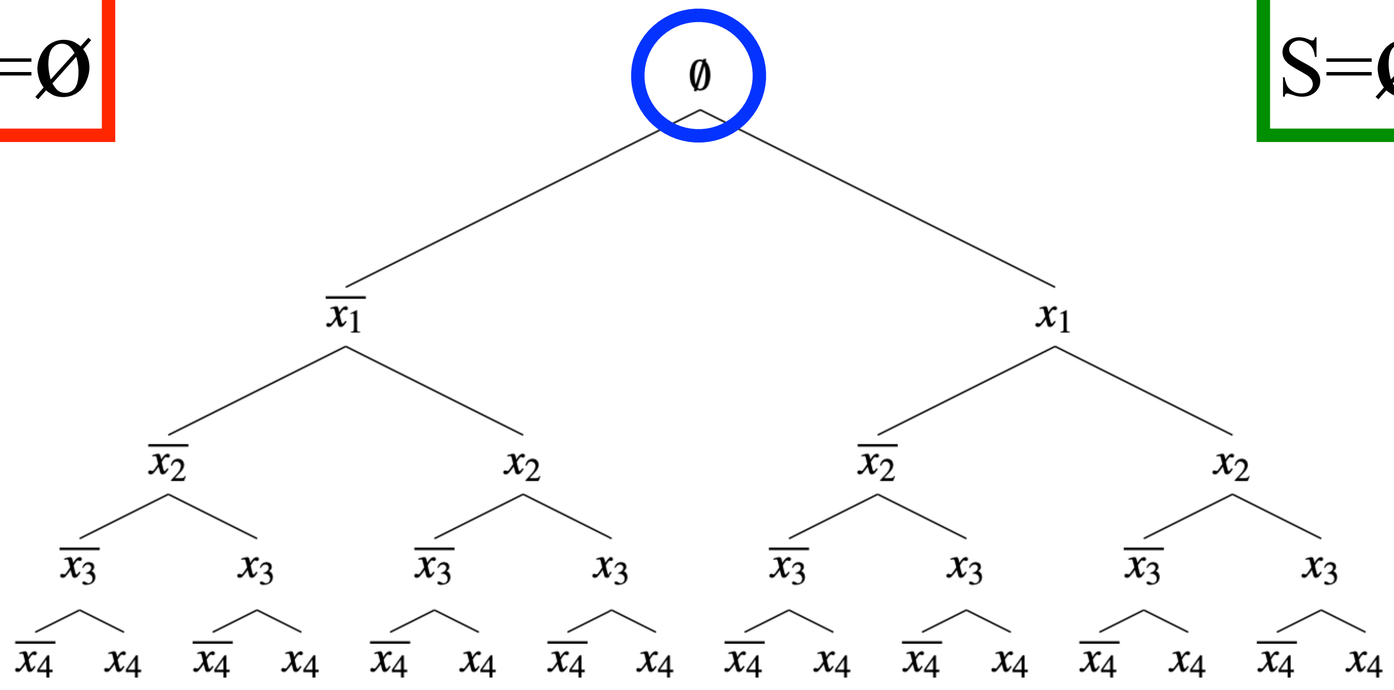

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

UB=16

i	1	2	3	4	5	6	7
z_i	2	3	6	7	5	9	4
p_i	6	5	8	9	6	7	3

LB=14

Vielen Dank!

s.fekete@tu-bs.de