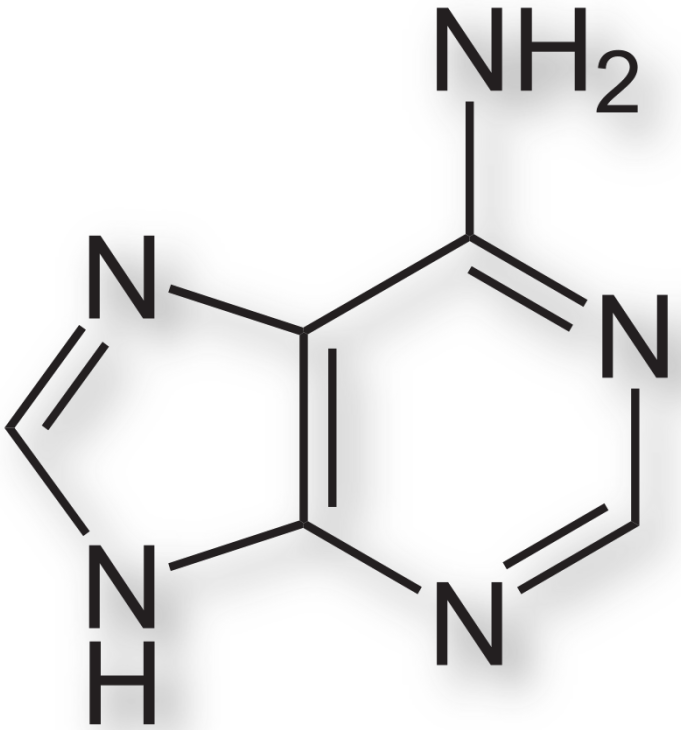
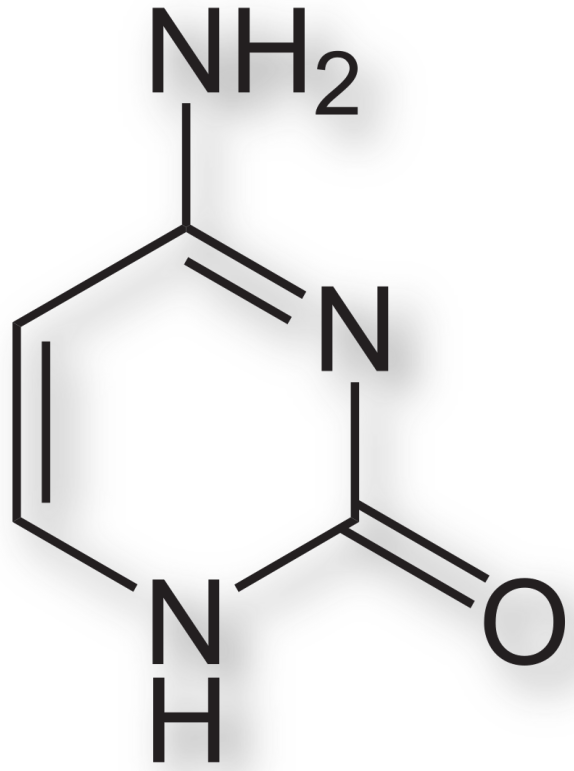
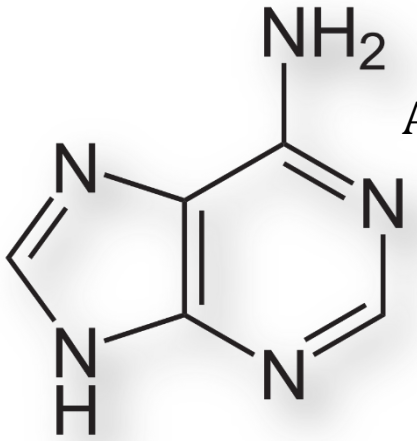
Technische Universität Braunschweig

# Algorithmen und Datenstrukturen 2 – Übung #2

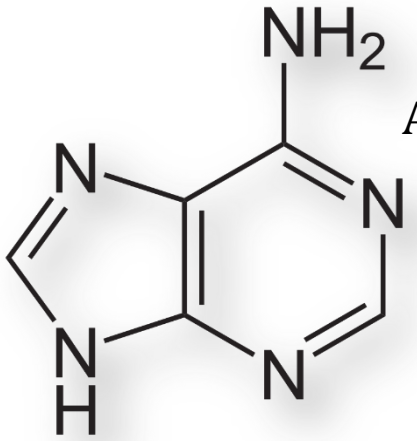
Phillip Keldenich
10.05.2023


NH2
N
N
N
N
H

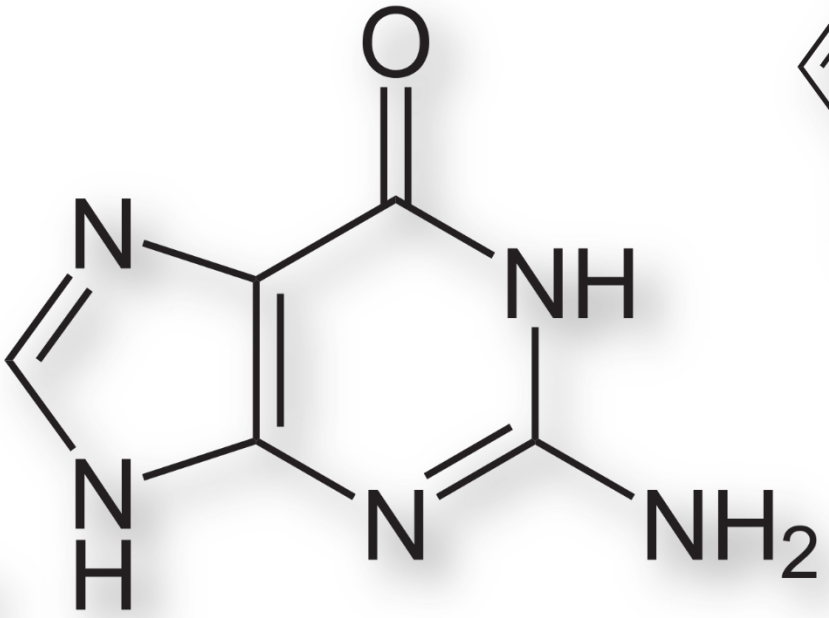


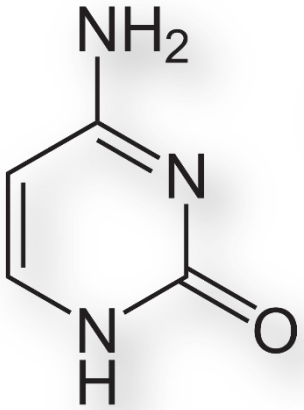




Adenin



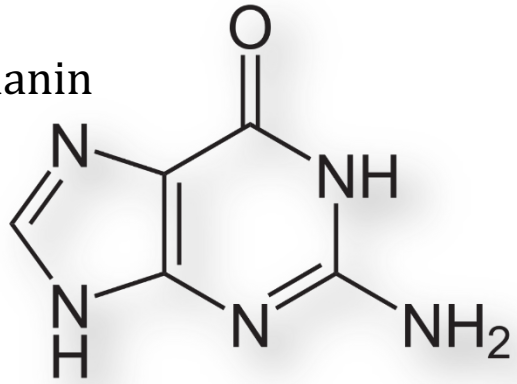

Adenin



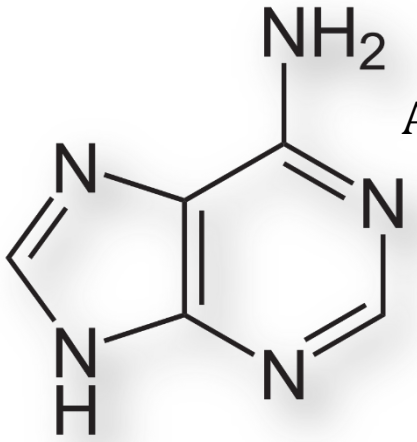



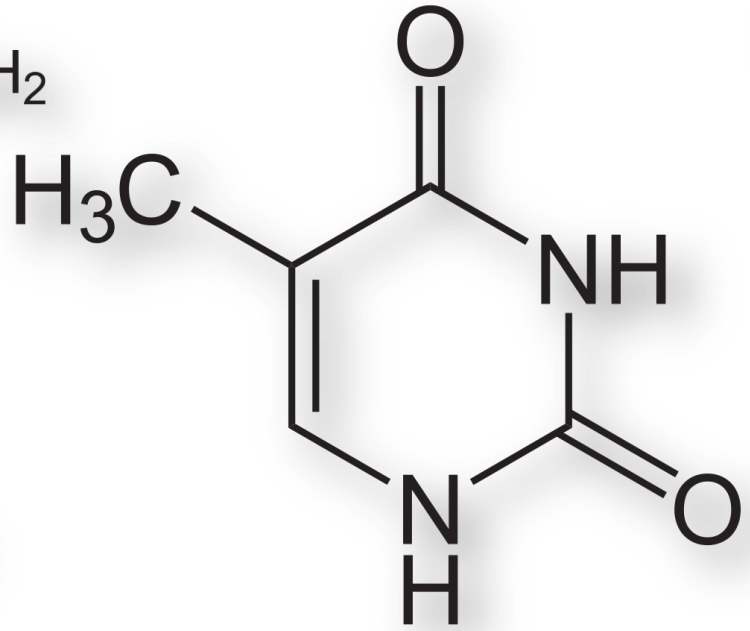

Cytosin

Guanin

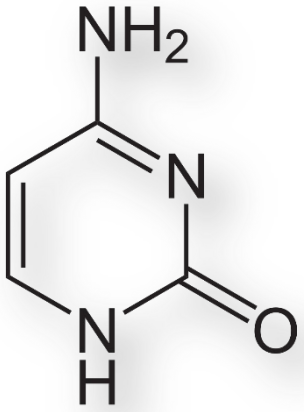



Adenin

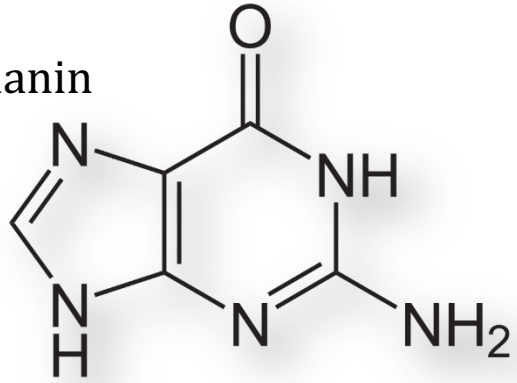


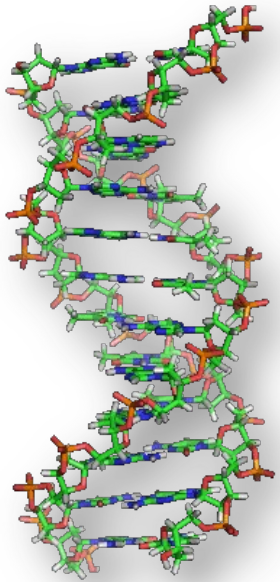




Cytosin

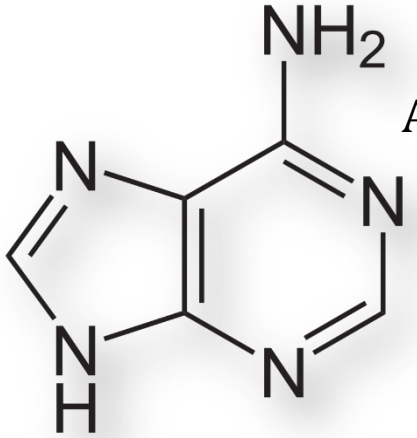


Guanin

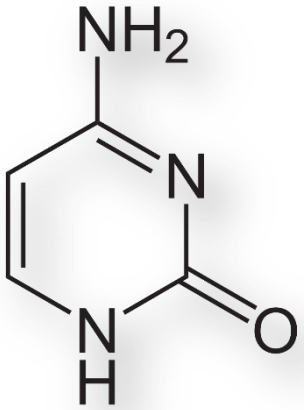


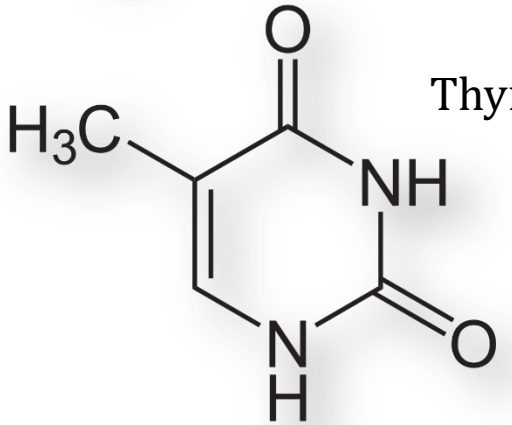
Adenin




Cytosin




Thymin



Sequenzierung von DNA Strängen

Wie ähnlich sind die beiden Wörter?

```
CAGTAGACCAGATTGACCGATAGATGCCCTTCCCTGGCTGGTTGTGGTCCGATTCC
TGTAGACCGTGCGTCGCTTGCCTACTGTTTACAGCGCCACTGACGCTAGTCATTAT
TGAGCACGCGGCCATACAACCCTTACTCTTCTAAGCCTGGATCTCTAAGGTTGACT
CCAAGAAGATGTAAAGTACTCCGTTAGCTCCGAAAAATTTAATTGGGACCATACG
AGCACTTTAGCACTTCTAGTTCTCATTTCGGCTAAGCGACCCCGAGAATACTTTTC
TGTCAACGACTGGACCTAGTGGGTCTATTTACTCATCGCCTGGCGGTCAGAGCCGT
GGATAGTTTGCGCTCGTTGGTAAGTGAACGGAGATAAAGGTGGTACCGAAGTATC
GGAAGGAATATACATCAATAGAACTGAAGTATATCGGCTTCTGTCGCCAATACAGT
ACCTTGAGCCGGATCGAAAGCGATTTGTGTGCGAACTAGGATGGATCTATTGTT
```

```
CTTGCTCATTGGAGTTGACAGAGTACTGTTTCACATCTGATCAAGGTTATGCTAGC
ACGTCCCAATGCAGGATAACTCAATATGAACTCCTTATAAGGCGATGAATTTGTTT
CTATGGTTGCCACGCAGCTCTTGGTCGGGTCAGAAGGGGTTTCCTAGGTGTGGCG
TCATGTCCTTTCTGCGGCCACAGGCGTTTGTGGTGGATCTGCACCACGTGGGTGT
CTGGCTACGCACCGTTTGTACATCTTCAAAAATCGAGCTTTGCACGGCTCAATTG
GCATAGACTGCCTGCCGTAATCTCGCTGAGTATAAGTTGATGTAATTTTCAAGACG
AGAGAGCTGGTATCCAGACAAGTCGGATGGTGAGGTTACTGAAGCGGATCCCGGA
CACTAGCTAAATATAATCGACGGATGAGACGAGGTGTAACAGGACTTTATCTCCGC
TTACGCCACACGTTCCCGGCCCTGCCGCTAGTTCCAGTTCCAATGTCCAAATGAGT
```

Sequenzierung von DNA Strängen

Wie ähnlich sind die beiden Wörter?

CAGTAGACCAGATTGACCGATAGATGCCCTTCCCTGGCTGGTTGTGGTCCGATTCC
TGTAGACCGTGCGTCGCTTGCCTACTGTTTACAGCGCCACTGACGCTAGTCATTAT
TGAGCACGCGGCCATACAACCCTTACTCTTCTAAGCCTGGATCTCTAAGGTTGACT
CCAAGAAGATGTAAAGTACTCCGTTAGCTCCGAAAAATTTAATTGGGACCATACG
AGCACTTTAGCACTTCTAGTTCTCATTTCGGCTAAGCGACCCCGAGAATACTTTTC
TGTCAACGACTGGACCTAGTGGGTCTATTTACTCATCGCCTGGCGGTCAGAGCCGT
GGATAGTTTGCGCTCGTTGGTAAGTGAACGGAGATAAAGGTGGTACCGAAGTATC
GGAAGGAATATACATCAATAGAACTGAAGTATATCGGCTTCTGTCGCCAATACAGT
ACCTTGAGCCGGATCGAAAGCGATTTGTGTGCGAACTAGGATGGATCTATTGTT

CTTGCTCATTGGAGTTGACAGAGTACTGTTTCACATCTGATCAAGGTTATGCTAGC
ACGTCCCAATGCAGGATAACTCAATATGAACTCCTTATAAGGCGATGAATTTGTTT
CTATGGTTGCCACGCAGCTCTTGGTCGGGTCAGAAGGGGTTTCCTAGGTGTGGCG
TCATGTCCTTTCTGCGGCCACAGGCGTTTGTGGTGGATCTGCACCACGTGGGTGT
CTGGCTACGCACCGTTTGTACATCTTCAAAAATCGAGCTTTGCACGGCTCAATTG
GCATAGACTGCCTGCCGTAATCTCGCTGAGTATAAGTTGATGTAATTTTCAAGACG
AGAGAGCTGGTATCCAGACAAGTCGGATGGTGAGGTTACTGAAGCGGATCCCGGA
CACTAGCTAAATATAATCGACGGATGAGACGAGGTGTAACAGGACTTTATCTCCGC
TTACGCCACACGTTCCCGGCCCTGCCGCTAGTTCCAGTTCCAATGTCCAAATGAGT

# Longest Common Subsequence

**Gegeben:**

- Alphabet $Z$
- Sequenzen
  - $X \coloneqq x_1 x_2 \dots x_n$ mit $x_i \in Z$
  - $Y \coloneqq y_1 y_2 \dots y_m$ mit $y_i \in Z$

**Gesucht:**

- Eine längstmögliche Sequenz $T$, die eine *Teilsequenz* von $X$ und $Y$ ist.

Eine Teilsequenz eines Wortes entsteht durch Weglassen von Buchstaben.

Beispiel:

ACTG

ist eine Teilsequenz von

ACCTATATGTT

# Longest Common Subsequence

**Gegeben:**

- Alphabet $Z$
- Sequenzen
  - $X \coloneqq x_1 x_2 \dots x_n$ mit $x_i \in Z$
  - $Y \coloneqq y_1 y_2 \dots y_m$ mit $y_i \in Z$

**Gesucht:**

- Eine längstmögliche Sequenz $T$, die eine *Teilsequenz* von $X$ und $Y$ ist.

Eine Teilsequenz eines Wortes entsteht durch Weglassen von Buchstaben.

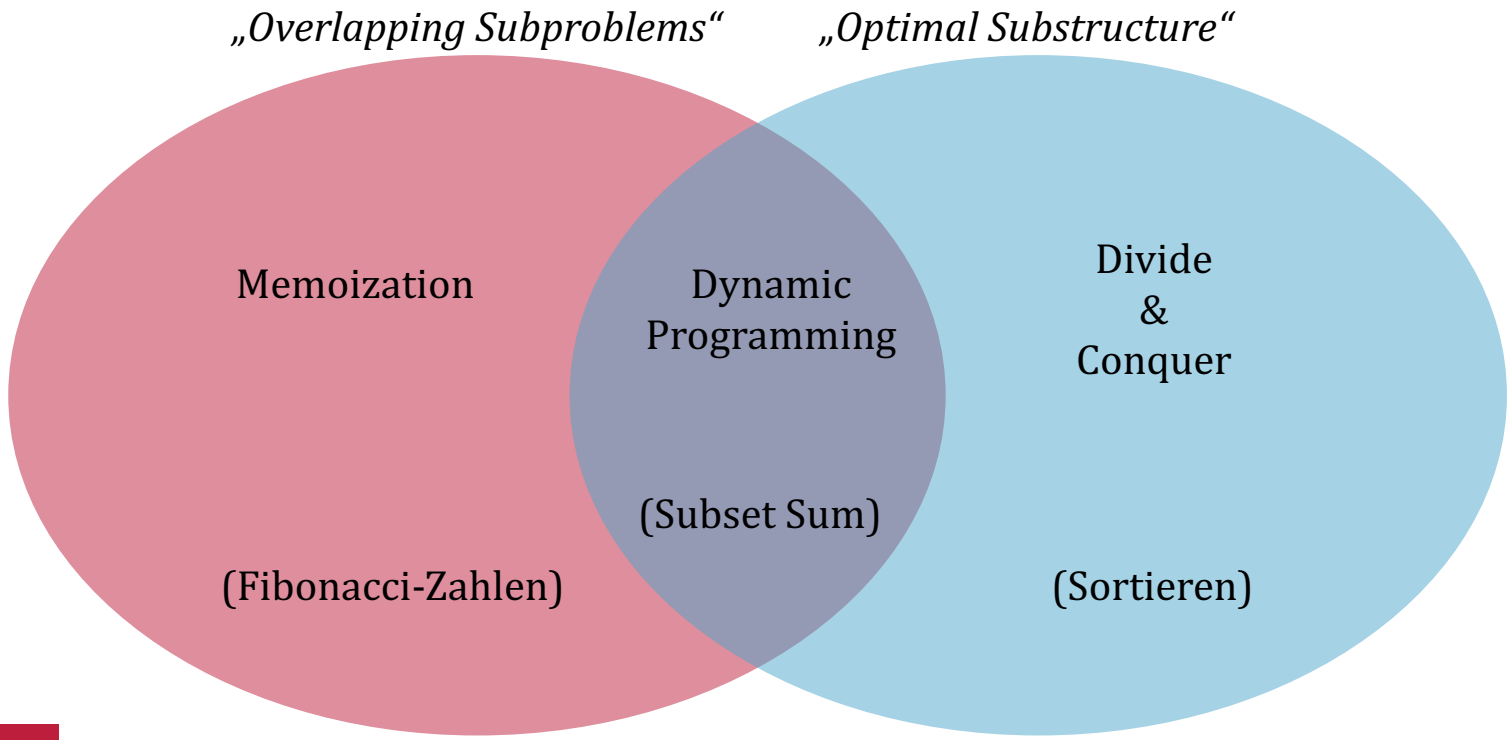
Beispiel:

ACTG

ist eine Teilsequenz von

ACCTATATGTT

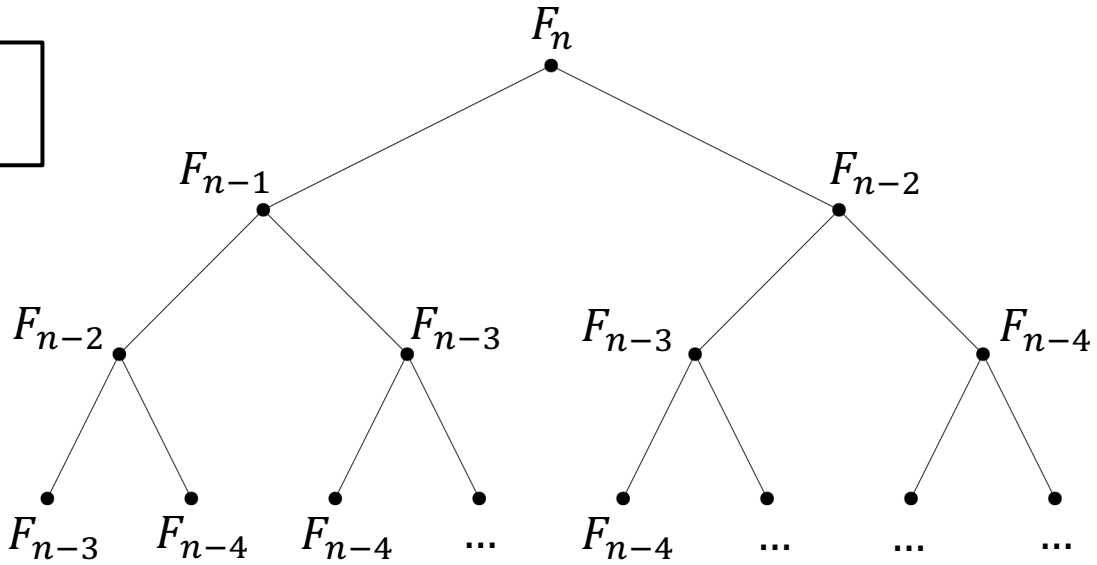
# Dynamic Programming



# Overlapping Subproblems

Fibonnaci-Zahlen:

$$F_n := F_{n-1} + F_{n-2}, \quad F_0 = 0, \quad F_1 = 1$$

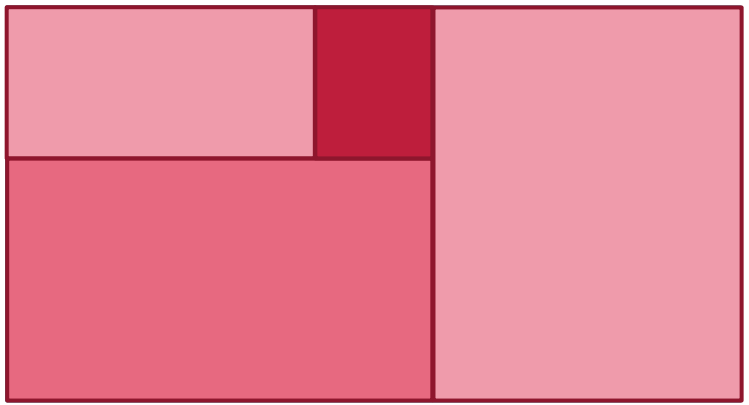



**Input :** $n \geq 0$
**Output :** $n$-te Fibonacci Zahl

**if** $n \leq 2$ **then**
    $f \leftarrow 1$
**else if** $\exists$memo$[n]$ **then**
    $f \leftarrow$ memo$[n]$
**else**
    $f \leftarrow$ FibonacciMemoization$(n-1)$ + FibonacciMemoization$(n-2)$
    memo$[n] \leftarrow f$
**return** $f$

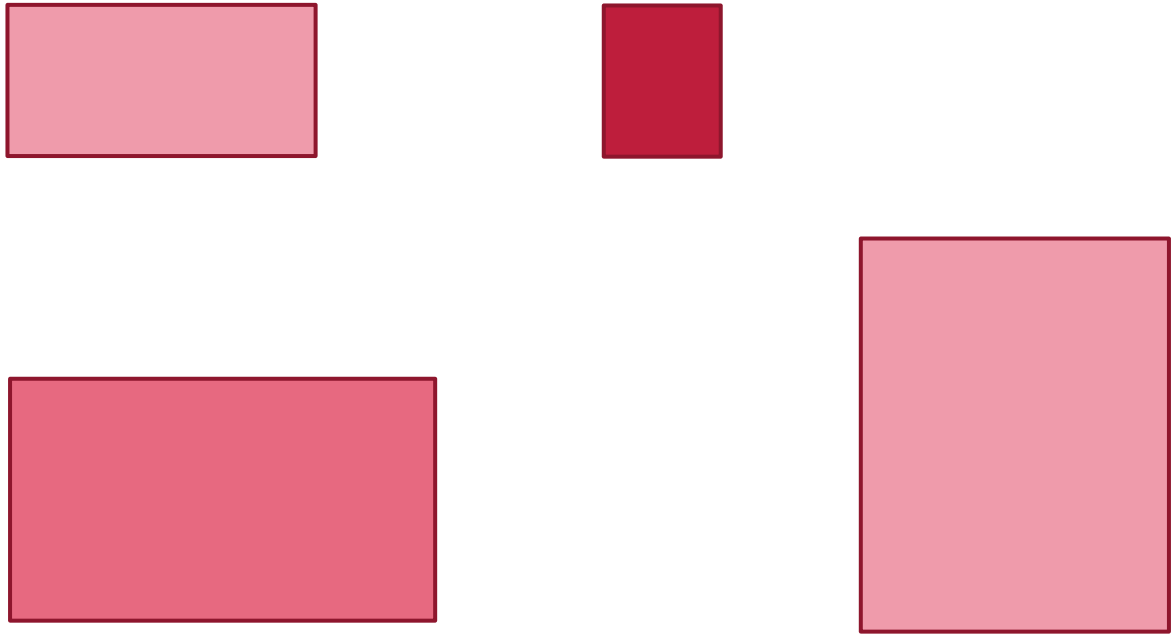
# Optimal Substructure

Aufteilen in Teilprobleme



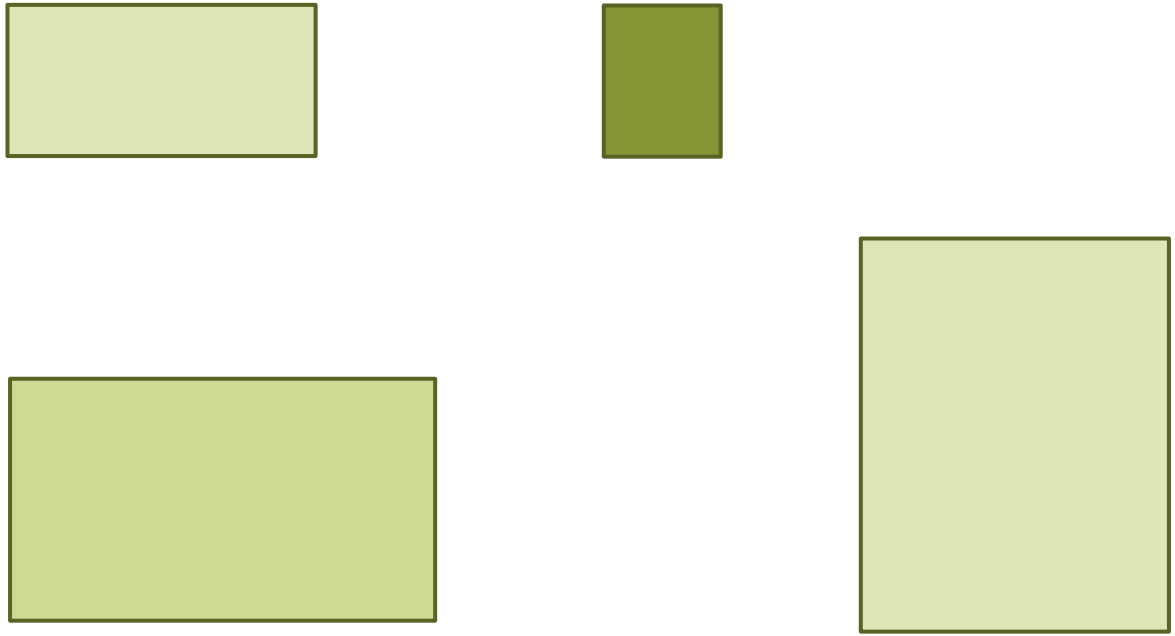
# Optimal Substructure

Lösen der Teilprobleme



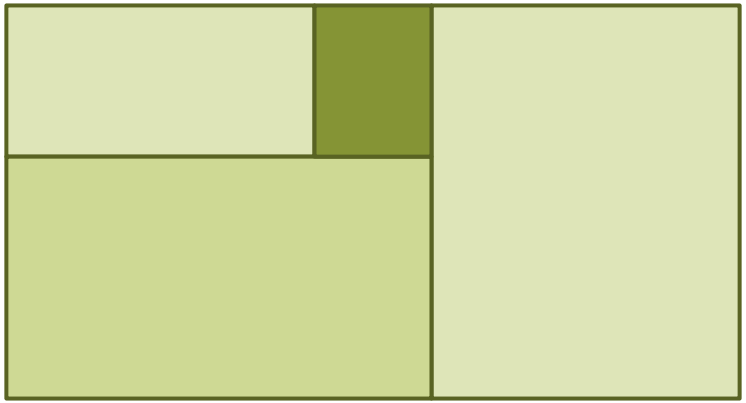
# Optimal Substructure

Lösen des Hauptproblems



# Optimal Substructure

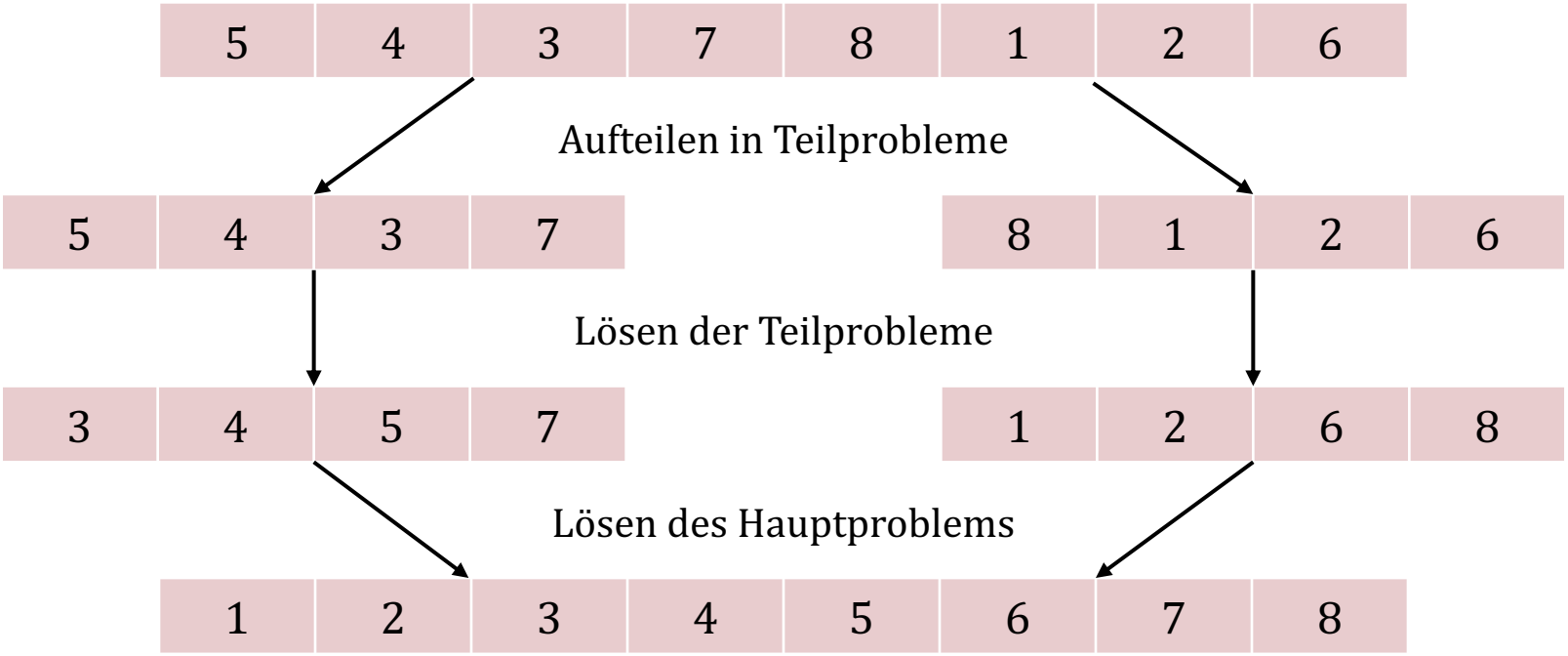
Lösen des Hauptproblems



# Optimal Substructure

Lösen des Hauptproblems

# Optimal Substructure – Beispiel Sortieren




Welcher Sortieralgorithmus ist das?

# Longest Common Subsequence

**Gegeben:**

- Alphabet $Z$
- Sequenzen
  - $X := x_1 x_2 \dots x_n$ mit $x_i \in Z$
  - $Y := y_1 y_2 \dots y_m$ mit $y_j \in Z$

**Gesucht:**

- Eine längstmögliche Sequenz $T$, die eine *Teilsequenz* von $X$ und $Y$ ist.

Eine Teilsequenz eines Wortes entsteht durch Weglassen von Buchstaben.

Beispiel:

ACTG

ist eine Teilsequenz von

ACCTATATGTT

# Dynamic Programming für LCS

*Seien*

$X^i := x_1 \dots x_i$

$Y^j := y_1 \dots y_j$

*Was sind die kleinsten Wörter, die wir lösen können?*

*Einfach:* $i = 0$ *und* $j = 0$

$LCS(X^0, Y^0) = 0$

Geht das allgemeiner?

Für alle $0 \leq i \leq n$ gilt

$LCS(X^i, Y^0) = 0$

und für alle $0 \leq j \leq m$ gilt

$LCS(X^0, Y^j) = 0$

Ja! Nur eines der beiden Wörter muss leer sein.

# Dynamic Programming für LCS

*Betrachte*

$x_1 \dots x_{i-1} x_i$

$y_1 \dots y_{j-1} y_j$

| $x_i \neq y_j$ | $x_i = y_j$ |
|---|---|
| $x_i$ oder $y_j$ ist nicht in LCS | Nutze den Match oder lass es sein. |
| $\Downarrow$ | $\Downarrow$ |
| $LCS(X^i, Y^j) = \max \begin{pmatrix} LCS(X^{i-1}, Y^j), \\ LCS(X^i, Y^{j-1}) \end{pmatrix}$ | $LCS(X^i, Y^j) = \max \begin{pmatrix} LCS(X^{i-1}, Y^{j-1}) + 1, \\ LCS(X^{i-1}, Y^j), \\ LCS(X^i, Y^{j-1}) \end{pmatrix}$ |
| | Braucht man die? |

# Dynamic Programming für LCS

**Fall $\mathbf{x_i = y_i}$:**

$$LCS(X^i, Y^j) = \max \begin{pmatrix} LCS(X^{i-1}, Y^{j-1}) + 1, \\ LCS(X^{i-1}, Y^j), \\ LCS(X^i, Y^{j-1}) \end{pmatrix} \rightarrow \text{vereinfache zu } LCS(X^i, Y^j) = LCS(X^{i-1}, Y^{j-1}) + 1.$$

$LCS(X^i, Y^{j-1}) \leq LCS(X^{i-1}, Y^{j-1}) + 1$: Betrachte LCS von $X^i$ und $Y^{j-1}$ und lasse ggf. $x_i$ weg.

$LCS(X^{i-1}, Y^j) \leq LCS(X^{i-1}, Y^{j-1}) + 1$: Analog.

$$LCS(X^i, Y^j) = \begin{cases} 0, & \text{falls } i = 0 \text{ oder } j = 0, \\ LCS(X^{i-1}, Y^{j-1}) + 1, & \text{falls } x_i = y_j, \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right), & \text{sonst.} \end{cases}$$

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0, & \text{falls } i = 0 \text{ oder } j = 0, \\ LCS(X^{i-1}, Y^{j-1}) + 1, & \text{falls } x_i = y_j, \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right), & \text{sonst.} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | | | | | | | | | | | |
| C | | | | | | | | | | | |
| A | | | | | | | | | | | |
| T | | | | | | | | | | | |
| G | | | | | | | | | | | |
| A | | | | | | | | | | | |
| C | | | | | | | | | | | |
| A | | | | | | | | | | | |
| T | | | | | | | | | | | |
| T | | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | | | | | | | | | | |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | | | | | | | | | | |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | | | | | | | | | | |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | | | | | | | | | | |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | | | | | | | | | | |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

T

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

T

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TT

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTAT

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTATA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTATA

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTATAC

| Y \ X | Ø | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ø | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

TTATAC

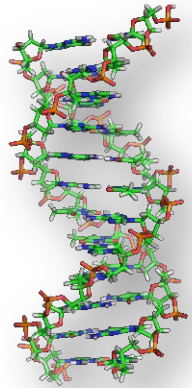
| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |

# Dynamic Programming für LCS

$$LCS(X^i, Y^j) = \begin{cases} 0 & \text{, falls } i = 0 \text{ oder } j = 0 \\ LCS(X^{i-1}, Y^{j-1}) + 1 & \text{, falls } x_i = y_j \\ \max\left(LCS(X^{i-1}, Y^j), LCS(X^i, Y^{j-1})\right) & \text{, sonst} \end{cases}$$

CATATT

| Y \ X | ∅ | A | C | C | T | A | T | A | T | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| T | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
| G | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| A | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| C | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| A | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |



CAGTAGACCAGATTGACCGATAGATGCCCTTCCCTGGCTGGTTGTGGTCCGATTCC
TGTAGACCGTGCGTCGCTTGCCTACTGTTTACAGCGCCACTGACGCTAGTCATTAT
TGAGCACGCGGCCATACAACCCTTACTCTTCTAAGCCTGGATCTCTAAGGTTGACT
CCAAGAAGATGTAAAGTACTCCGTTAGCTCCGAAAAATTTAATTGGGACCATACG
AGCACTTTAGCACTTCTAGTTCTCATTTCGGCTAAGCGACCCCGAGAATACTTTTC
TGTCAACGACTGGACCTAGTGGGTCTATTTACTCATCGCCTGGCGGTCAGAGCCGT
GGATAGTTTGCGCTCGTTGGTAAGTGAACGGAGATAAAGGTGGTACCGAAGTATC
GGAAGGAATATACATCAATAGAACTGAAGTATATCGGCTTCTGTCGCCAATACAGT
ACCTTGAGCCGGATCGAAAGCGATTTGTGTGCGAACTAGGATGGATCTATTGTT

CTTGCTCATTGGAGTTGACAGAGTACTGTTTCACATCTGATCAAGGTTATGCTAGC
ACGTCCCAATGCAGGATAACTCAATATGAACTCCTTATAAGGCGATGAATTTGTTT
CTATGGTTGCCACGCAGCTCTTGGTCGGGTCAGAAGGGGTTTCCTAGGTGTGGCG
TCATGTCCTTTCTGCGGCCACAGGCGTTTGTGGTGGATCTGCACCACGTGGGTGT
CTGGCTACGCACCGTTTGTACATCTTCAAAAATCGAGCTTTGCACGGCTCAATTG
GCATAGACTGCCTGCCGTAATCTCGCTGAGTATAAGTTGATGTAATTTTCAAGACG
AGAGAGCTGGTATCCAGACAAGTCGGATGGTGAGGTTACTGAAGCGGATCCCGGA
CACTAGCTAAATATAATCGACGGATGAGACGAGGTGTAACAGGACTTTATCTCCGC
TTACGCCACACGTTCCCGGCCCTGCCGCTAGTTCCAGTTCCAATGTCCAAATGAGT

LCS: 323

# Matrix Chain Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f & g \\ h & i & j \end{pmatrix}$$

Wie viele Multiplikationen werden benötigt?

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{2\times 2} \times \underbrace{\begin{pmatrix} e & f & g \\ h & i & j \end{pmatrix}}_{2\times 3} = \begin{pmatrix} ae+bh & af+bi & ag+bj \\ ce+dh & cf+di & cg+dj \end{pmatrix}$$

$$2 \cdot 2 \cdot 3 = 12 \text{ Multiplikationen}$$

# Matrix Chain Multiplication

$$\underset{A}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \times \underset{B}{\begin{pmatrix} e & f & g \\ h & i & j \end{pmatrix}} \times \underset{C}{\begin{pmatrix} k & l \\ m & n \\ o & p \end{pmatrix}}$$

Wie viele Multiplikationen werden benötigt?

| Operation | Dimensionen | Multiplikationen |
|---|---|---|
| $A \times B$ | $(2 \times 2) \times (2 \times 3)$ | 12 |
| $(A \times B) \times C$ | $(2 \times 3) \times (3 \times 2)$ | 12 |
| | | **24** |

Alternative

| Operation | Dimensionen | Multiplikationen |
|---|---|---|
| $B \times C$ | $(2 \times 3) \times (3 \times 2)$ | 12 |
| $A \times (B \times C)$ | $(2 \times 2) \times (2 \times 2)$ | 8 |
| | | **20** |

# Matrix Chain Multiplication

**Gegeben:**

- Matrizen $M_1 \times M_2 \times \cdots \times M_n$ wobei $M_i$ eine Matrix der Form $d_i \times d_{i+1}$ ist.

**Gesucht:**

- Eine Klammerung für die Multiplikation der Matrizen, sodass die Summe der benötigten Multiplikationen minimal ist.

Die Anzahl der möglichen Klammerungen von $n$ Matrizen kann durch die Folge der Catalan-Zahlen $C_n$ beschrieben werden.

Die Folge beginnt mit

1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,…

# Dynamic Programming für Matrix Chain Multiplication

$OPT(i,j)$ ist die Anzahl Multiplikationen, die für Matrizen $M_i$ bis $M_j$ benötigt werden.

Was ist das kleinste Teilproblem, welches wir lösen können?

Für $i = j$ müssen keine Multiplikationen durchgeführt werden.

Für alle $i \in \{1, \dots n\}$ gilt
$$OPT(i,i) = 0$$

# Dynamic Programming für Matrix Chain Multiplication

Betrachte

$$\ldots\Big((M_i \times M_{i+1} \times \cdots \times M_k) \times (M_{k+1} \times \cdots \times M_{j-1} \times M_j)\Big) \ldots$$

$M: d_i \times d_{k+1}$
$M': d_{k+1} \times d_{j+1}$

Wir brauchen $d_i \cdot d_{k+1} \cdot d_{j+1}$ Multiplikationen für die Multiplikation von $M$ und $M'$.

Wir brauchen insgesamt $d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j)$ Multiplikationen.

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | | | | | |
| $M_2$ | | | | | |
| $M_3$ | | | | | |
| $M_4$ | | | | | |
| $M_5$ | | | | | |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | | | | |
| $M_2$ | | 0 | | | |
| $M_3$ | | | 0 | | |
| $M_4$ | | | | 0 | |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | | | | |
| $M_2$ | | 0 | | | |
| $M_3$ | | | 0 | | |
| $M_4$ | | | | 0 | |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 2$

- $k = 1: 3 \cdot 5 \cdot 10 + 0 + 0 = 150$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 → | 150 | | | |
| $M_2$ | | ↑ 0 | | | |
| $M_3$ | | | 0 | | |
| $M_4$ | | | | 0 | |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 2$
- $k = 1: 3 \cdot 5 \cdot 10 + 0 + 0 = 150$

$i = 2; j = 3$
- $k = 2: 5 \cdot 10 \cdot 2 + 0 + 0 = 100$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | | | |
| $M_2$ | | 0 → | 100 | | |
| $M_3$ | | | ↑ 0 | | |
| $M_4$ | | | | 0 | |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 2$
- $k = 1: 3 \cdot 5 \cdot 10 + 0 + 0 = 150$

$i = 2; j = 3$
- $k = 2: 5 \cdot 10 \cdot 2 + 0 + 0 = 100$

$i = 3; j = 4$
- $k = 3: 10 \cdot 2 \cdot 4 + 0 + 0 = 80$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | | | |
| $M_2$ | | 0 | 100 | | |
| $M_3$ | | | 0 → | 80 | |
| $M_4$ | | | | 0 | |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 2$
- $k = 1: 3 \cdot 5 \cdot 10 + 0 + 0 = 150$

$i = 2; j = 3$
- $k = 2: 5 \cdot 10 \cdot 2 + 0 + 0 = 100$

$i = 3; j = 4$
- $k = 3: 10 \cdot 2 \cdot 4 + 0 + 0 = 80$

$i = 4; j = 5$
- $k = 4: 2 \cdot 4 \cdot 3 + 0 + 0 = 24$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | | | |
| $M_2$ | | 0 | 100 | | |
| $M_3$ | | | 0 | 80 | |
| $M_4$ | | | | 0 → | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 3$

- $k = 1: 3 \cdot 5 \cdot 2 + 0 + 100 = 130$
- $k = 2: 3 \cdot 10 \cdot 2 + 150 + 0 = 210$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | | |
| $M_2$ | | 0 | 100 | 140 | |
| $M_3$ | | | 0 | 80 | |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 3$
- $k = 1: 3 \cdot 5 \cdot 2 + 0 + 100 = 130$
- $k = 2: 3 \cdot 10 \cdot 2 + 150 + 0 = 210$

$i = 2; j = 4$
- $k = 2: 5 \cdot 10 \cdot 4 + 0 + 80 = 280$
- $k = 3: 5 \cdot 2 \cdot 4 + 100 + 0 = 140$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | | |
| $M_2$ | | 0 | 100 | 140 | |
| $M_3$ | | | 0 | 80 | |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 3$
- $k = 1: 3 \cdot 5 \cdot 2 + 0 + 100 = 130$
- $k = 2: 3 \cdot 10 \cdot 2 + 150 + 0 = 210$

$i = 2; j = 4$
- $k = 2: 5 \cdot 10 \cdot 4 + 0 + 80 = 280$
- $k = 3: 5 \cdot 2 \cdot 4 + 100 + 0 = 140$

$i = 3; j = 5$
- $k = 3: 10 \cdot 2 \cdot 3 + 0 + 24 = 84$
- $k = 4: 10 \cdot 4 \cdot 3 + 80 + 0 = 200$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | | |
| $M_2$ | | 0 | 100 | 140 | |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 4$

- $k = 1: 3 \cdot 5 \cdot 4 + 0 + 140 = 200$
- $k = 2: 3 \cdot 10 \cdot 4 + 140 + 80 = 340$
- $k = 3: 3 \cdot 2 \cdot 4 + 130 + 0 = 154$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | 154 | |
| $M_2$ | | 0 | 100 | 140 | |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 4$

- $k = 1: 3 \cdot 5 \cdot 4 + 0 + 140 = 200$
- $k = 2: 3 \cdot 10 \cdot 4 + 140 + 80 = 340$
- $k = 3: 3 \cdot 2 \cdot 4 + 130 + 0 = 154$

$i = 2; j = 5$

- $k = 2: 5 \cdot 10 \cdot 3 + 0 + 84 = 234$
- $k = 3: 5 \cdot 2 \cdot 3 + 100 + 24 = 154$
- $k = 4: 5 \cdot 4 \cdot 3 + 140 + 0 = 200$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | 154 | |
| $M_2$ | | 0 | 100 | 140 | 154 |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 5$

- $k = 1: 3 \cdot 5 \cdot 3 + 0 + 154 = 199$
- $k = 2: 3 \cdot 10 \cdot 3 + 150 + 84 = 324$
- $k = 3: 3 \cdot 2 \cdot 3 + 130 + 24 = 172$
- $k = 4: 3 \cdot 4 \cdot 3 + 154 + 0 = 190$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | 154 | 172 |
| $M_2$ | | 0 | 100 | 140 | 154 |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 5$

- $k = 1: 3 \cdot 5 \cdot 3 + 0 + 154 = 199$
- $k = 2: 3 \cdot 10 \cdot 3 + 150 + 84 = 324$
- $k = 3: 3 \cdot 2 \cdot 3 + 130 + 24 = 172$
- $k = 4: 3 \cdot 4 \cdot 3 + 154 + 0 = 190$

Ergebnis: 172 Multiplikationen

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | 154 | 172 |
| $M_2$ | | 0 | 100 | 140 | 154 |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

Online-Demo: https://rosulek.github.io/vamonos/demos/matrix-chain.html

# Dynamic Programming für Matrix Chain Multiplication

$$OPT(i,j) = \begin{cases} 0 & \text{, falls } i = j \\ \min\limits_{i \le k < j} d_i \cdot d_{k+1} \cdot d_{j+1} + OPT(i,k) + OPT(k+1,j) & \text{, sonst} \end{cases}$$

| $i$ | $d_i$ |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 10 |
| 4 | 2 |
| 5 | 4 |
| 6 | 3 |

$i = 1; j = 5$

- $k = 1: 3 \cdot 5 \cdot 3 + 0 + 154 = 199$
- $k = 2: 3 \cdot 10 \cdot 3 + 150 + 84 = 324$
- $k = 3: 3 \cdot 2 \cdot 3 + 130 + 24 = 172$
- $k = 4: 3 \cdot 4 \cdot 3 + 154 + 0 = 190$

| i\j | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $M_1$ | 0 | 150 | 130 | 154 | 172 |
| $M_2$ | | 0 | 100 | 140 | 154 |
| $M_3$ | | | 0 | 80 | 84 |
| $M_4$ | | | | 0 | 24 |
| $M_5$ | | | | | 0 |

Ergebnis: 172 Multiplikationen
Reihenfolge: $(M_1 \times (M_2 \times M_3)) \times (M_4 \times M_5)$

Online-Demo: https://rosulek.github.io/vamonos/demos/matrix-chain.html

# Dynamic Programming

1. Geeignete Teilprobleme definieren
2. Lösung für kleinste Teilprobleme definieren (Initialisierung)
3. Lösung für Problem aus gelösten Teilprobleme bestimmen (Rekursionsgleichung)
4. Algorithmus schreiben (wie läuft man durch die Tabelle?)

Weitere Fragen:

- Wie bekommt man die Lösung und nicht nur den Wert?
- Gibt es mehrere mögliche dynamische Programme?
- Was ist mit der Effizienz?