

Sheet 0

You do not have to hand in this exercise sheet. The solutions will be discussed in the small tutorial on Monday, the 29th of April 2024.

Exercise 1 (MEMORY):

In this exercise, we consider a single-player version of the card game MEMORY. There is a deck of n pairs of cards lying face down on the table in front of the player. The player has to remove the cards by uncovering matching pairs of cards in as few moves as possible.

In each move, the player selects a first card and turns it around to see its face. After that, he selects and uncovers a second card. If the two cards match, they are removed. Otherwise, the cards are turned face down and remain on the table.

Knowing all cards' positions, the optimal offline algorithm removes the cards in n moves.

(Hint: Start with a proper, formal definition of the problem.)

- (a) Design an online algorithm for MEMORY that requires at most $2n - 1$ moves.
- (b) Prove that there is no online algorithm that requires at most $2n - 2$ moves.
- (c) Can you design a $3/2$ -competitive online algorithm for MEMORY?
- (d) In the two-player game, a player gets another free move after uncovering a matching pair of cards. In the single player variant, we can apply the same rule to provide free moves.

Give a c -competitive online algorithm for some constant $c > 1$ for this version of MEMORY, or prove that no such algorithm exists.