Satz: $F_n = \frac{1}{\sqrt{5}} \cdot \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$

n-te Fibonacci-Zahl

Def:

$F_n := F_{n-1} + F_{n-2}\ ,\ F_0 = 0,\ F_1 = 1$

NR:

$$1 + \frac{1+\sqrt{5}}{2} = \frac{3+\sqrt{5}}{2} \quad \Big| \cdot \frac{2}{2}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$= \frac{1+2\sqrt{5}+5}{4}$$

$$= \left(\frac{1+\sqrt{5}}{2} \right)^2$$

Beweis: Per Induktion über n.

IA: $n = 0: F_n = 0 \overset{?}{=} \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^0 - \left(\frac{1-\sqrt{5}}{2} \right)^0 \right)$ ✓

$n = 1: F_n = 1 \overset{?}{=} \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5} - 1 + \sqrt{5}}{2} \right) = \frac{1}{\cancel{\sqrt{5}}} \cdot \left(\frac{\cancel{2}\cancel{\sqrt{5}}}{\cancel{2}} \right) = 1$

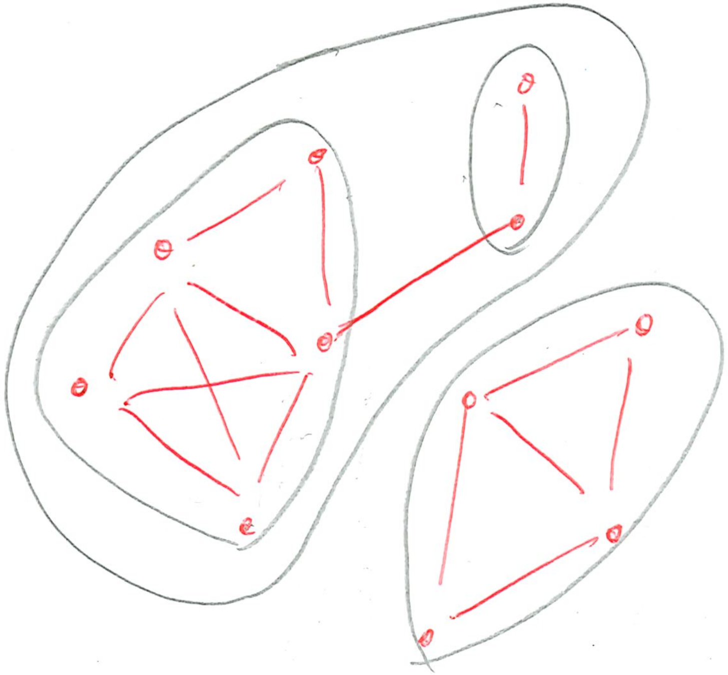
IV: Aussage gilt für bel., aber festes $n \geq 0$

IS: $n \to n+1$

$$F_{n+1} = F_n + F_{n-1} = \underbrace{\frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)}_{F_n} + \underbrace{\frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n-1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n-1} \right)}_{F_{n-1}}$$

$$= \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n-1} \cdot \left(1 + \frac{1+\sqrt{5}}{2} \right) - \left(\frac{1-\sqrt{5}}{2} \right)^{n-1} \cdot \left(1 + \frac{1-\sqrt{5}}{2} \right) \right)$$

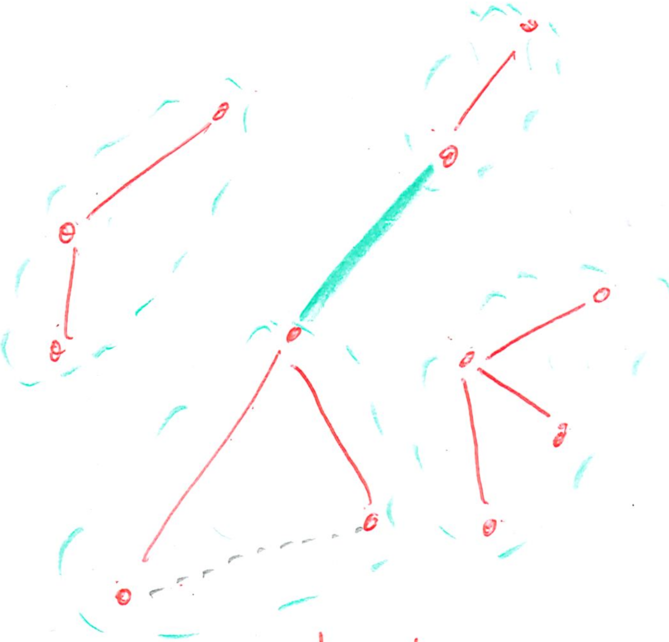
$$= \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} \right) \qquad \square$$



$k=3$

$n=12$

$m=|E|=9$



$k=4$

$n=12$

$|E|=8=12-4$