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ON THE IDENTIFICATION OF THE CONVEX HULL OF A FINITE SET OF POINTS IN THE PLANE

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convex hull algorithm

1. Introduction

This paper presents an extremely simple algorithm for identifying the convex hull of a finite set of points in the plane in essentially, at most $n(m+1)$ operations for n points in the set and $m \leq n$ points on the convex hull. In most cases far less than $n(m+1)$ operations are necessary because of a powerful point deletion mechanism that can easily be included. The operations are themselves trivial (computationally inexpensive) and consist of angle comparisons only. Even these angle comparisons need not be actually carried out if an improvement suggested in a later section is implemented. Although Graham's algorithm [1] requires no more than $(n \log n)/\log 2 + Cn$ operations*, the operations are themselves more complex than those of the method presented here; in particular, Graham's method would not be as efficient for low m.

2. Geometric interpretation

The underlying method of the algorithm can be described simply: find an origin point outside the point set and swing a radius arm in an arbitrary direction until a point of the set is met; this point becomes the first point on the hull. Make this the new origin point and swing a radius arm from this point in the same direction as before till the next hull point is found. Repeat until the points are enclosed by the convex hull. Delete points from further consideration if

(i) they have already been identified as being on the convex hull,

(ii) they lie in the area enclosed by a line from the first to the last convex hull point found and the lines joining the convex hull points in the sequence found.

Fig. 1 illustrates this geometric interpretation.

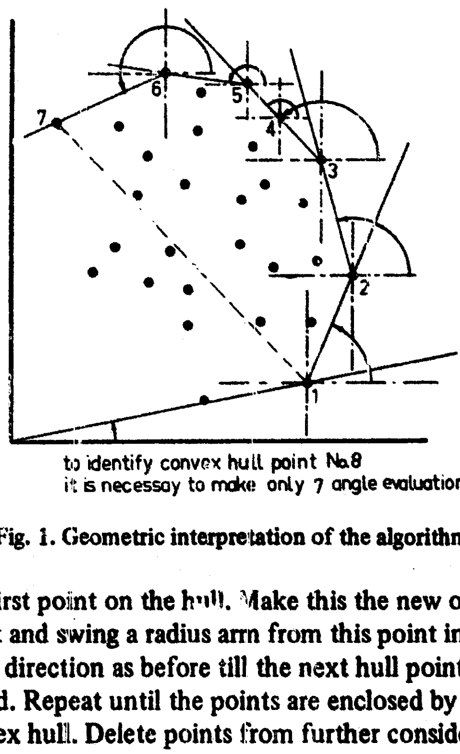

Fig. 1. Geometric interpretation of the algorithm.

* To quote Graham, "C is a small positive constant which depends on what is meant by an 'operation'". In fact, C is distributed over the five basic steps of Graham's algorithm and his paper should be consulted for detailed interpretation.