Algorithms Group **Winter 2022/2023**
Department of Computer Science - IBR
TU Braunschweig

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Computational Geometry
Homework Assignment 1
Dec 08, 2022

Solutions are due on Thursday, the 12th of January 2023, until 15:00 in the homework cupboard. You can also hand in your solution in person before the big tutorial on the 12th begins or via e-mail to `kramer@ibr.cs.tu-bs.de` with CC to `perk@ibr.cs.tu-bs.de`. Please do not email us scanned versions of your handwritten solutions.

We consider these problems in the two-dimensional variant. Please explain your answers in a few sentences. Sometimes it may be helpful to provide us with a drawing.

Exercise 1 (Convex hull):

a) What is a definition of the convex hull of a set of points in the plane?

b) What is your favorite algorithm for computing the convex hull? Why?

c) Given a convex hull $conv(P)$ of size k, how can you decide in $O(\log k)$ time whether a given point is inside $conv(P)$?

(5 + 5 + 5 = 15 pts.)

Exercise 2 (Closest and farthest point pairs):
We assume that the distance between two points is the Euclidean distance.

a) What is the basic idea of the divide and conquer algorithm for computing the closest pair of a set of n points?

b) What is the key observation in the merging step of Bentley's and Shamos' algorithm?

c) Let $P = (p_1, \ldots, p_n)$ be a sorted point set (either by $\leq_x$ or $\leq_y$) and p_i, p_j be the closest pair. Without loss of generality, let $i < j$. Is $j = i + 1$ always true?

d) How is the farthest point pair related to the convex hull?

e) Explain the intuition behind the rotating calipers method. How fast can we compute the diameter of a convex polygon?

(10 + 5 + 5 + 5 + 10 = 35 pts.)

Exercise 3 (Voronoi diagram):

a) Give an intuitive description of a Voronoi diagram

b) What relationships are there between Voronoi cells, Voronoi vertices, and Voronoi edges?

c) How can you efficiently store a Voronoi diagram in memory?

d) What is your favorite property of a Voronoi diagram? Why?

(5 + 7 + 8 + 5 = 25 pts.)