Technische
Universität
Braunschweig

Computational Geometry – Exercise Meeting #1

November 18th, 2021

A paper-and-pencil game – Rules

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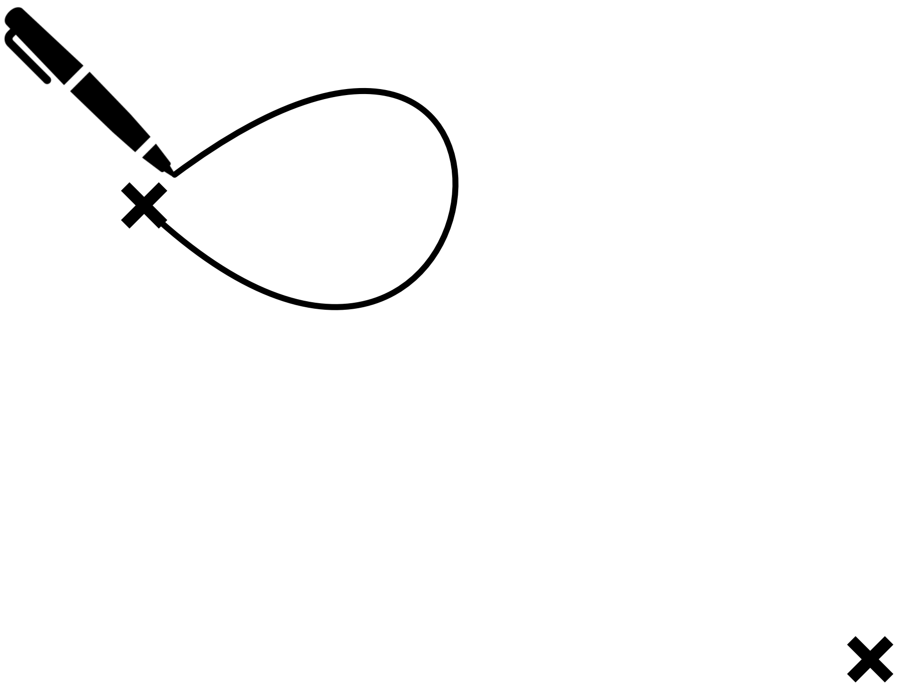

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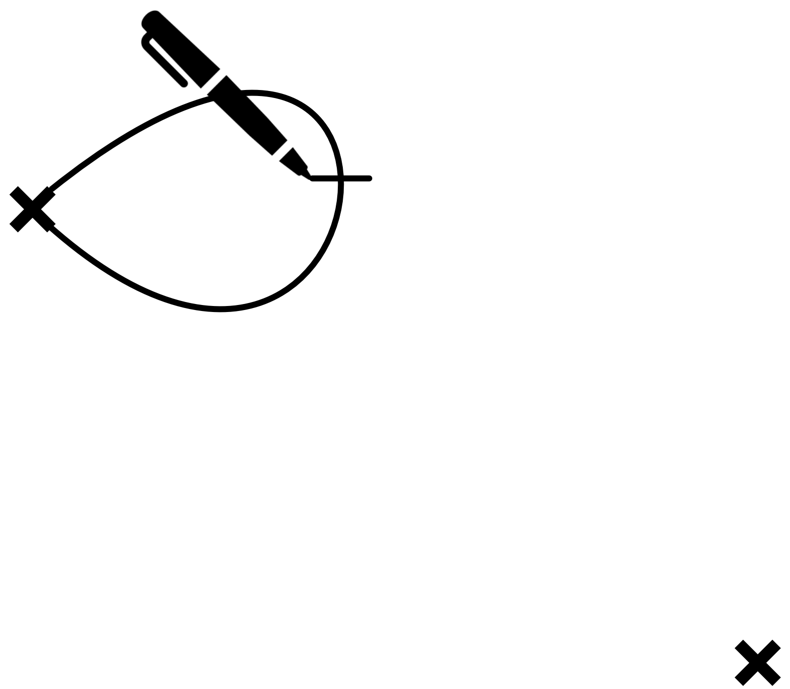

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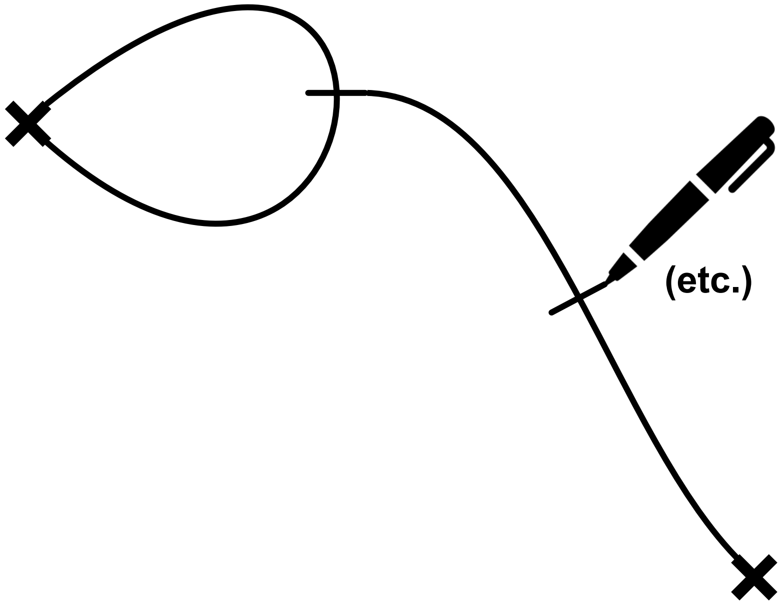

A paper-and-pencil game – Example

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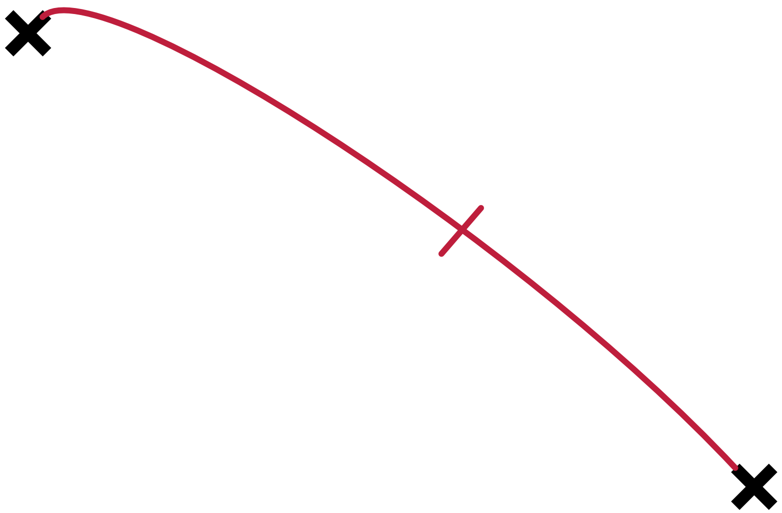

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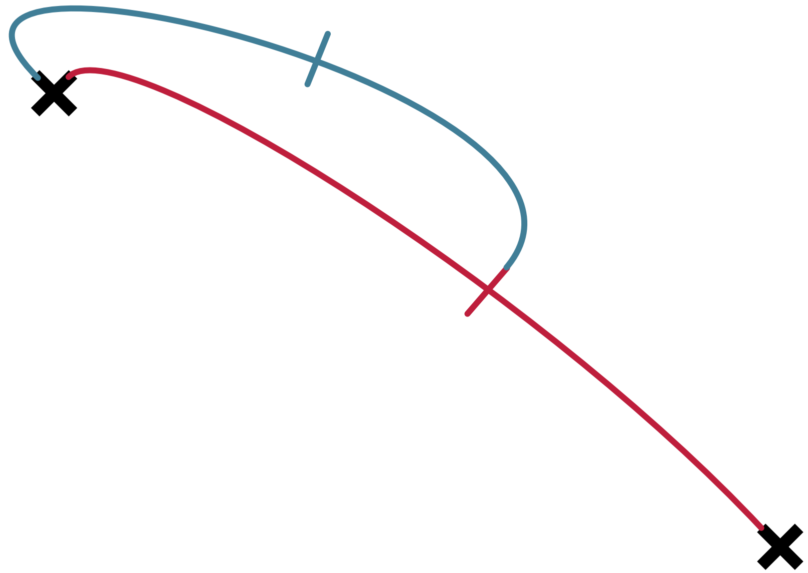

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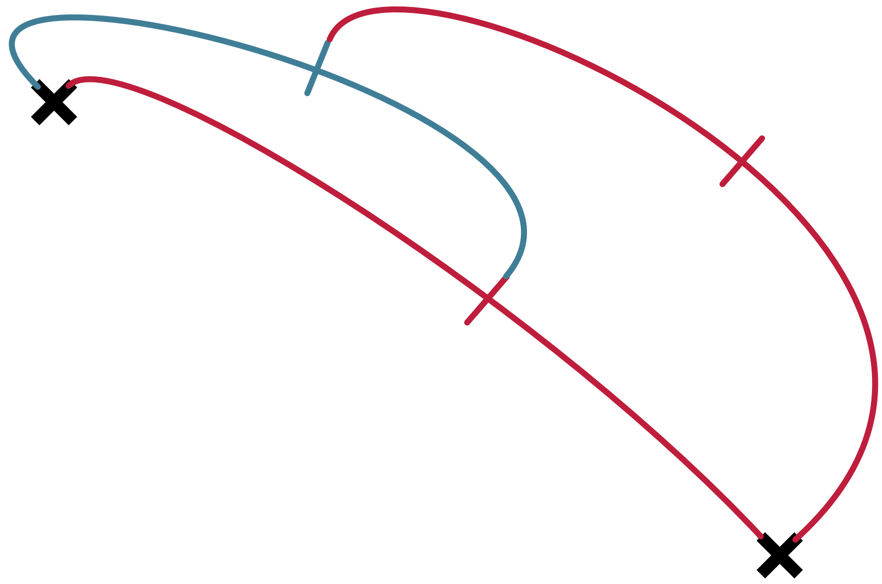

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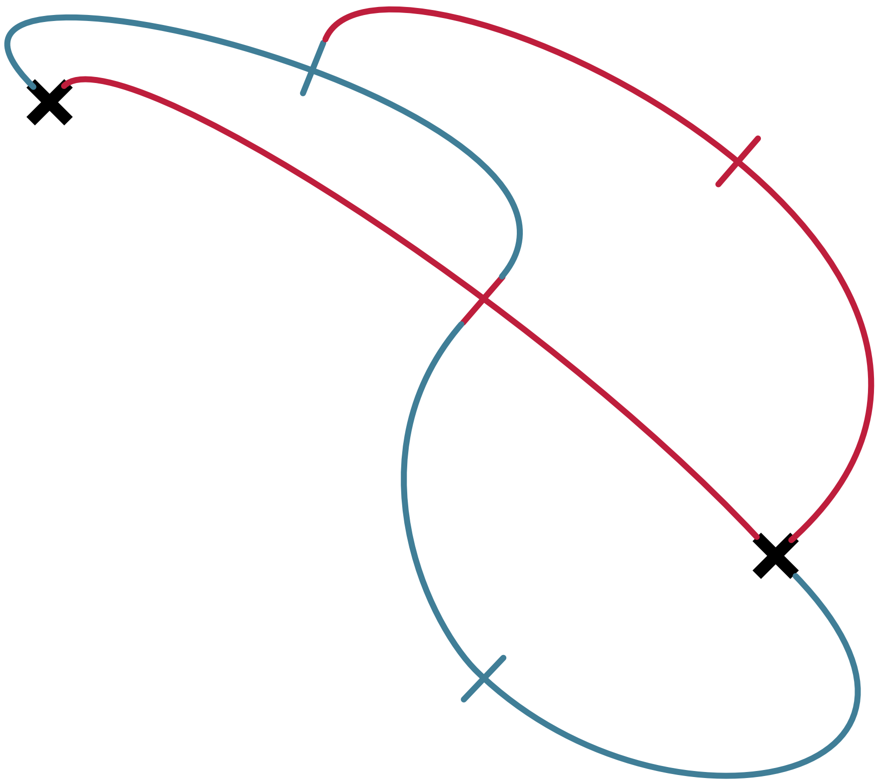

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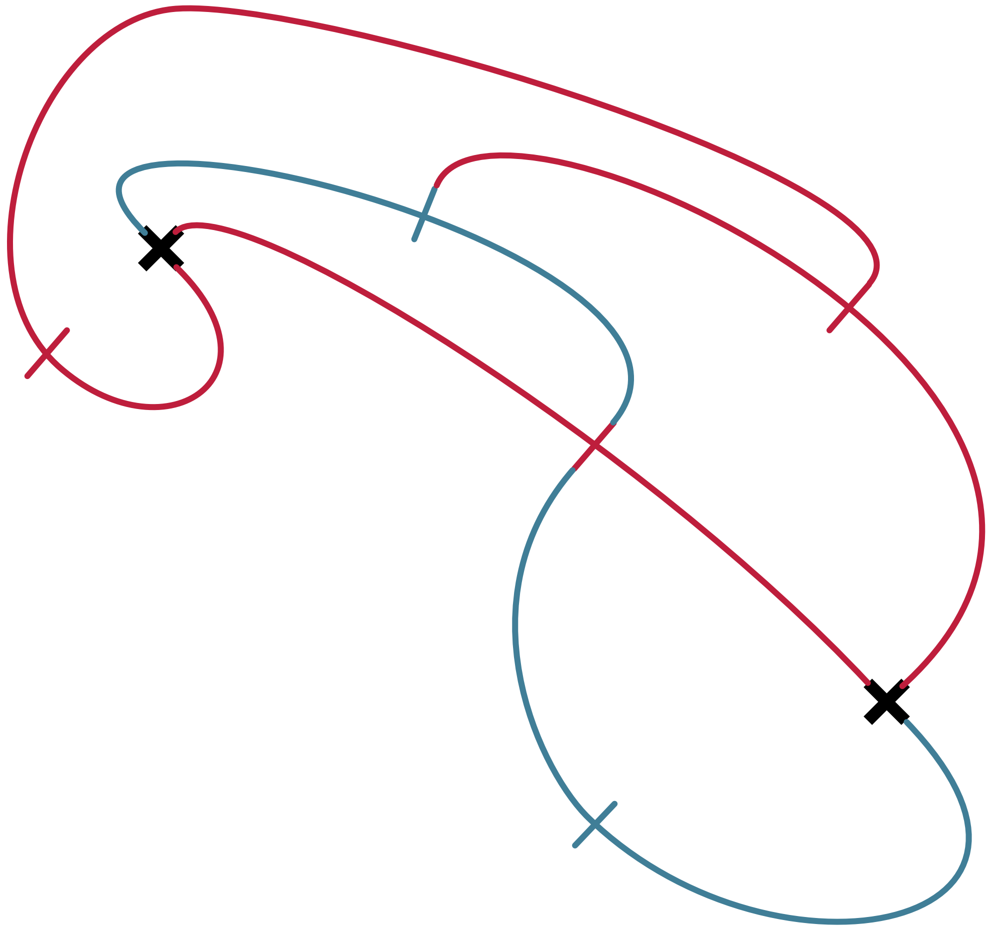

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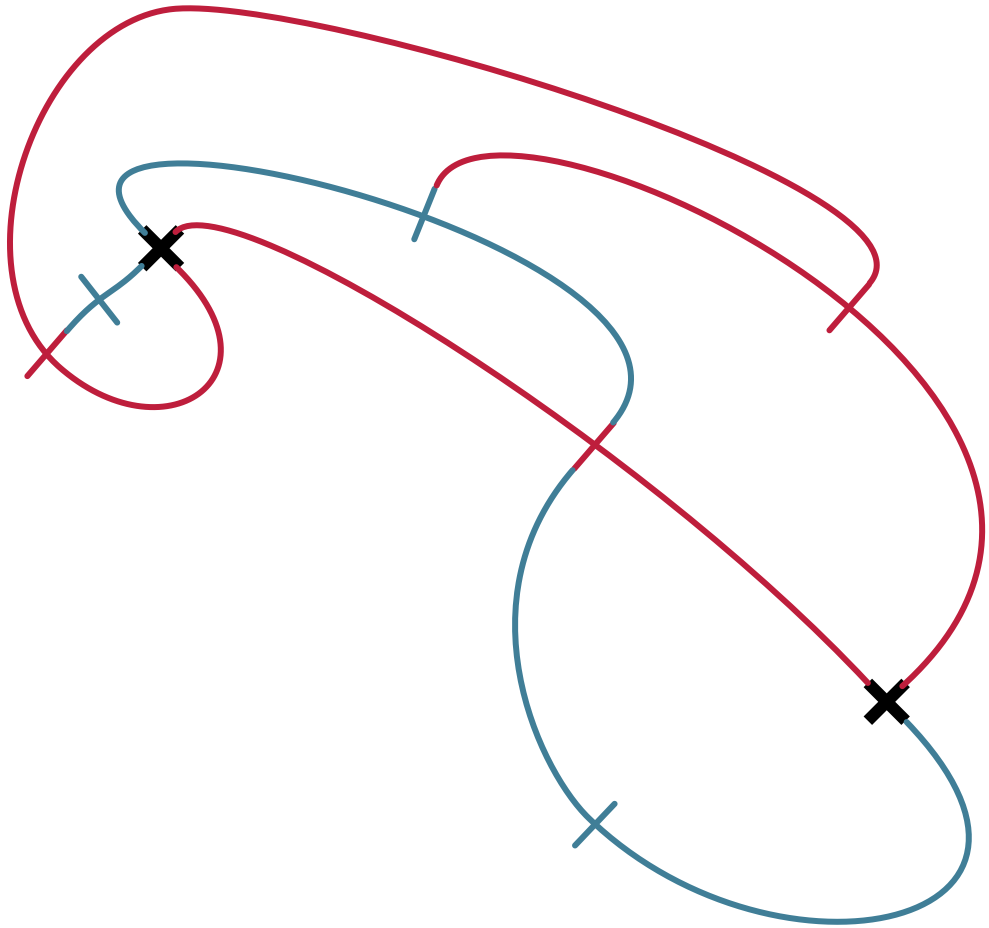

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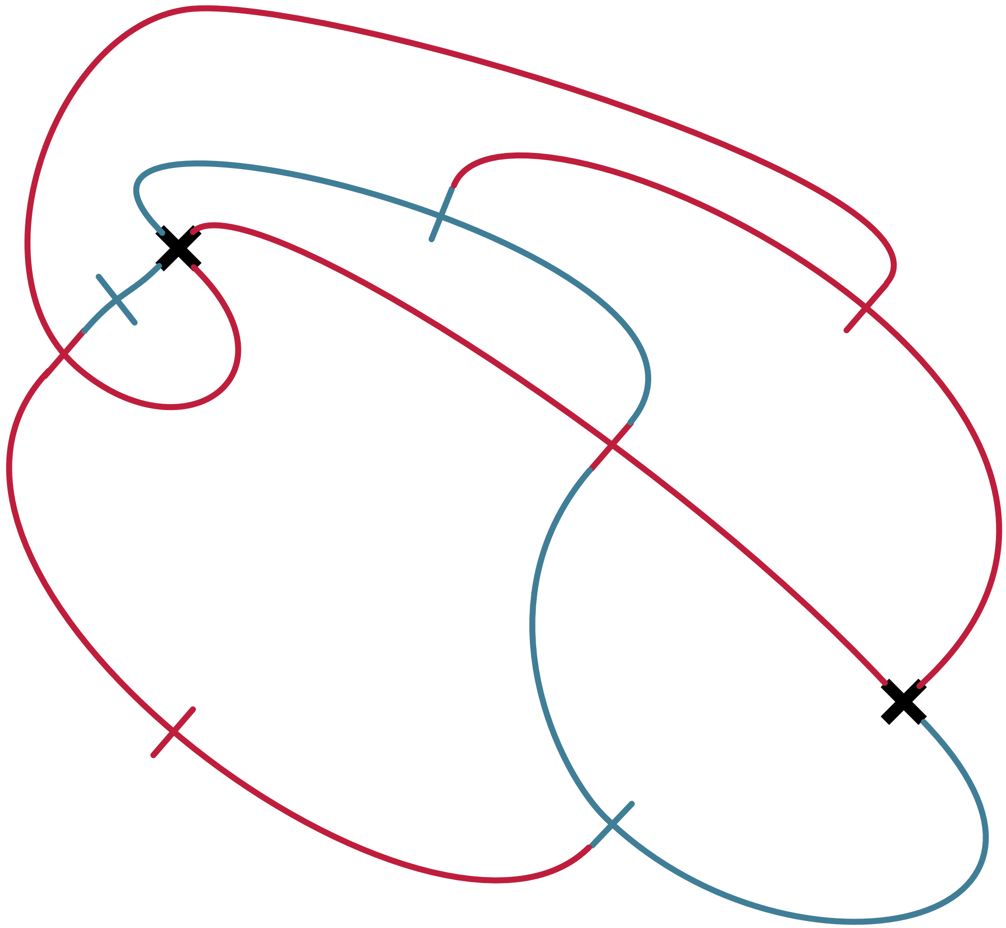

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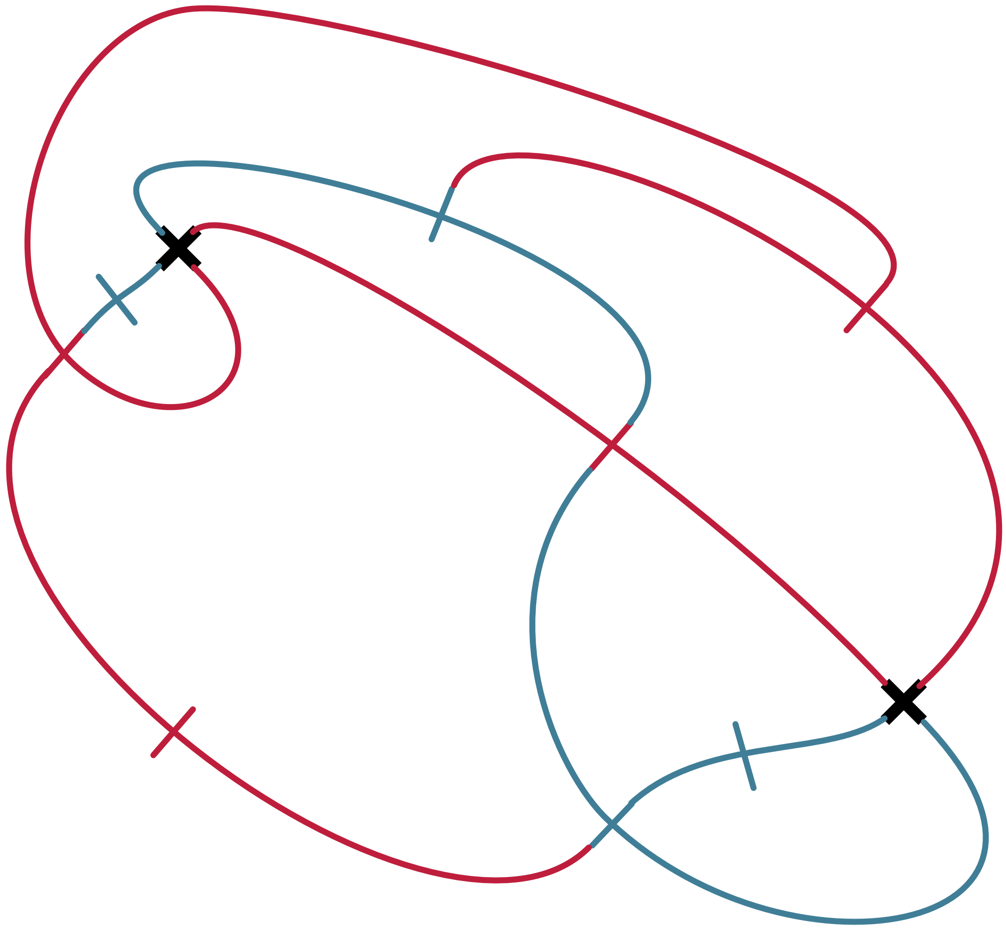

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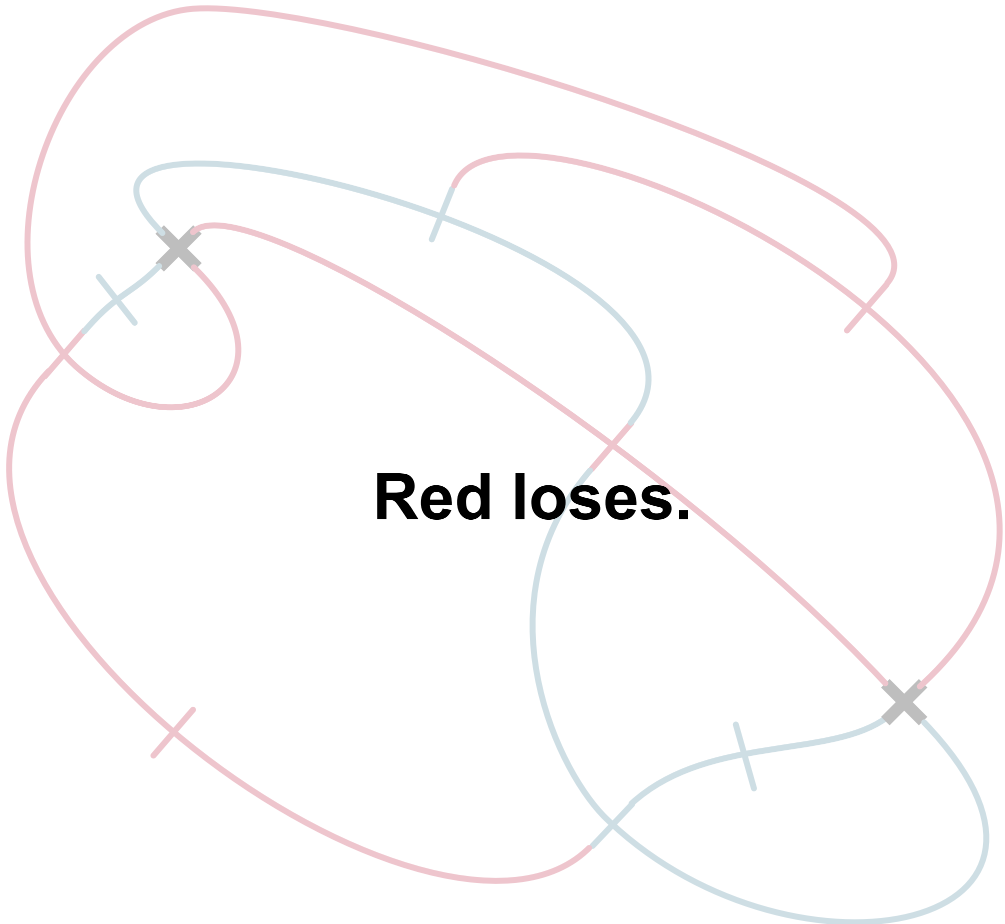
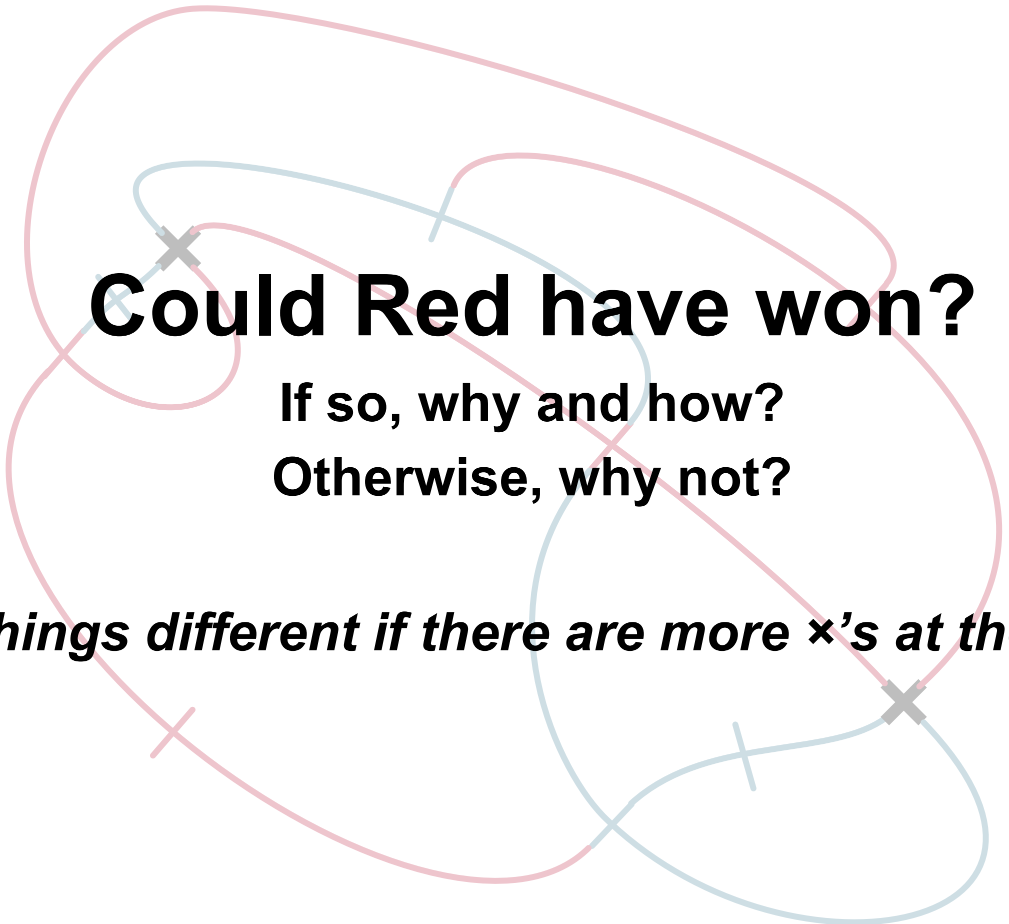


Red loses.

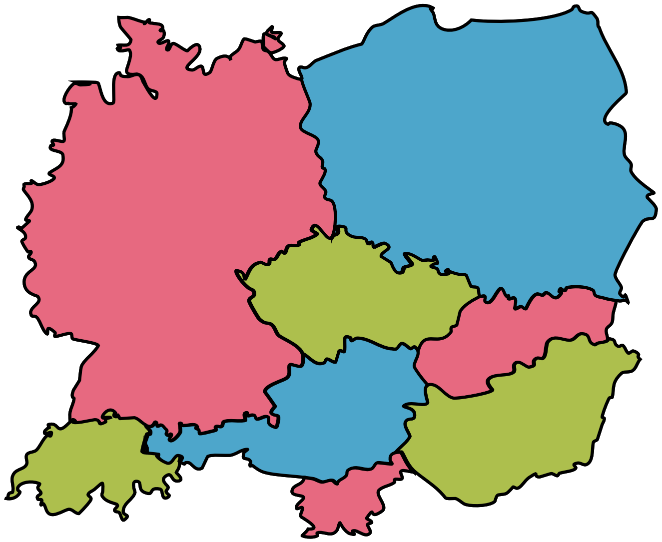


Could Red have won?

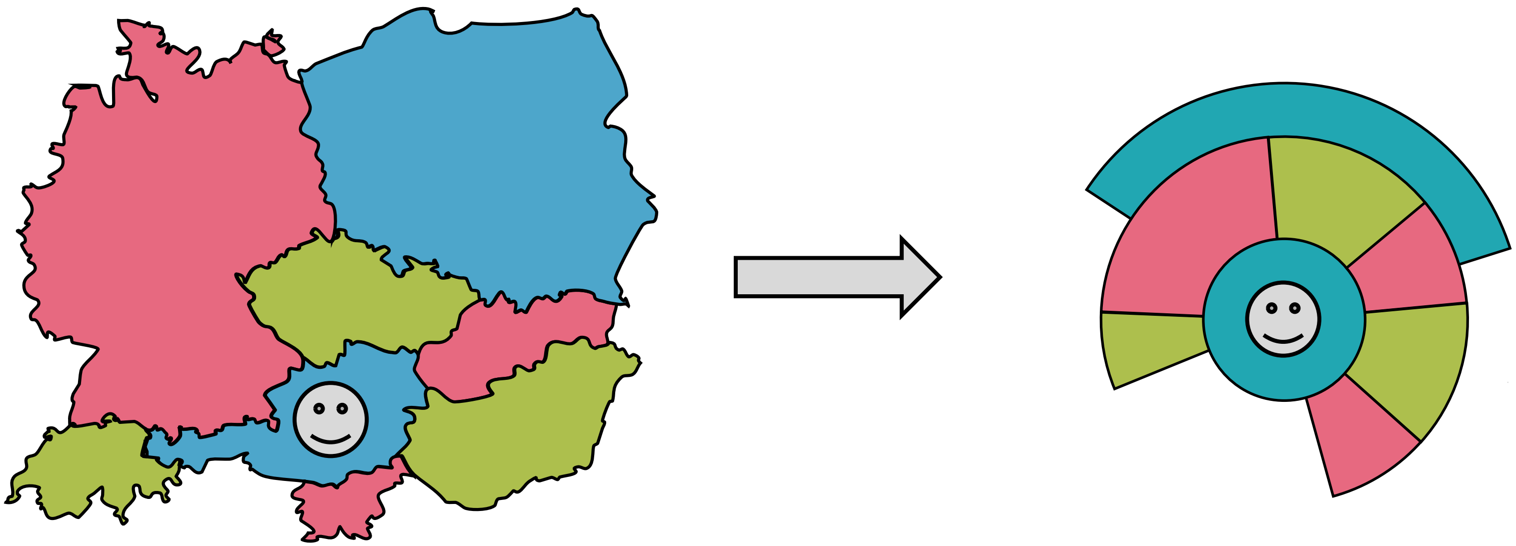
If so, why and how?

Otherwise, why not?

(Are things different if there are more ×'s at the start?)



Three colors are sufficient for this map!

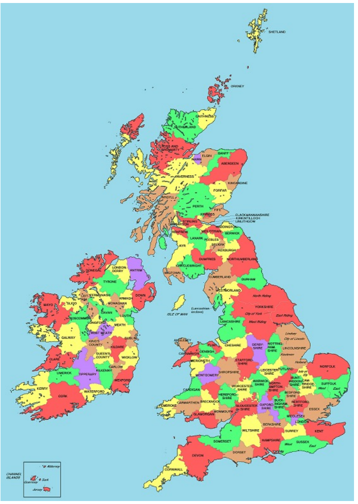


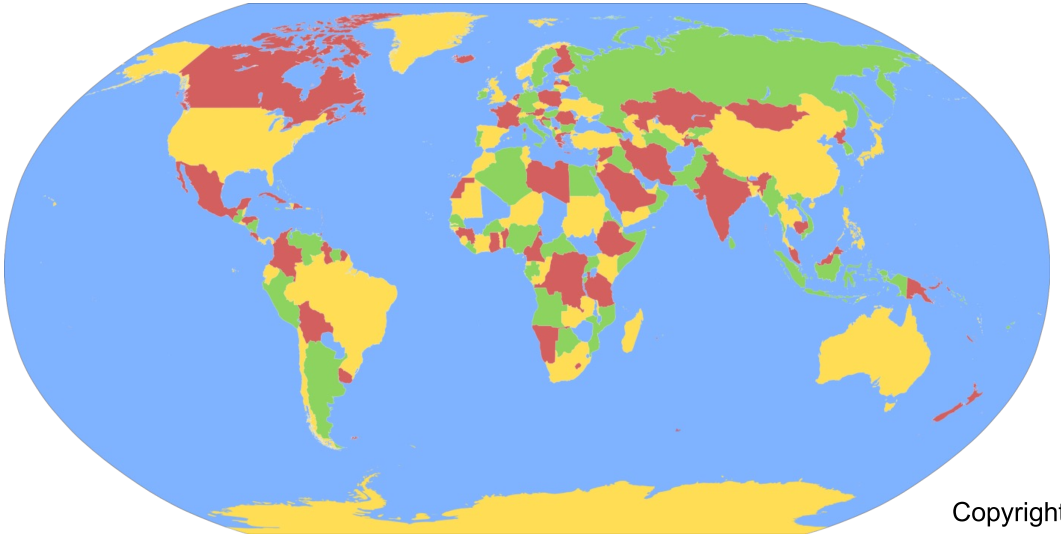
Can you find a map that needs more than three colors?

Can we find a number k such that **every** map can be colored with k colors?

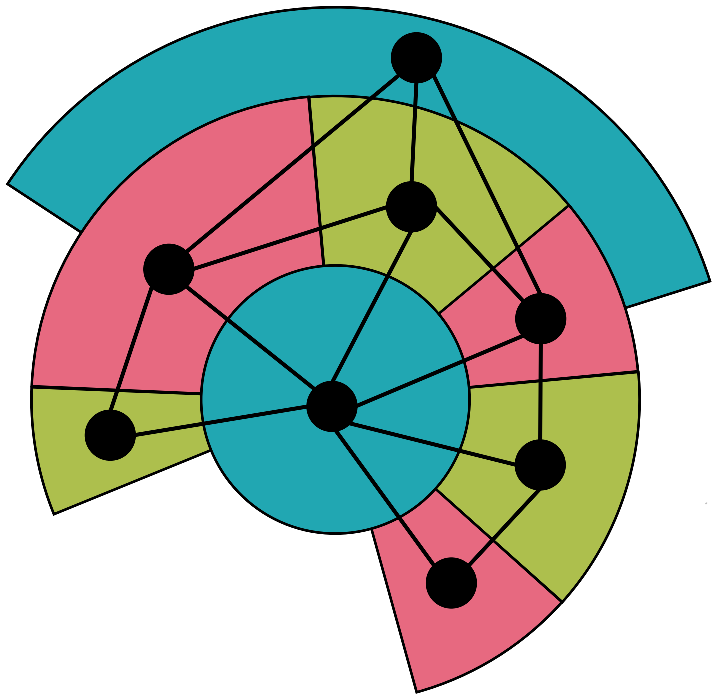
Francis Guthrie

Augustus De Morgan

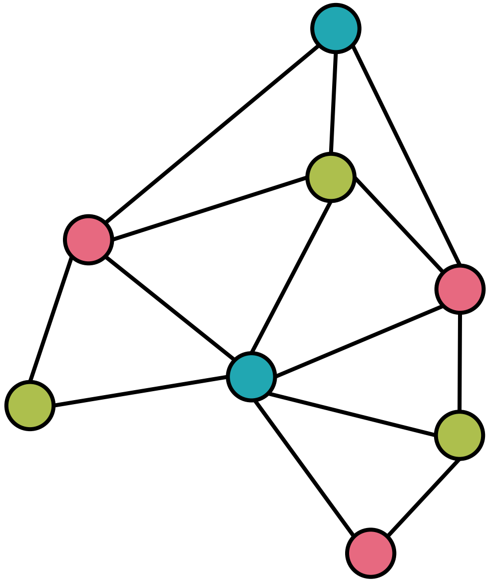




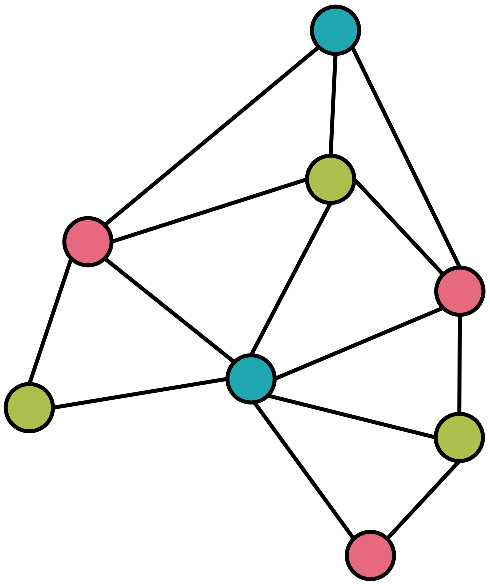
Dual Graph



Dual Graph



Dual Graph



Graph properties:

- Connected
- Planar
- Loopless

The question of coloring map becomes the identification of the *chromatic number* $\chi(G)$ of this graph.

Color the vertices of G such that two adjacent vertices do not share the same color.

Minimum degree of planar graphs

Theorem 1.1

Every connected planar graph with $n \geq 3$ has at least one vertex with degree at most 5.

Proof

First note that by Euler‘s formula: $|E| \leq 3|V| - 6$

Suppose there exists a planar graph G with with $\mathrm{d}(\mathrm{v}) \geq 6 \quad \forall v \in V$

$$\sum_{v \in V} d(v) = ? \;= 2\,|E| \;\leq 6\,|V| - 12$$

$$\sum_{v \in V} d(v) \geq 6|V|$$

How many colors are always sufficient?

Can we find a number k such that **every** planar graph can be colored with k colors?

We can prove that $k \leq 6$: **see board** ☺

Theorem 1.2

For a loopless planar graph G, its chromatic number is $\chi(G) \leq 4$

Kenneth Ira Appel
University of New Hampshire

Wolfgang Haken
© Tori Egherman

So actually four colors are sufficient for **every** map you can think of!