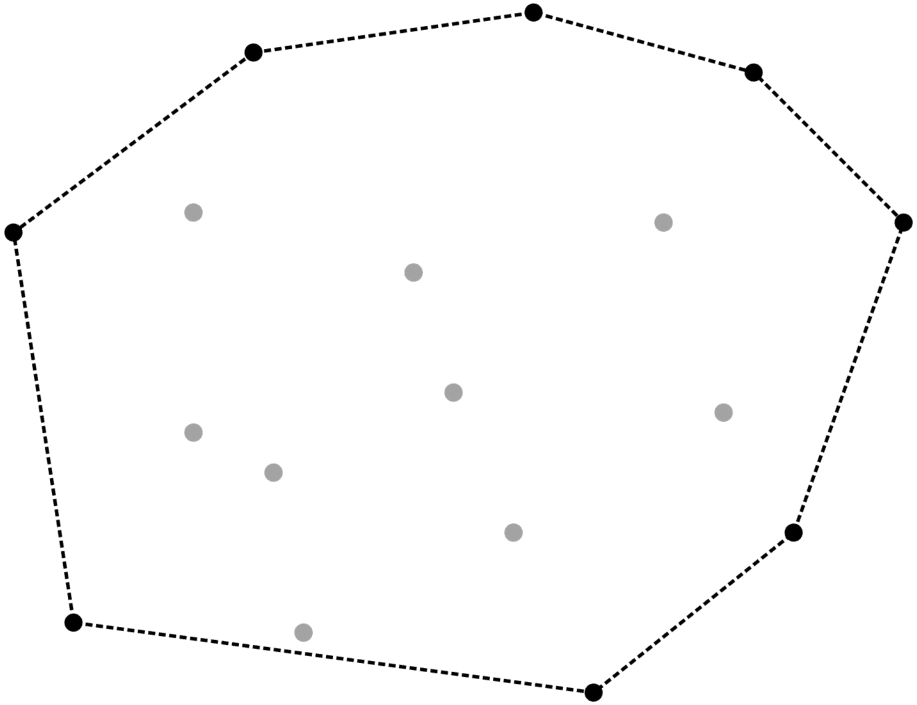
Technische Universität Braunschweig

Computational Geometry – Exercise Meeting #3

December 8th, 2022

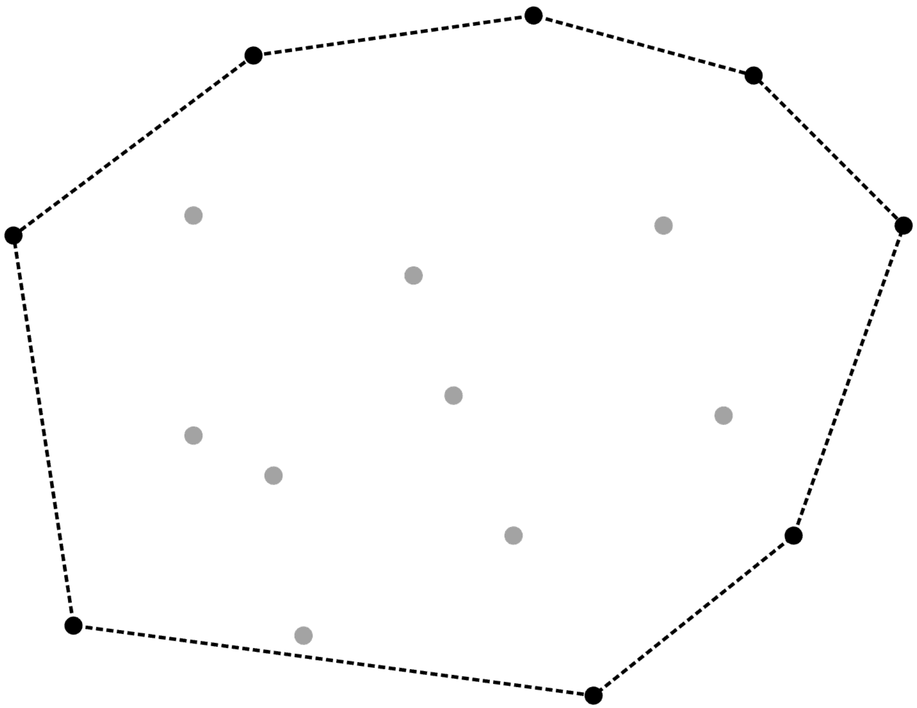
Farthest Pairs

$\mathcal{O}(n\log(n))$

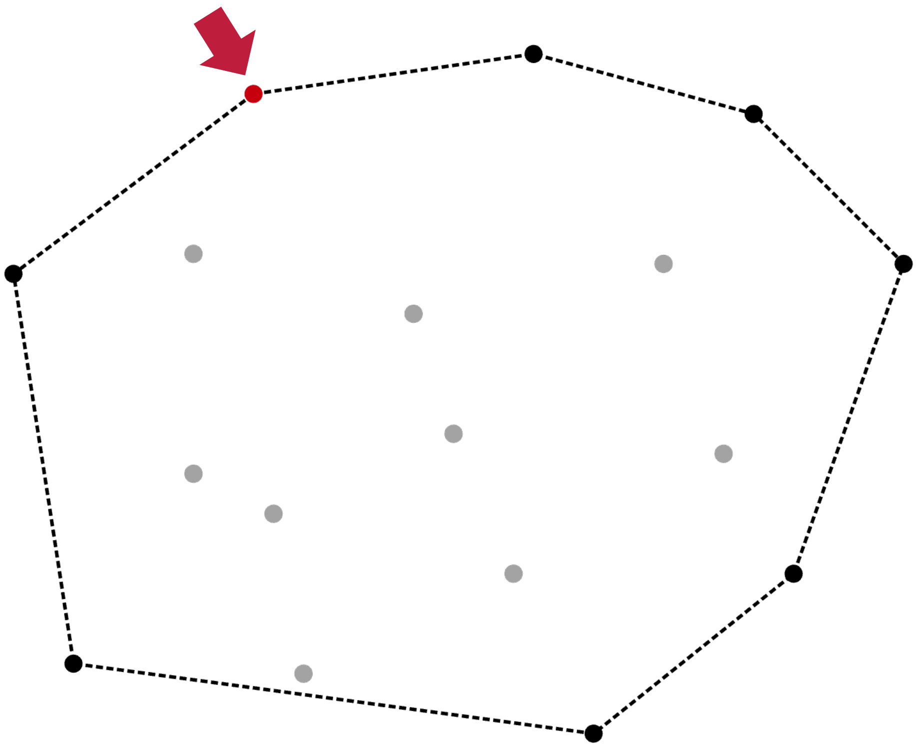


Farthest Pairs

$$\mathcal{O}(n \log(n))$$

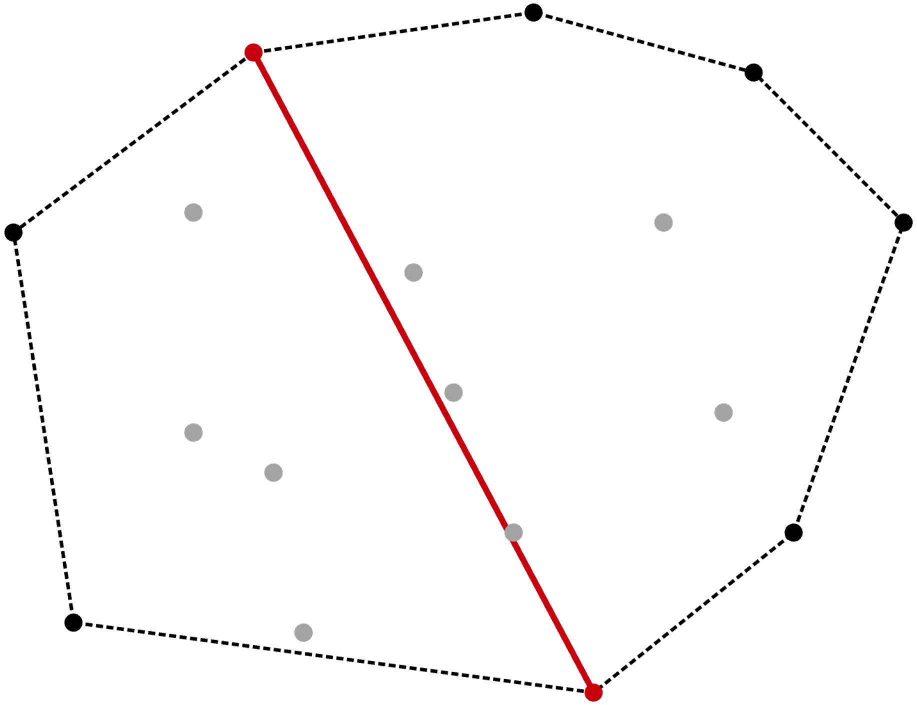


Farthest Pairs

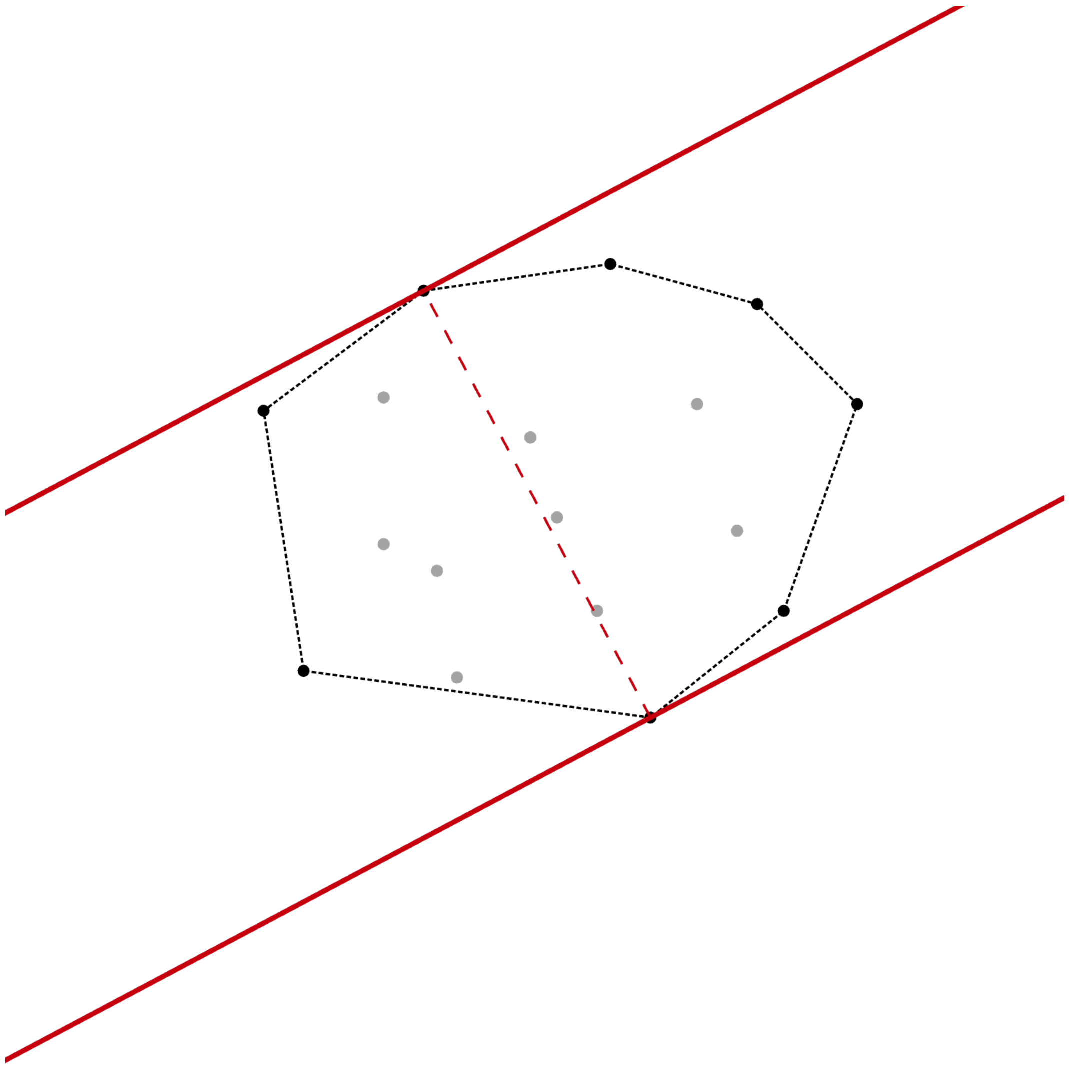


Farthest Pairs

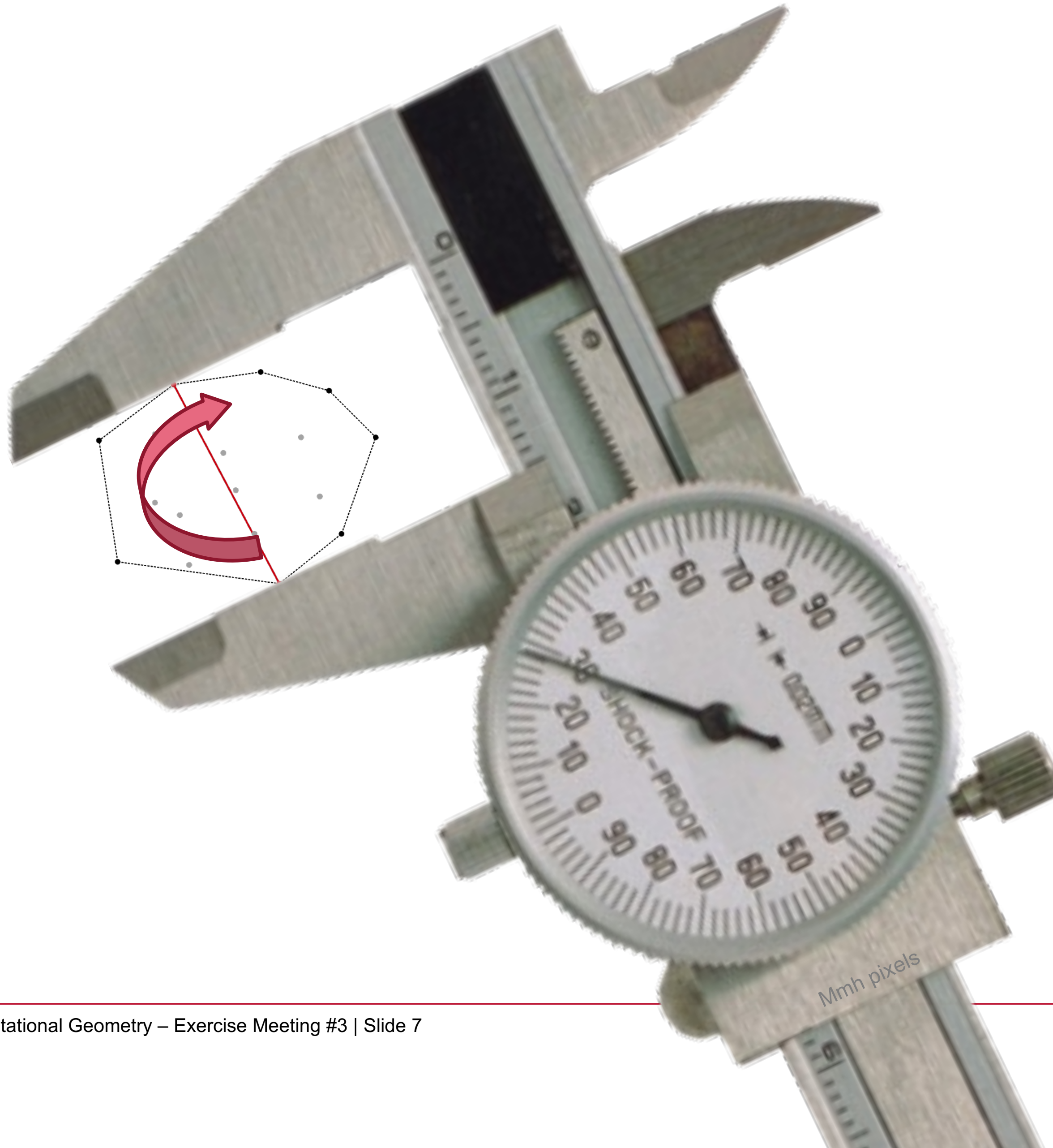
$\mathcal{O}(n)$



Farthest Pairs

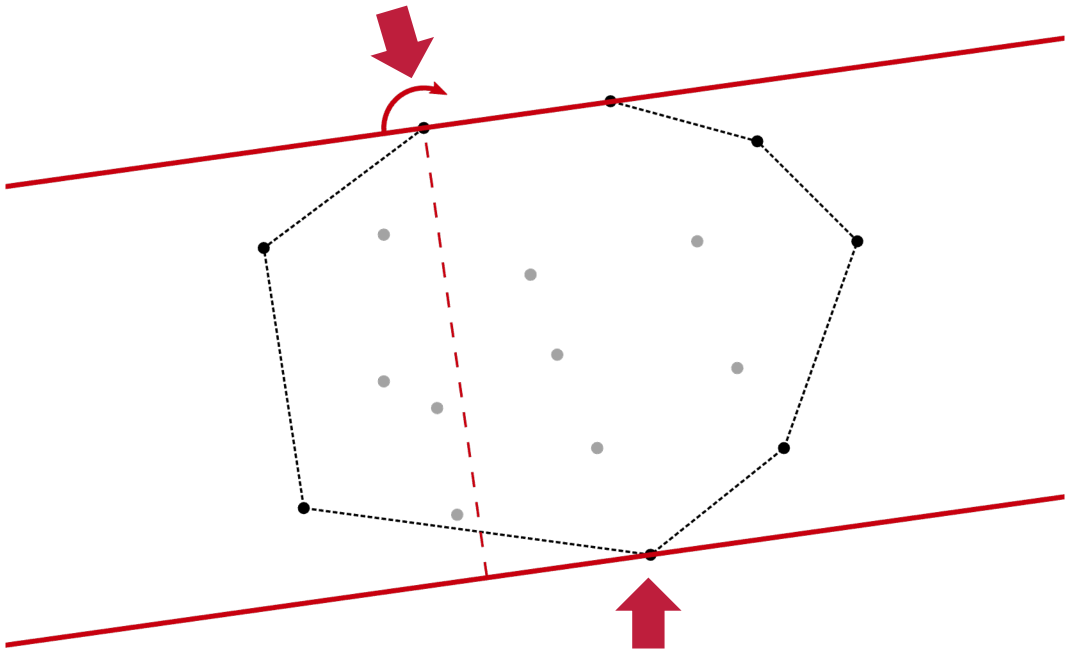


Farthest Pairs

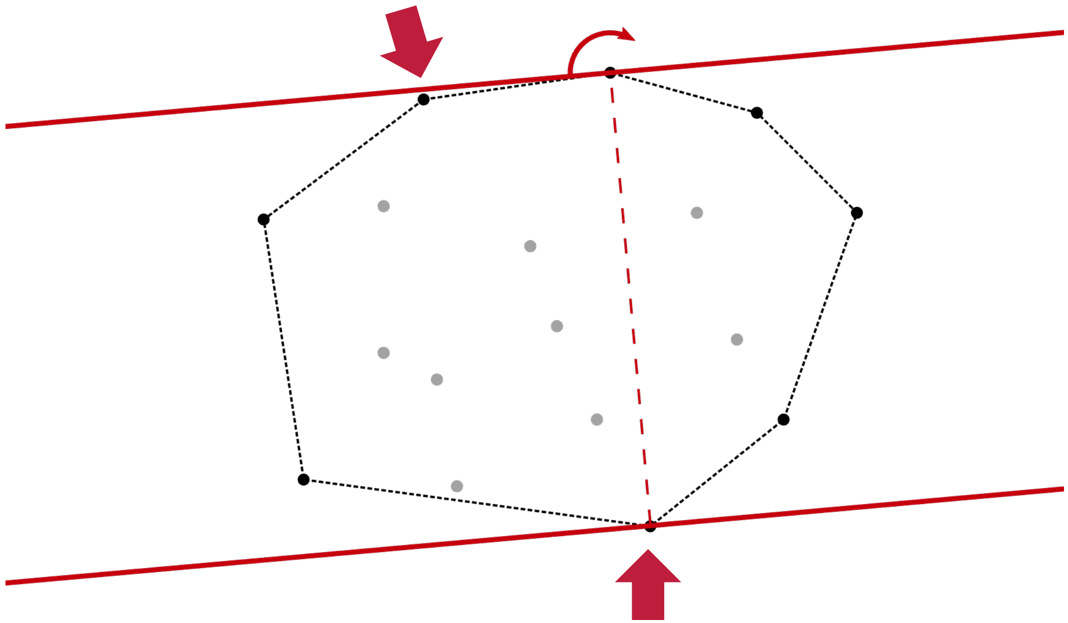


Source: www.richter-messzeuge.de

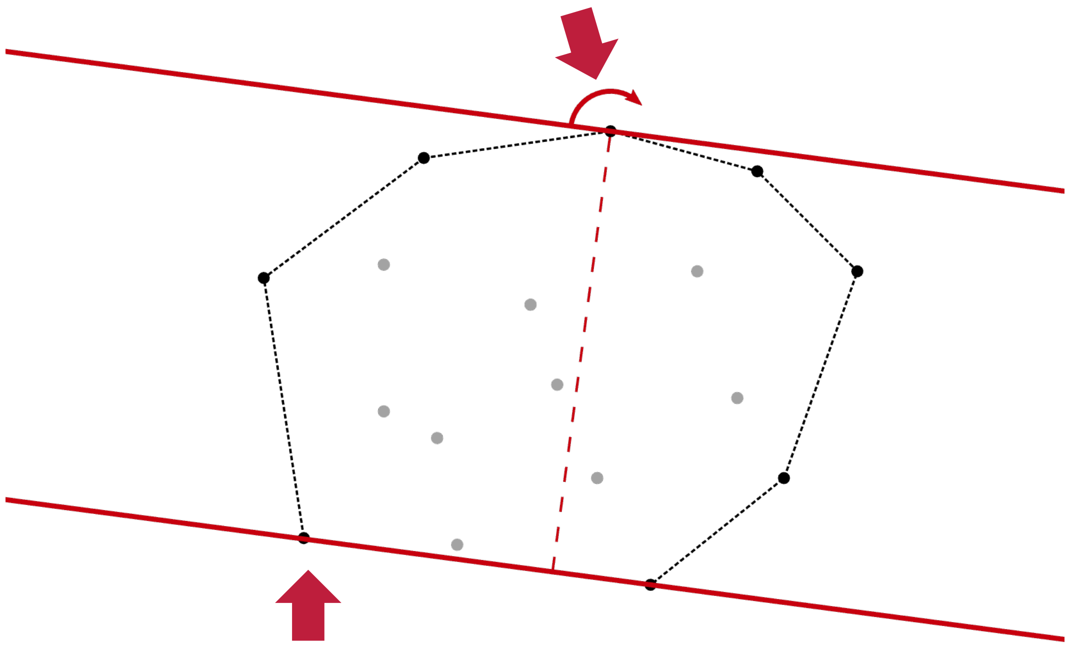
Farthest Pairs



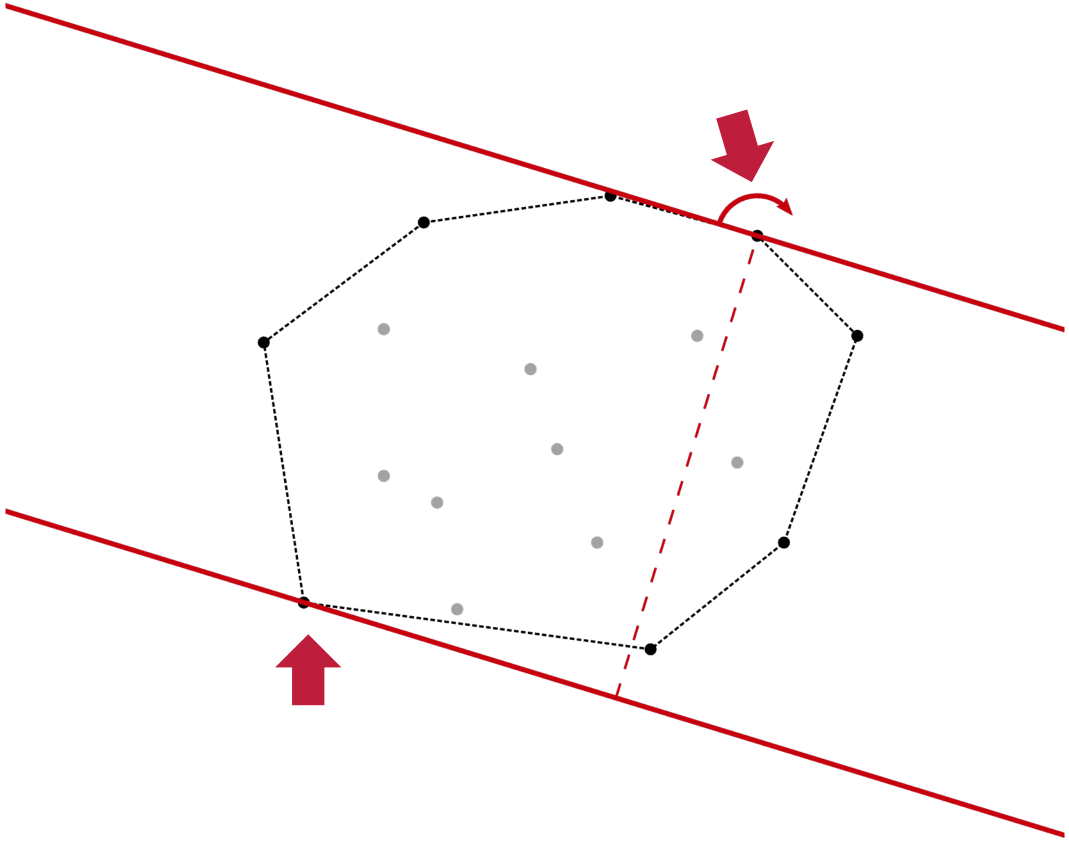
Farthest Pairs



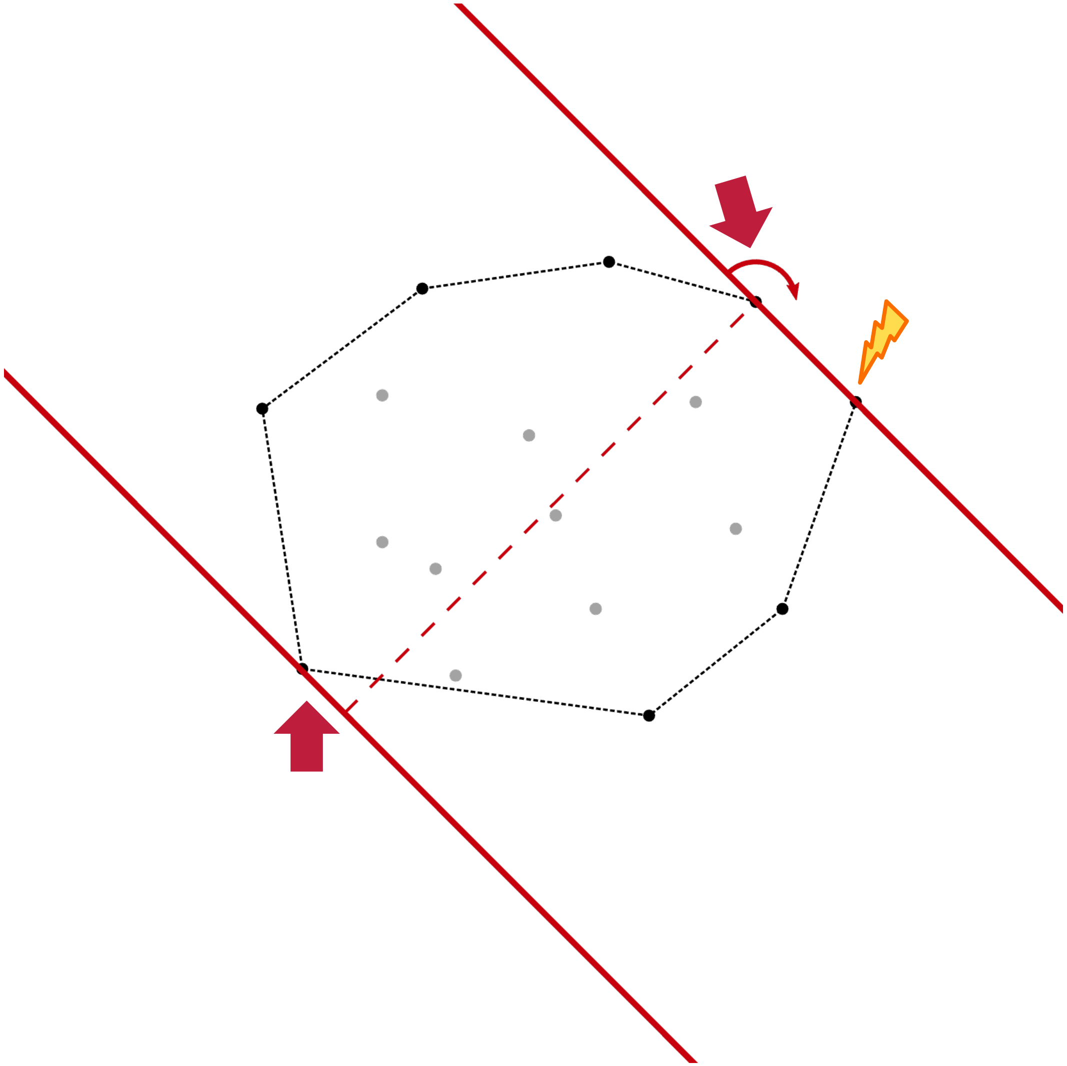
Farthest Pairs



Farthest Pairs



Farthest Pairs



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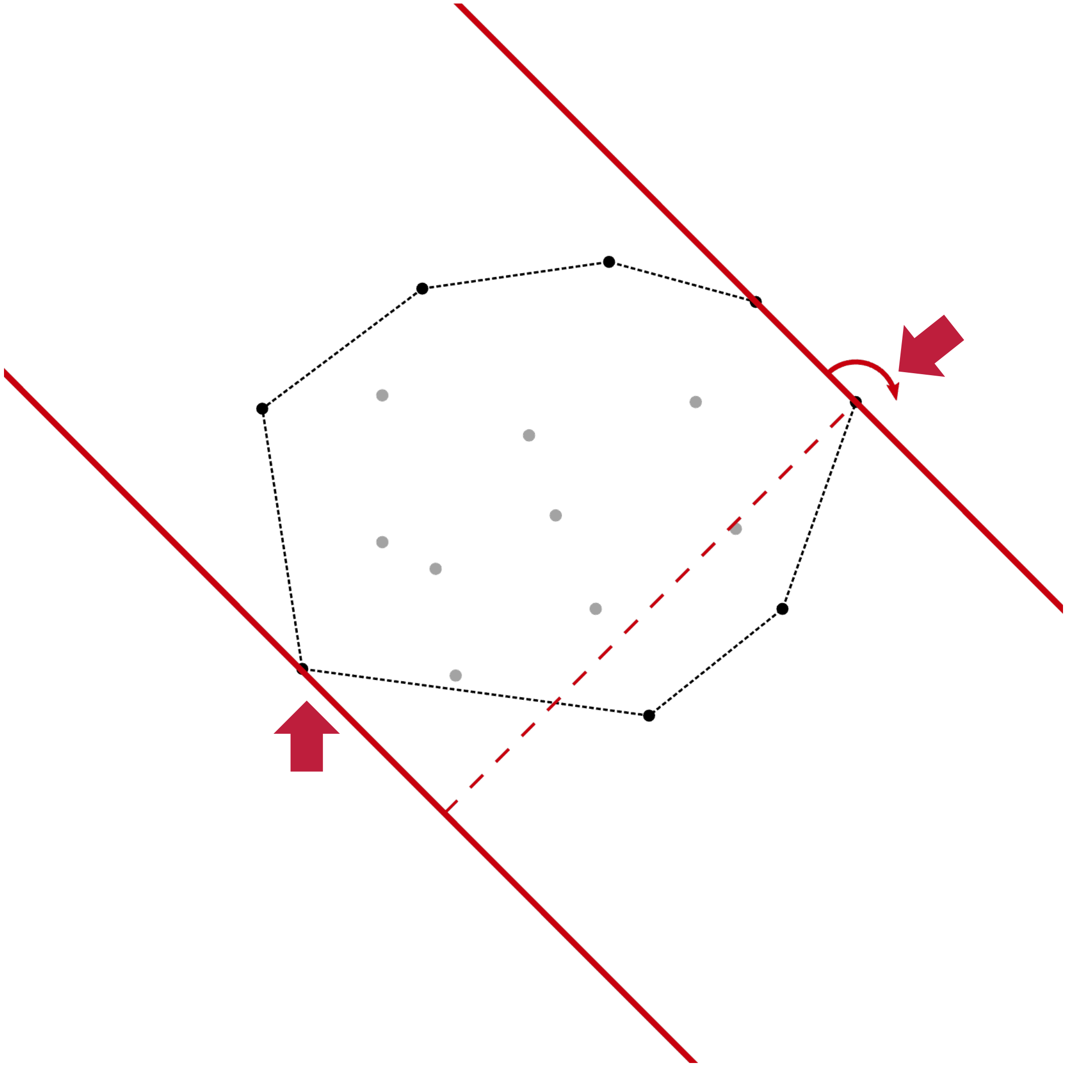
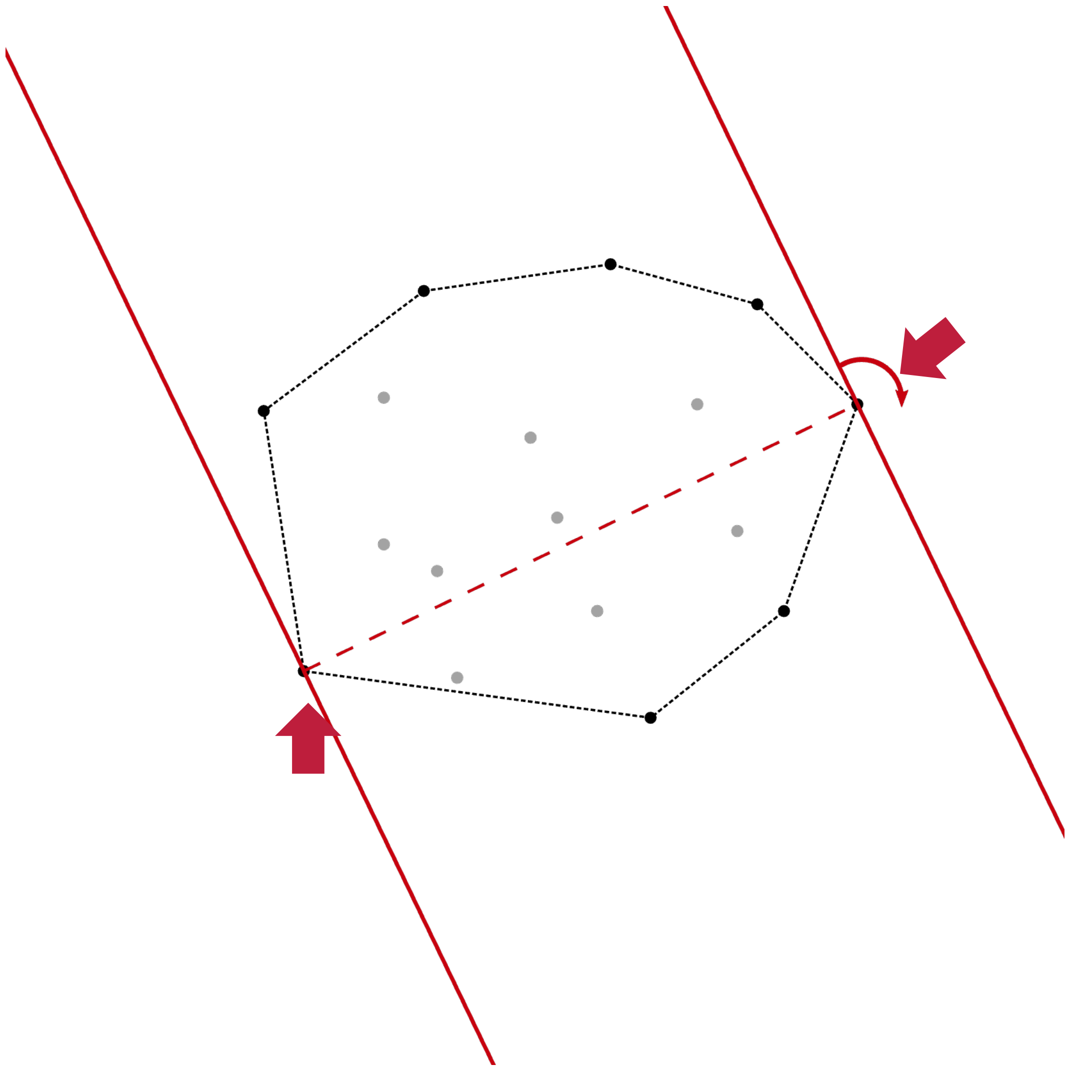
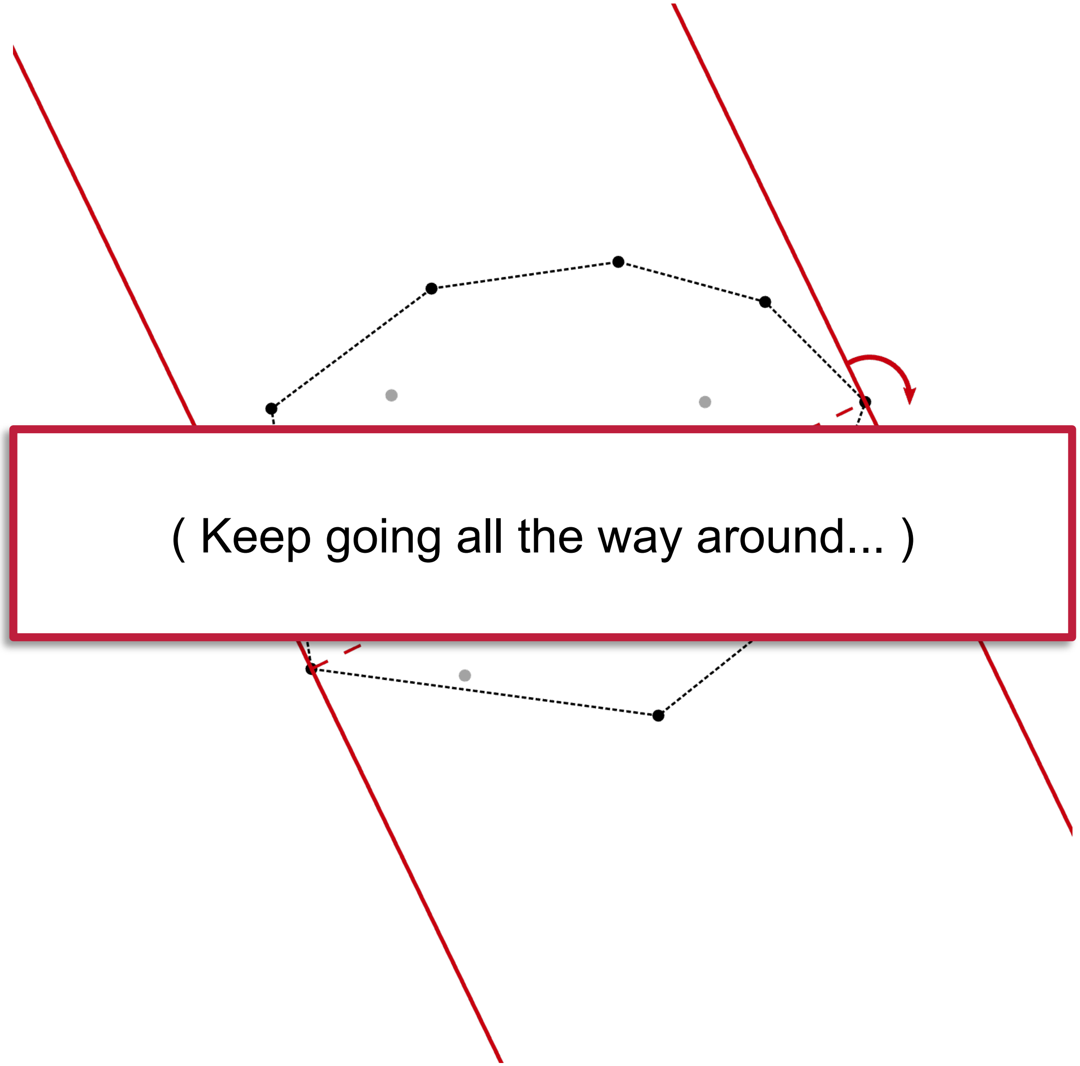


Farthest Pairs



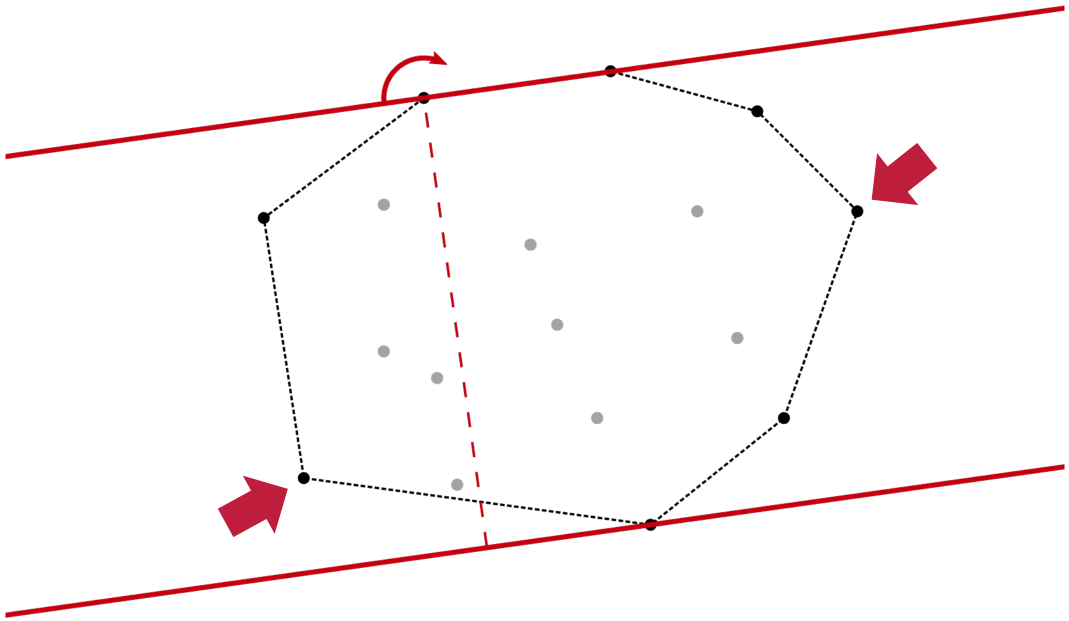
Farthest Pairs



(Keep going all the way around...)

Farthest Pairs

$\mathcal{O}(n \log(n))$ total



Rotating Calipers

```
/* p[] is in standard form, ie, counter clockwise order,
     distinct vertices, no collinear vertices.
   ANGLE(m, n) is a procedure that returns the clockwise angle
     swept out by a ray as it rotates from a position parallel
     to the directed segment Pm,Pm+1 to a position parallel to Pn, Pn+1
   We assume all indices are reduced to mod N (so that N+1 = 1).
*/
GetAllAntiPodalPairs(p[1..n])
    // Find first anti-podal pair by locating vertex opposite P1
    i = 1
    j = 2
    while angle(i, j) < pi
        j++
    yield i, j

    /* Now proceed around the polygon taking account of
         possibly parallel edges. Line L passes through
         Pi, Pi+1 and M passes through Pj, Pj+1
    */

    // Loop on j until all of P has been scanned
    current = i
    while j != n
        if angle(current, i + 1) <= angle(current, j + 1)
            j++
            current = j
        else
            i++
            current = i
        yield i, j
```

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Rotating Calipers – Other Applications

Distances [edit]

- Diameter (maximum width) of a convex polygon[6][7]
- Width (minimum width) of a convex polygon[8]
- Maximum distance between two convex polygons[9][10]
- Minimum distance between two convex polygons[11][12]
- Widest empty (or separating) strip between two convex polygons (a simplified low-dimensional variant of a problem arising in support vector machine based machine learning)
- Grenander distance between two convex polygons[13]
- Optimal strip separation (used in medical imaging and solid modeling)[14]

Bounding boxes [edit]

- Minimum area oriented bounding box
- Minimum perimeter oriented bounding box

Triangulations [edit]

- Onion triangulations
- Spiral triangulations
- Quadrangulation
- Nice triangulation
- Art gallery problem
- Wedge placement optimization problem[15]

Multi-polygon operations [edit]

- Union of two convex polygons
- Common tangents to two convex polygons
- Intersection of two convex polygons[16]
- Critical support lines of two convex polygons
- Vector sums (or Minkowski sum) of two convex polygons[17]
- Convex hull of two convex polygons

Traversals [edit]

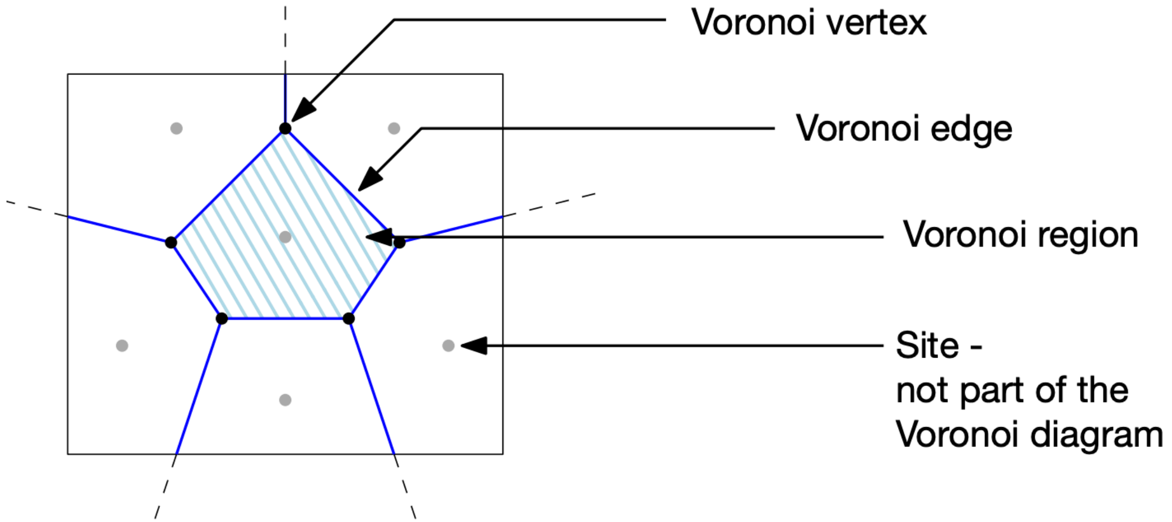
- Shortest transversals[18][19]
- Thinnest-strip transversals[20]

Others [edit]

- Non parametric decision rules for machine learned classification[21]
- Aperture angle optimizations for visibility problems in computer vision[22]
- Finding longest cells in millions of biological cells[23]
- Comparing precision of two people at firing range
- Classify sections of brain from scan images

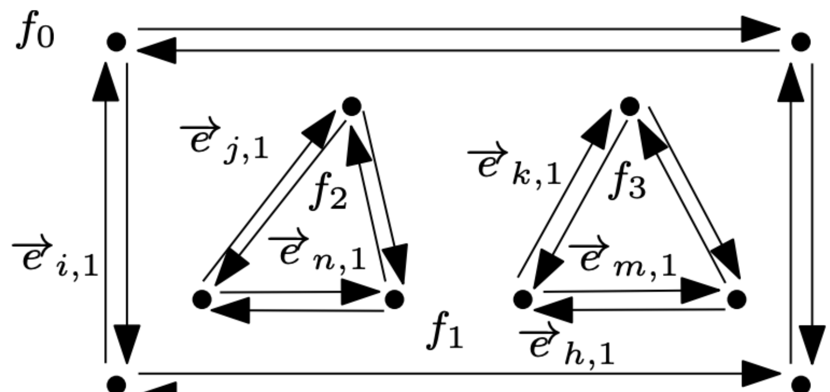
https://en.wikipedia.org/wiki/Rotating_calipers#Applications

Voronoi Basics



Theorem 4.7

$Vor(\mathcal{P})$ has precisely n Voronoi regions, at most $2n-5$ Voronoi vertices and at most $3n-6$ Voronoi edges.



Lukasz Kobus/European Commission, Attribution, via Wikimedia Commons

EU-KOMMISSIONSCHEFIN

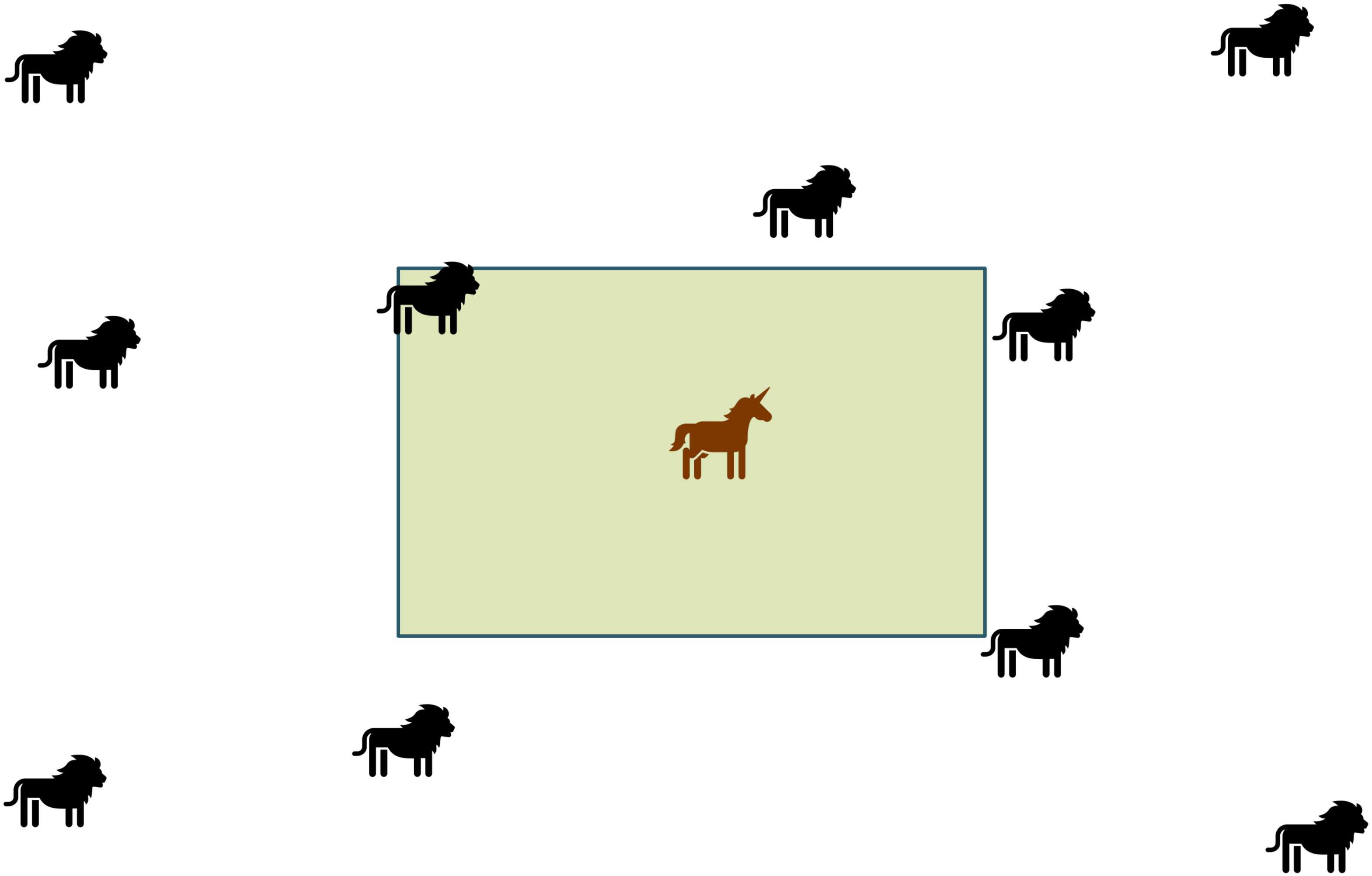
Wolf tötet Ursula von der Leyens Pony – jetzt will sie den geschützten Tieren an den Kragen

ERGEBNIS DES GENTESTS

Problemwolf GW 950m hat von der Leyens Pony „Dolly“ gerissen

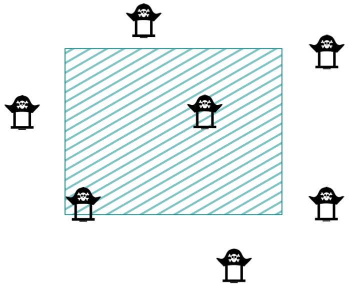
User:Mas3cf, CC BY-SA 4.0, via Wikimedia Commons

URSULAVONDERLEYEN/INSTAGRAM



Maximizing distance to sites in a bounded area

Provided a set of **sites $\boldsymbol{P}$** in the plane and an axis-aligned **rectangle $\boldsymbol{R}$**, find a point inside of R with **maximal distance to the nearest member of $\boldsymbol{P}$**. Assume that no three points in P are collinear.



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