Computational Geometry

Tutorial #7 — Exam preparation

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February 22, 2024

Organisation
Exam: Date, Time & Place

- **On March 14th, at 2pm in SN 19.1**
- **Permitted aids:** ruler, (optionally colored) pens ***(NO red ink)***

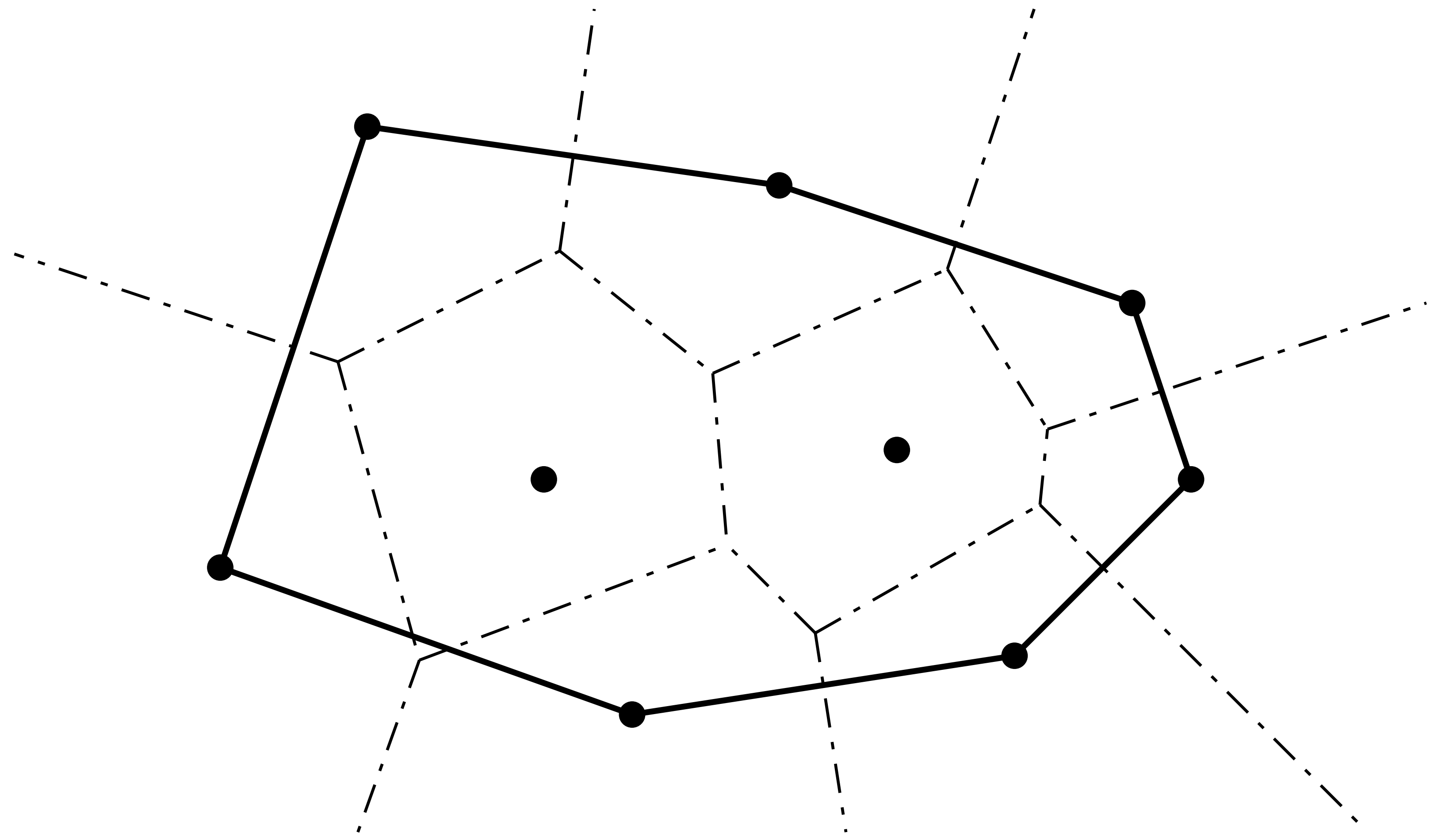
- **Total time:** 120 minutes

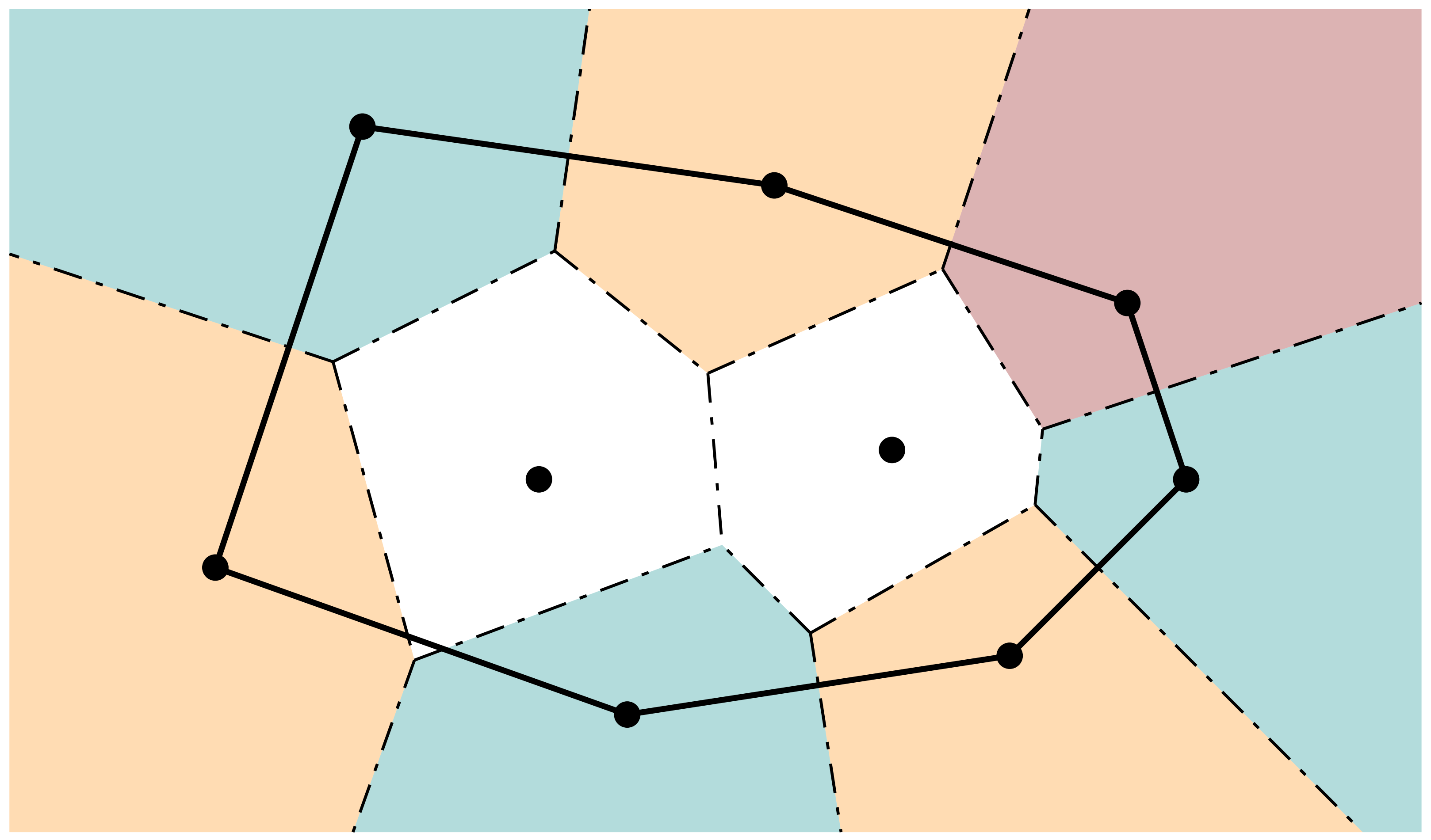
- Covers *all* chapters of the lecture and homework:
  - Convex hull, Closest pairs, Voronoi diagrams + games, Polygon + Point triangulation, Location problems, Tours, Milling, …

Common mistakes on Sheet #2

Voronoi diagram
Dual graph

(S01.3c) "Is there a relationship between the convex hull of a point set and its Voronoi diagram?"





Voronoi diagrams

Higher-order Voronoi diagrams

(1a) *"For $k \geq 1$, what does the kth order Voronoi diagram represent?"*

- First order: Which one of the n sites is closest?
- Second order: Which two of the n sites are closest?

$\ldots$

- $(n-1)$th order: Which $(n-1)$ of the n sites are closest?

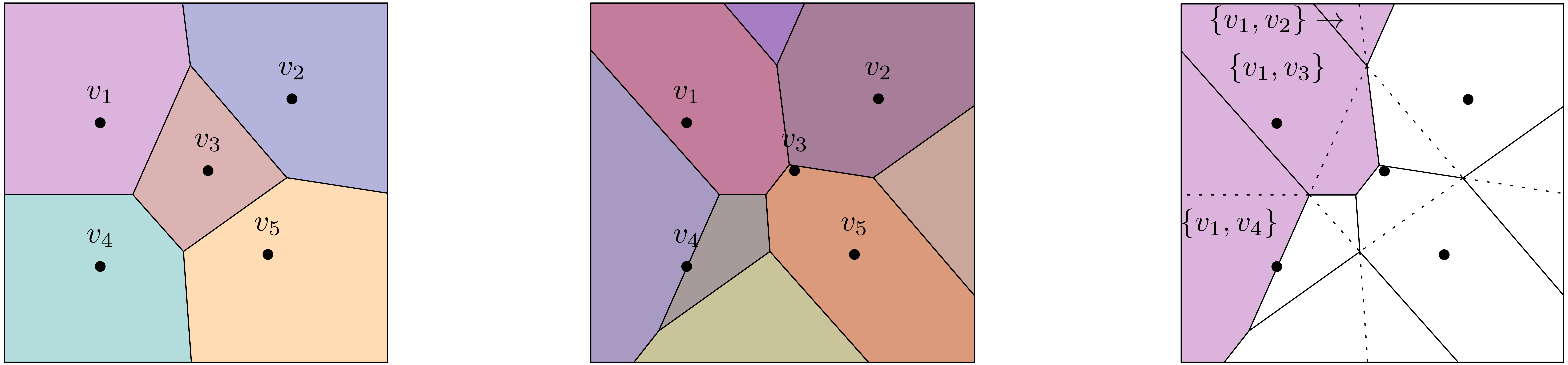
… which is eqivalent to:

- "Farthest point": Which one of the n sites is farthest?

Voronoi diagrams

Higher-order Voronoi diagrams

(1a) *"For $k \geq 1$, what does the kth order Voronoi diagram represent?"*



Voronoi diagrams
Higher-order Voronoi diagrams

(1b) “Consider a region of the k*th order Voronoi diagram. Argue into how many regions it will be split in the* $(k+1)$*th order Voronoi diagram.”*

- Idea here: Bound from above, the exact number depends on the points!
- “True” bound: kth order has $\mathcal{O}(k(n-k))$ Voronoi regions
- Simple upper bound per region: $(n-k)$ new regions, as this is how many options we have to pick a $(k+1)$th point to add to the existing ones.

Voronoi diagrams

Higher-order Voronoi diagrams

(1c) "Argue why for $n \geq 3$*, the* $(n-1)$*th order Voronoi diagram [is] a tree."*

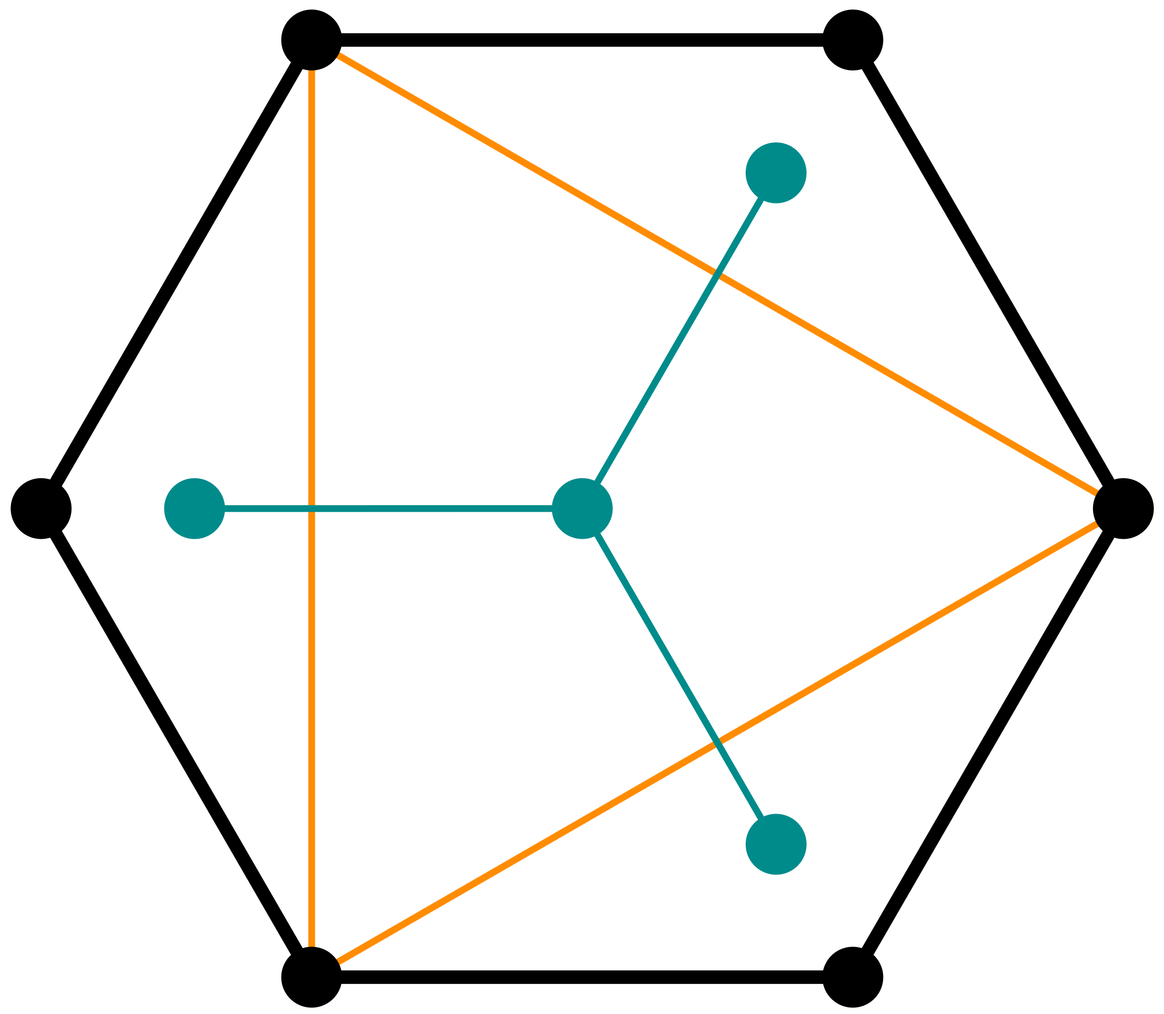
- *Recall:* Farthest pairs lie on the convex hull.
- Farthest point Voronoi diagram has one site per convex hull vertex, each of these is an unbounded region (*).
- *Argument:* A cycle can only exist if there exists a bounded region, therefore (*) implies that the $(n-1)$th order Voronoi diagram is acyclic, i.e., a tree.

Polygon triangulations

Convex polygons

(2a) *"Argue that every convex polygon permits a triangulation that has a dual graph with maximal vertex degree* 2*."*

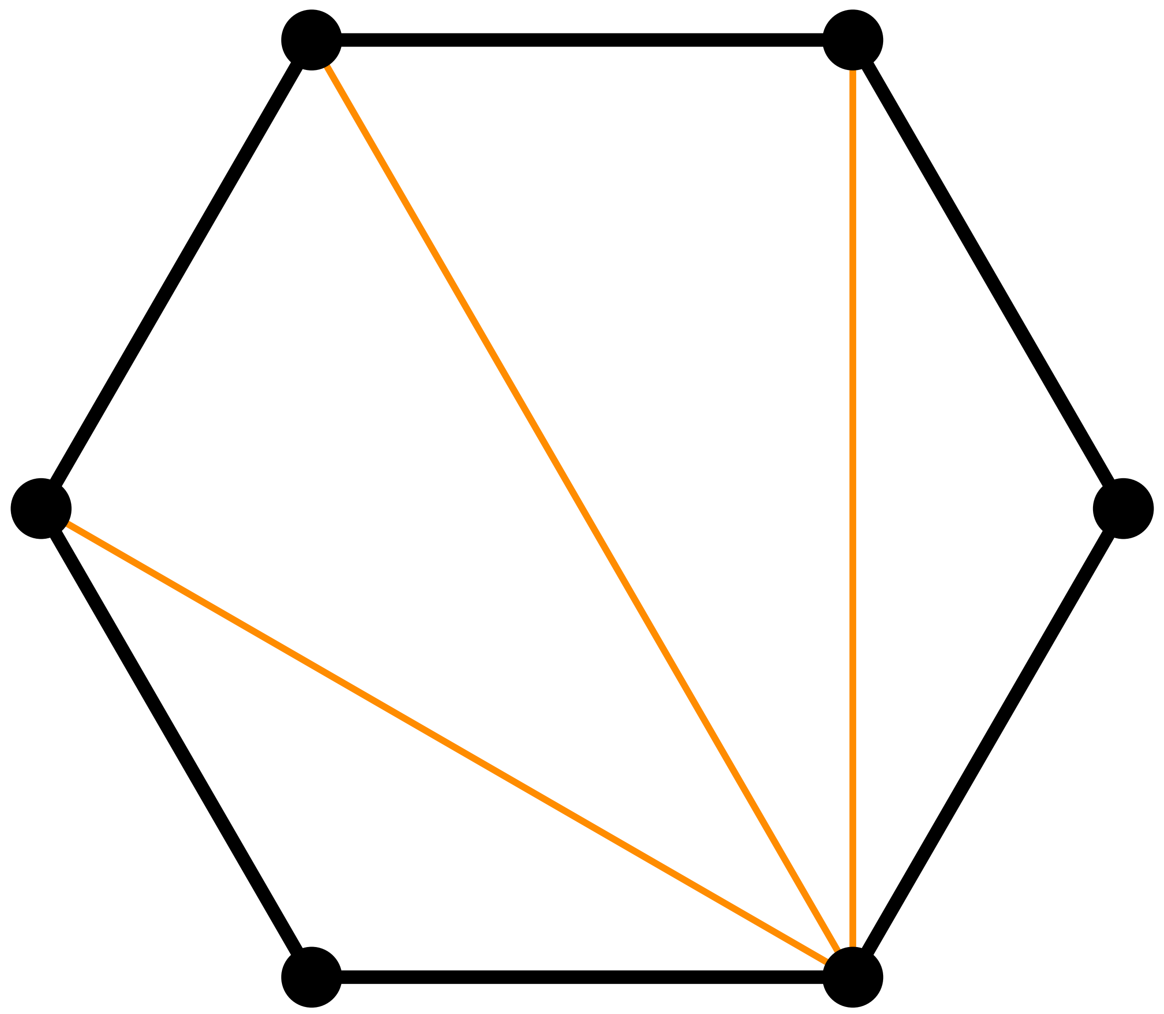
- *Common error: Attempting to prove this bound for any such triangulation.*



Polygon triangulations
Convex polygons

(2a) “Argue that every convex polygon permits a triangulation that has a dual graph with maximal vertex degree 2.”

- *Constructive proof:*



Polygon triangulations
Convex polygons

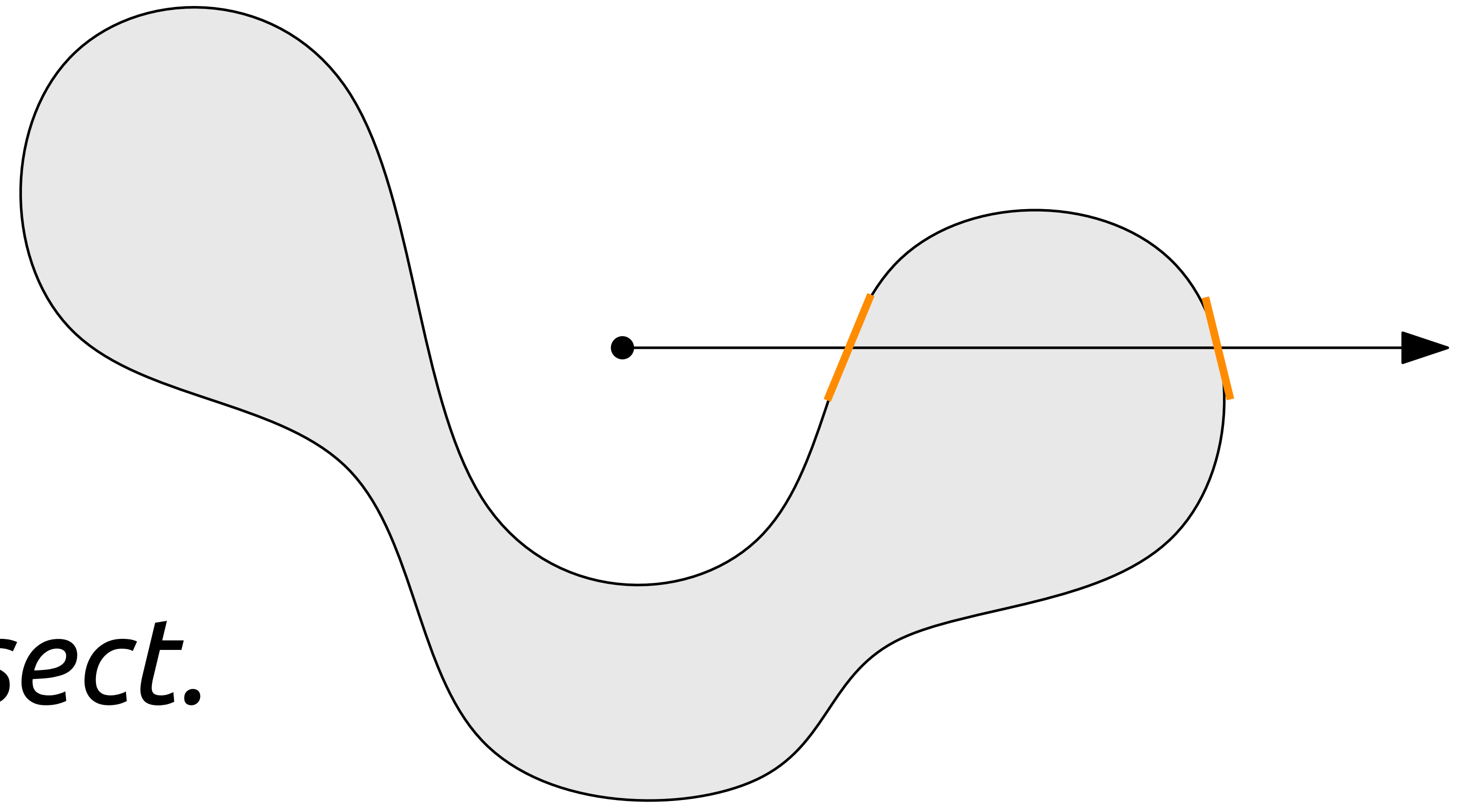
(2a) "Argue that every convex polygon permits a triangulation that has a dual graph with maximal vertex degree 2*."*

- *Constructive proof:*
  - The line segment connecting any two vertices of a convex polygon P is fully contained in it.
  - Two line segments ending in the same point cannot intersect.
  - Therefore: Connect one vertex to all others, obtain a triangulation T.
  - Every triangle of T shares an edge with P, implying the desired bound of two on the degree of vertices in the dual graph.

Polygon triangulations
Point Location Problem

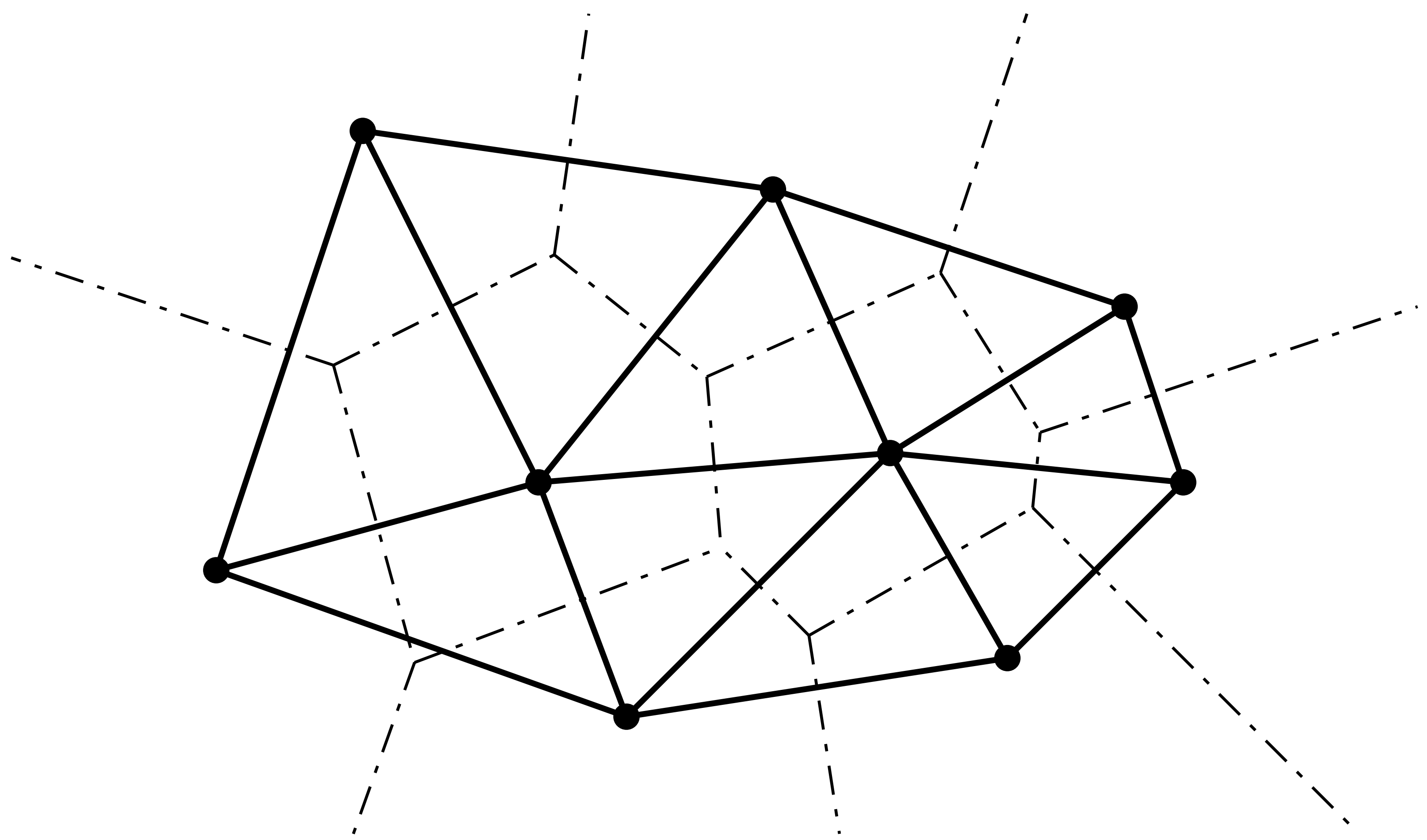
(2b) "How can you decide in $\mathcal{O}(n \log n)$, if a given point p is inside of [a simple] polygon P [of n vertices]?"

- "Expected", simple approach:
  "Triangulate in ___ or ______, then check each of the resulting ___ triangles."
- Alternate approach commonly taken:
  - *Ray casting algorithm:*
    1. *Define a ray r from p in any direction*
    2. *For each edge e of P, check if r and e intersect.*
    3. *Count number of intersections. If odd: p is inside of P.*



Voronoi diagram
Dual graph

(3a) *"Briefly, argue why the dual graph of a point sets Voronoi diagram is a Delauney Triangulation."*



Voronoi diagram

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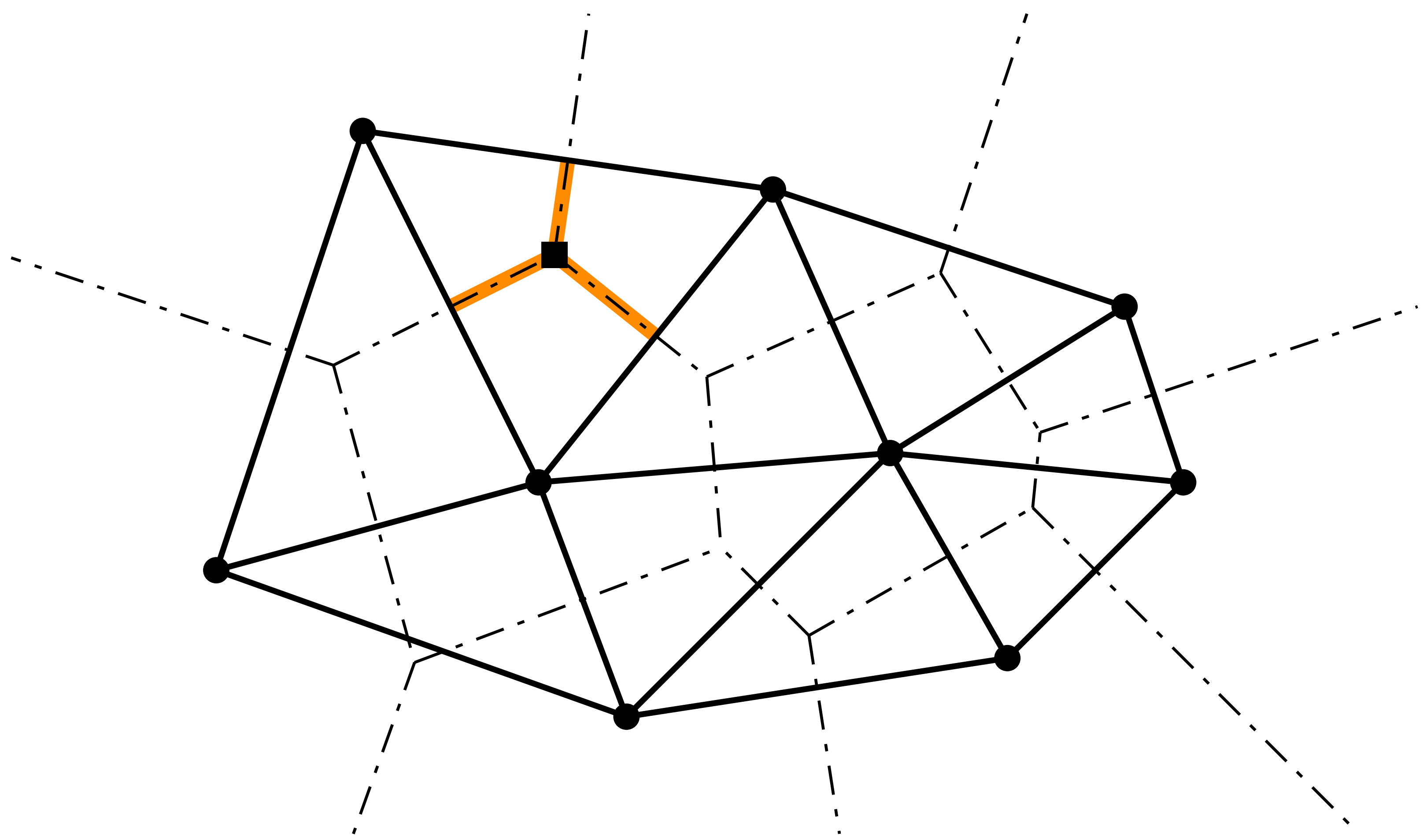

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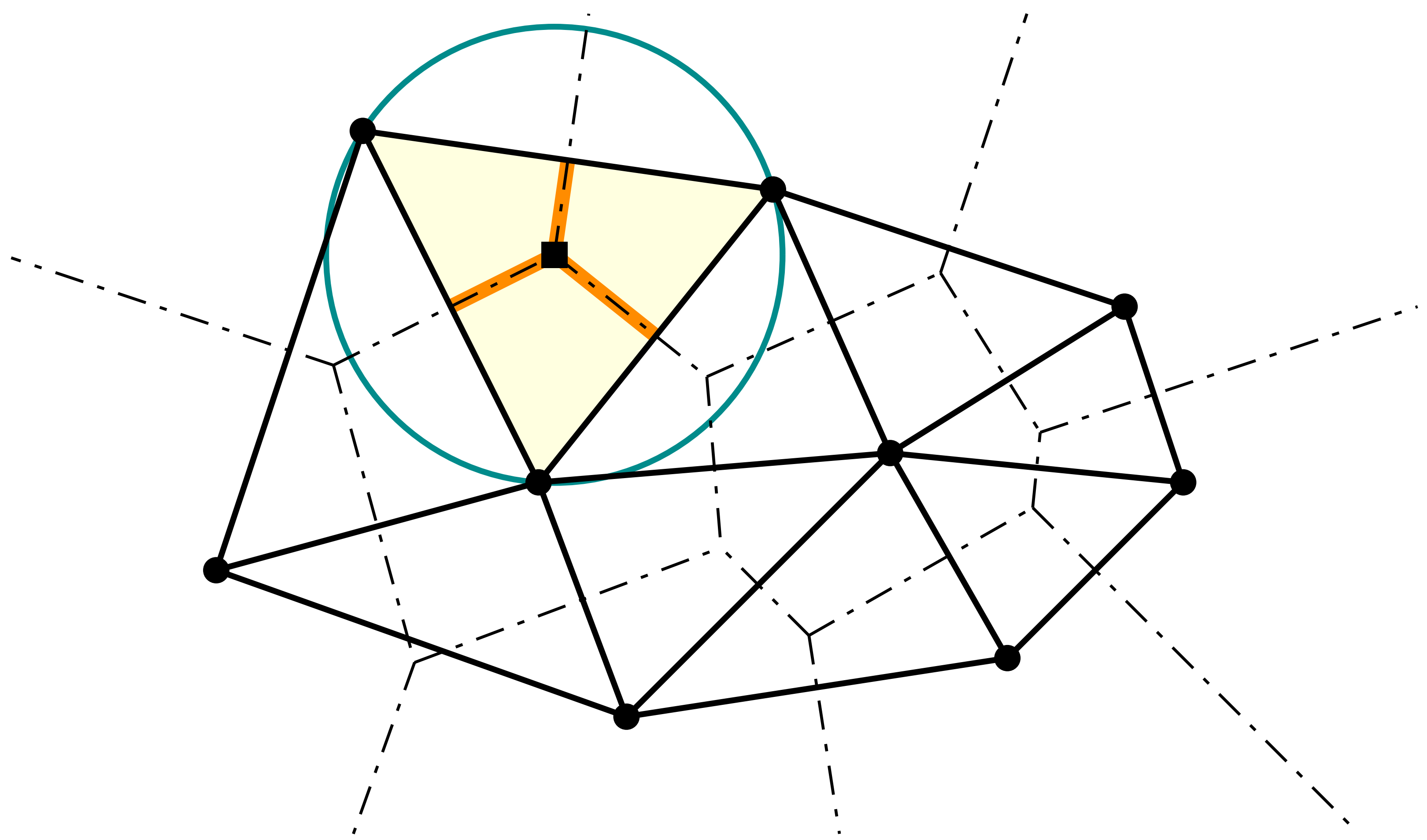

Polygon triangulations

Point Location Problem

(4) "Explain the concept of sweep-line algorithms for geometric problems[…]. What are its components and requirements? […] Name examples!"

Polygon triangulations

Point Location Problem

(4) *"Explain the concept of sweep-line algorithms for geometric problems[…]. What are its components and requirements? […]"*

- **Requirements:**
  - Sortable geometry in some sense (e.g. along an axis or angular) such that items have distinct "intervals of influence" along the sweep
  - Discrete, identifiable events defined based on discrete components.
    - "When does a geometric primitive become relevant to the sweep, when does it stop being relevant?"
    - "When do two (or more) geometric primitives interact and change the state?"
- **Components:**
  - Efficient data structure to track sweep line state (e.g., AVL Tree or constant-size state)
  - Ordered list of insertion and removal events to the state-tracking structure
  - Protocols to detect and handle interaction between components
  - Output structure that can efficiently be appended to