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Due 14.11.2024
Discussion 21.11.2024

Please submit your handwritten answers in pairs, *using the box in front of* IZ338 before *the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in.*

In accordance with the **guidelines** *of the TU Braunschweig, using AI tools to solve any part of the exercises is* ***not permitted***.

Exercise 1. **(5 points)**

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex, i.e., for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$,

$$\mathrm{conv}(\mathrm{conv}(\mathcal{P}) \cap \mathrm{conv}(\mathcal{Q})) = \mathrm{conv}(\mathcal{P}) \cap \mathrm{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). **(15 points)**

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \ldots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$ and a point $q \in \mathbb{R}^2$, does P contain q? You may assume that the point set $\{p_1, p_2, \ldots, p_n, q\}$ obeys general position.

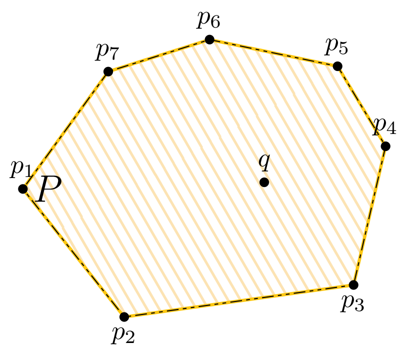

Figure 1: Example: the point q is located inside the convex polygon P.

Exercise 3 (Farthest Point Pairs). **(5+10 points)**

Given a set $\mathcal{P}$ of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in $\mathcal{P}$ if

$$\forall\, u, v \in \mathcal{P}: \quad |p - q| \;\geq\; |u - v|.$$

The Euclidean distance between p and q is then also called the *diameter* of $\mathcal{P}$.

a) Prove that all farthest pairs in $\mathcal{P}$ are vertices of the convex hull $\mathrm{conv}(\mathcal{P})$.

b) Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of $\mathcal{P}$ up to a constant factor. Argue its correctness, approximation factor, and runtime!