Computational Geometry – Sheet 2 **Winter 2024/2025**

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Due 28.11.2024
Discussion 05.12.2024

Please submit your handwritten answers in pairs, *using the box in front of IZ338* before *the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in. In accordance with the* ***guidelines*** *of the TU Braunschweig, using AI tools to solve any part of the exercises is* ***not permitted****.*

Exercise 1 (Geometric Predicates). (5 points)

Using only the leftTurn and rightTurn predicates from Lecture 1, design a geometric predicate for the Euclidean plane that decides whether a line segment $\overline{pq}$ intersects a triangle $\triangle(u, v, w)$:

$$\operatorname{conv}(p, q) \cap \operatorname{conv}(u, v, w) = \varnothing\ ?$$

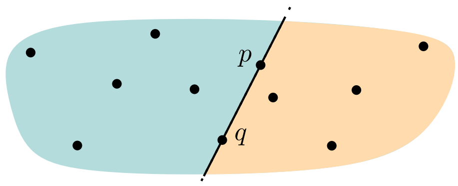
You may assume that (u, v, w) are in counterclockwise order and that no three points are collinear. Please explain your solution and briefly argue its correctness.

Exercise 2 (Partitioning Points). (15 points)

Consider a set $\mathcal{P}$ in the Euclidean plane $\mathbb{R}^2$ in general position according to *Definition E1.*

a) Prove that there exist points $p, q \in \mathcal{P}$ that divide $\mathcal{P}$ evenly based on left-/rightTurn:

$$|\{\, r \in \mathcal{P} \quad | \quad \text{leftTurn}(p, q, r) = \texttt{true} \,\}| = {}^{|\mathcal{P}|}/_{2} \pm 1.$$



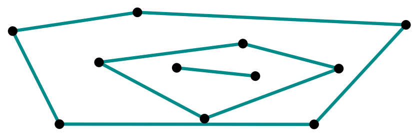
b) Design an algorithm that finds p and q in $\mathcal{O}(n)$ time for $n = |\mathcal{P}|$.

(Hint: Start with **b)**, a good correctness proof can also give you a constructive proof of existence.)

Exercise 3 (Convex layers). (10 points)

The *convex layers* of a finite point set $\mathcal{P}$ in the plane correspond to a decomposition of $\mathcal{P}$ into nested, convex polygons (*layers*). The outermost layer L_0 consists exactly of the extremal points defining $\operatorname{conv}(P)$. The next layer is recursively defined as points defining $\operatorname{conv}(P \setminus L_0)$, meaning

$$L_i = \mathcal{P} \cap \delta \operatorname{conv}\Big(\mathcal{P} \setminus \bigcup_{j \in [0,i]} L_j\Big).$$



Design an algorithm which computes the convex layers of n points in the Euclidean plane, in $\mathcal{O}(n^2)$ time. Briefly argue its runtime and correctness.