Computational Geometry – **Sheet 4** **Winter 2024/2025**

Prof. Dr. Sándor P. Fekete **Due** 09.01.2025

Peter Kramer **Discussion** 16.01.2025

Please submit your handwritten answers in pairs, *using the box in front of* `IZ338` before *the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in. In accordance with the* `guidelines` *of the TU Braunschweig, using AI tools to solve any part of the exercises is* ***not permitted****.*

This (final) homework sheet is centered entirely on polygon partitions, especially triangulations. A sum of 70 (i.e., 50%) of the total points across all sheets suffice to pass the coursework.

You may assume general position for all tasks on this sheet, i.e., that no three vertices of the polygons are collinear.

Exercise 1 (Convex vertices). (7 points)

Let P be any simple polygon of $n \geq 3$ vertices. Prove that P has at least three convex vertices.

Exercise 2 (Reflex vertices). (5 points)

Let P be any simple polygon with k reflex vertices. What is the *minimal* number of subpolygons into which P must be divided by cutting along diagonals such that each subpolygon is convex?

Exercise 3 (Triangulations of polygons with holes). (3+5 points)

Let P be any simple polygon with h holes, and let n be the total number of its vertices (including vertices of the holes). Find a formula for the number of triangles in a triangulation of P and prove its correctness.

Exercise 4 (Dual graphs of triangulations). (5 points)

The *dual graph* of a triangulation contains one vertex per triangle, and an edge between a pair of vertices exactly if the corresponding triangles share an edge.

Let P be any convex polygon. Prove or disprove: There exists a triangulation of P such that the dual graph is a path, i.e., every vertex of the dual graph has degree at most two.

Exercise 5 (Colorful triangulations). (8+4 points)

Let P be a convex polygon with n vertices $p_1, \dots, p_n$. We assign each vertex p_i one of two colors such that $c(p_i) \in \{\texttt{red}, \texttt{blue}\}$. A triangulation of P is then *colorful* exactly if every triangle contains at least one vertex of each color.

a) Prove that there exists a colorful triangulation for any coloring of P that uses both colors.

b) Show that this does not extend to general (simple) polygons.