

Computational Geometry

Tutorial #1 — Organisation and Convex Hulls

Organisation

Website

The screenshot shows a university website for the course 'Computational Geometry (Algorithmische Geometrie)'. The page is part of the Institute of Operating Systems and Computer Networks at TU Braunschweig. It features a navigation menu, a breadcrumb trail, and a sidebar with social media icons and a navigation menu. The main content area displays course details for the Winter 2024/2025 semester, including the lecturer Prof. Dr. Sándor P. Fekete, assistant Peter Kramer, and Hiwi Kai Kobbe. It also lists the number of credits (5), hours (2+1+1), and the time and place of lectures and tutorials.

Technische Universität Braunschweig

Study & Teaching Research International TU Braunschweig Organisation

Quicklinks DE EN

Organisation > Faculties > Carl-Friedrich-Gauß-Fakultät > Institutes > Institute of Operating Systems and Computer Networks > Courses > Winter 2024/2025

Institute of Operating Systems and Computer Networks

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^ About us

^ Connected and Mobile Systems

^ Reliable System Software

^ Algorithms

Microprocessor Lab

^ Education

Winter 2024/2025

Summer 2024

Theses

^ Services

Spin-Offs

^ Research Cooperations

Semester Winter 2024/2025

Programmes Computer Science Master, Business Information Systems Master

IBR Group ALG (Prof. Fekete)

Type Lecture & Exercise

Lecturer

Prof. Dr. Sándor P. Fekete
Abteilungsleiter
✉ s.fekete@tu-bs.de
☎ +49 531 3913111
📍 Room 335

Assistant

Peter Kramer
Wissenschaftlicher Mitarbeiter
✉ kramer@ibr.cs.tu-bs.de
☎ +49 531 3913113
📍 Room 332

Hiwi

Kai Kobbe
✉ kobbe@ibr.cs.tu-bs.de

Credits 5

Hours 2+1+1

Time & Place Lecture: Tuesdays, 3:00 p.m. - 4:30 p.m. (Room SN 19.2) ☺
Plenary tutorial: Thursdays, 3:00 p.m. - 4:30 p.m. (Room PK3.4)

Start The first lecture will be held on 22.10.2024 in SN 19.2.
The first plenary tutorial will be held on 24.10.2024 in PK 3.4.

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Institute of Operating Systems and Computer Networks

Computational Geometry (Algorithmische Geometrie)

Semester Winter 2024/2025

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Material

We provide video recordings of the lecture from 2021 below. These alone cannot replace attendance in the **ongoing semester**: The course structure may differ.

Topic

Lecture Material

References

Introduction

[PDF] [Video] ↗

Postorius Sign In Sign Up

CG cg@ibr.cs.tu-bs.de

Summary

Computational Geometry - WS 2024/2025

To contact the list owners, use the following email address: cg-owner@ibr.cs.tu-bs.de

You have to sign in to visit the archives of this list.

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Subscribe

Tutorial

- **Every Thursday at 3pm** in this room, either a big or a small tutorial.
 - Depending on attendance, we might switch rooms.
- **Big tutorial:** Expand upon and put concepts from the lecture to use.
- **Small tutorial:** Homework discussion.

October						
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

November						
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

- 13 Lectures
- 7 Big Tutorials
- 4 Small Tutorials
- 4 Sheets

December						
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

January						
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Exam prep?

Tutorial

- **Every Thursday at 3pm** in this room, either a big or a small tutorial.
 - Depending on attendance, we might switch rooms.
- **Big tutorial:** Expand upon and put concepts from the lecture to use.
- **Small tutorial:** Homework discussion.
- **Studienleistung:** Four homework sheets.
 - Biweekly assignments to solve in pairs.
 - 50% of total possible points on the sheets; roughly 70 - 75 points.

Exam...

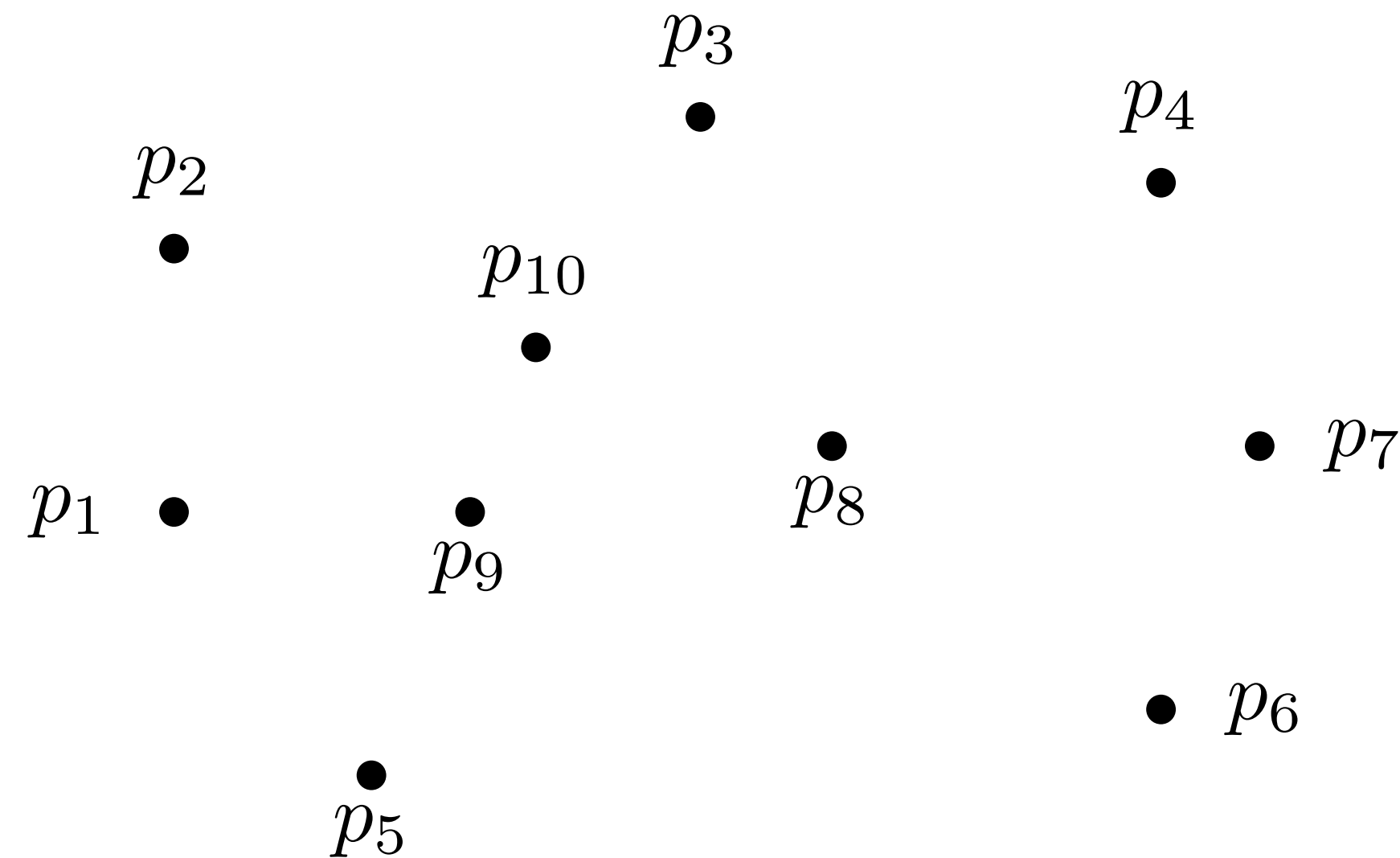
**The form of exam depends on the number of attending students.
Announcement in an upcoming lecture.**

Convex Hulls

The convex hull

... is a point set? A convex polygon? A subset?

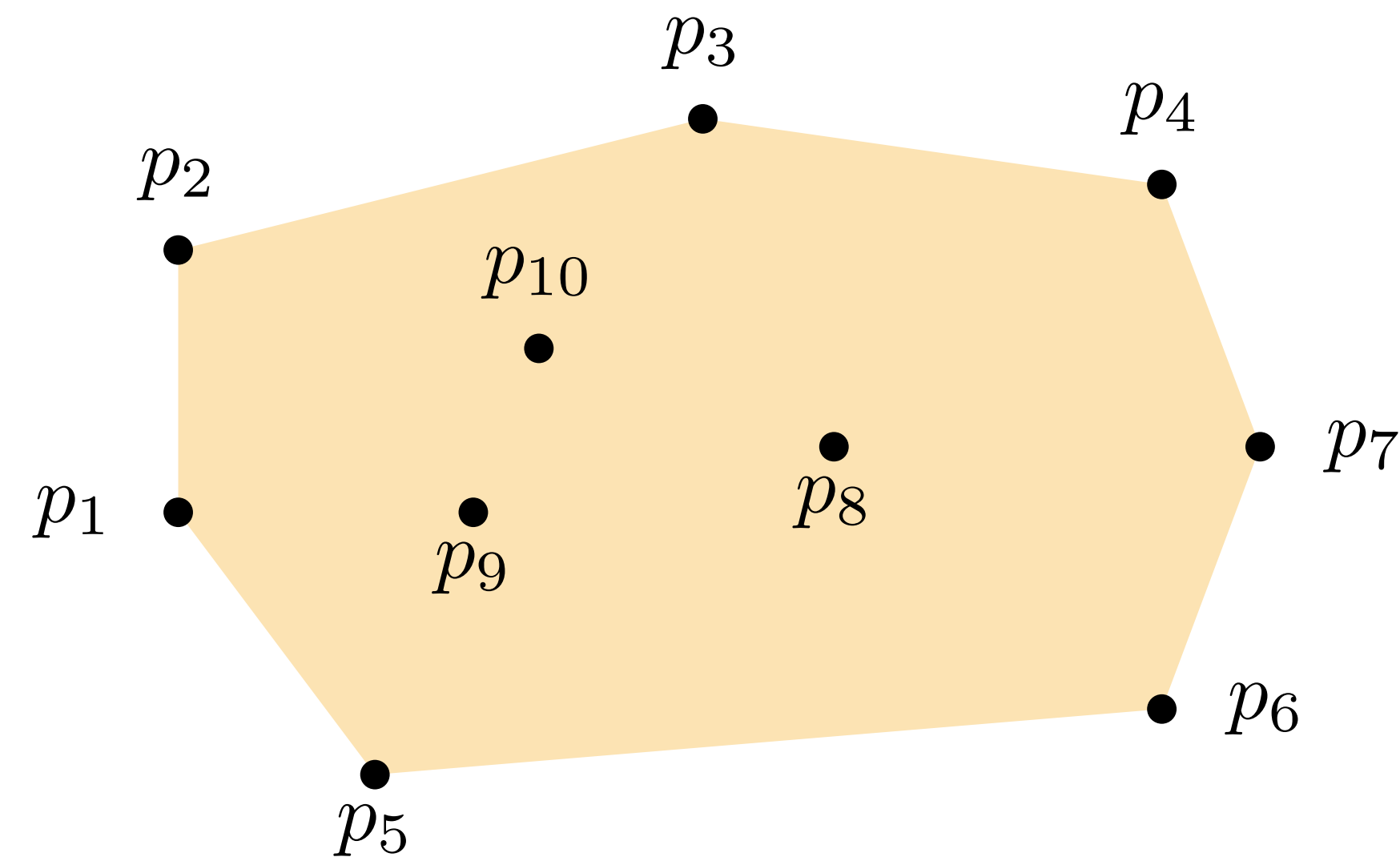
- For a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$, $\text{conv}(\mathcal{P}) = \dots?$



The convex hull

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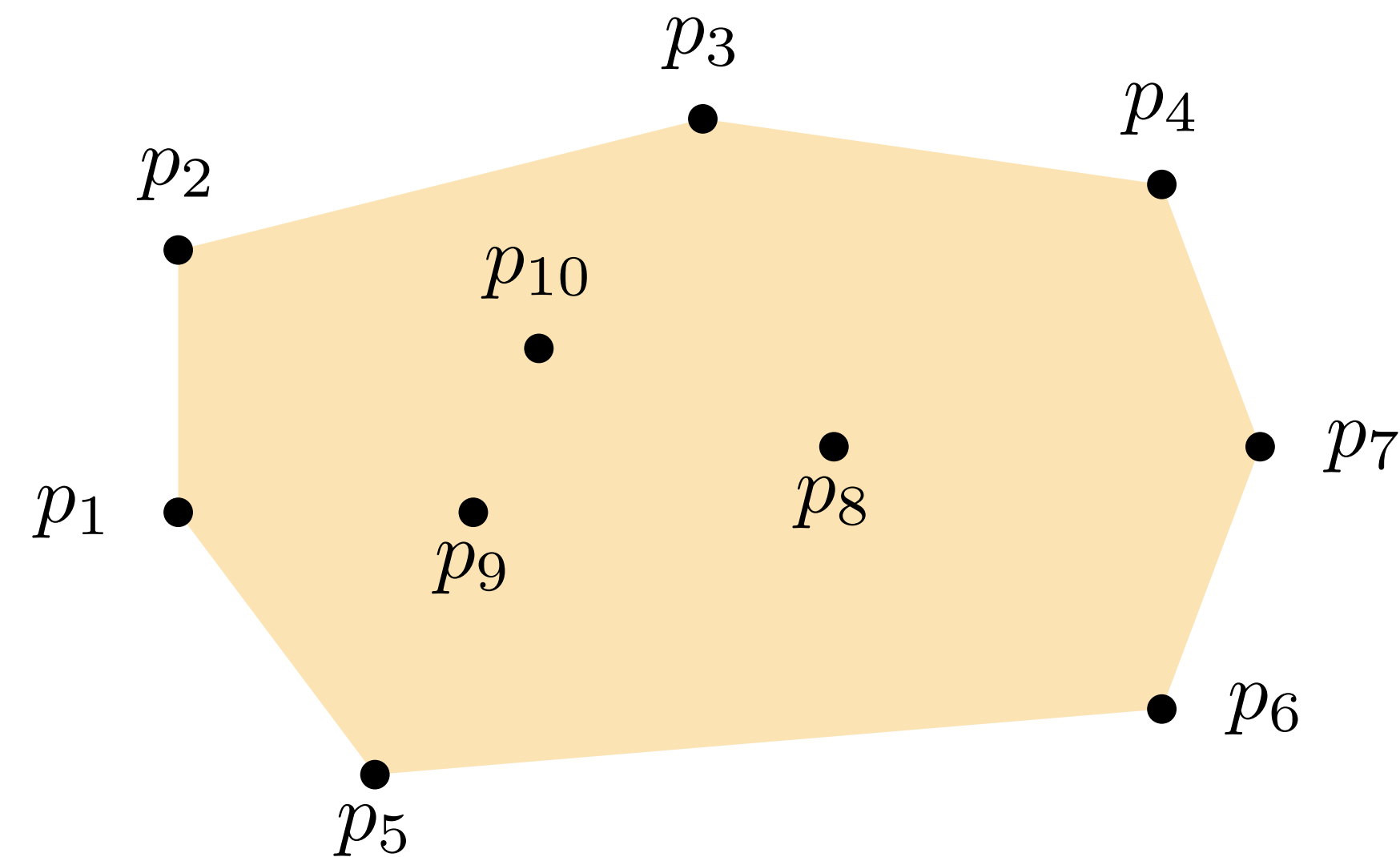
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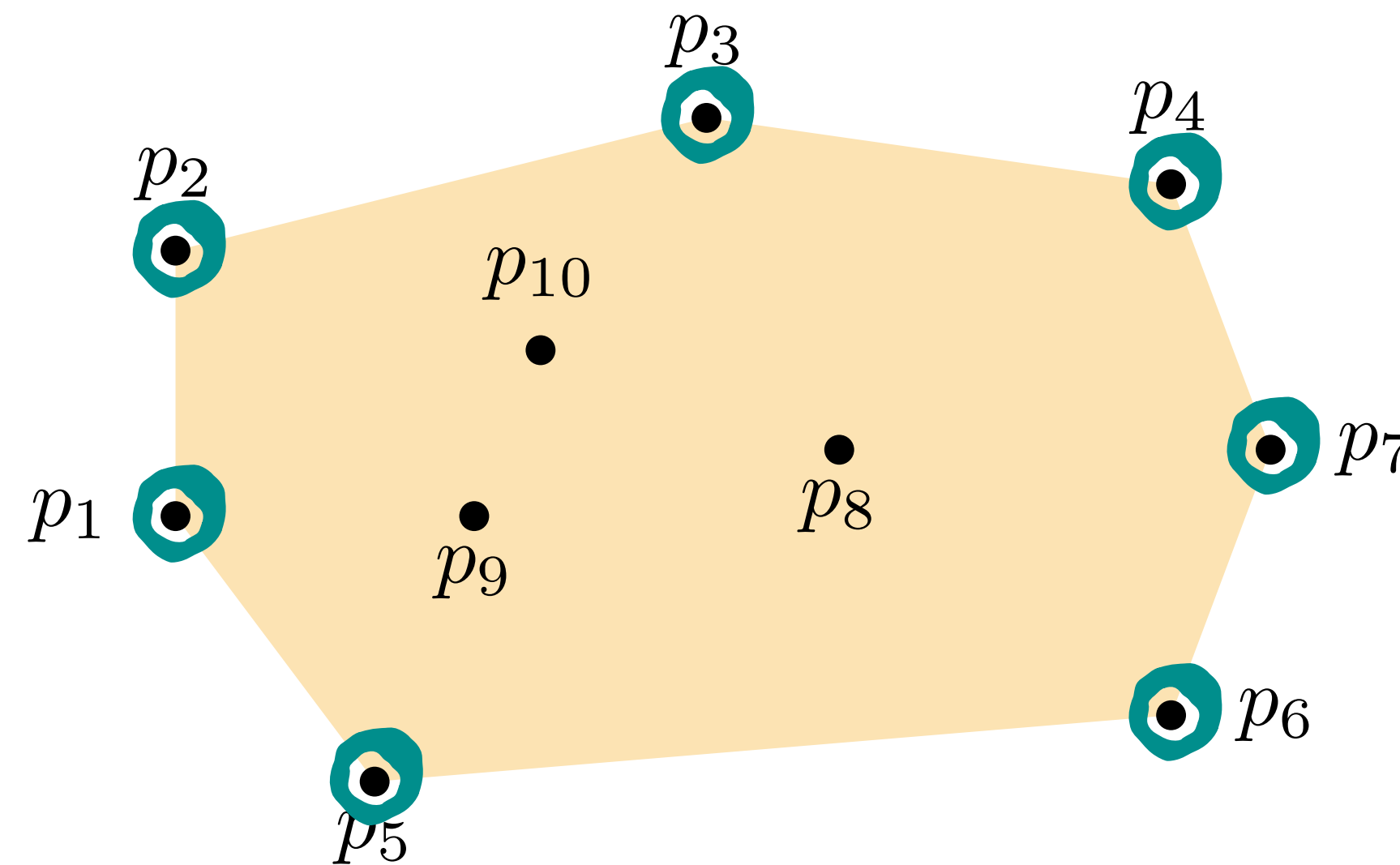
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- *Definition 2.3:* $\text{conv}(\mathcal{P}) = \{x \in \mathbb{R}^2 \mid x \text{ is a convex combination of } \mathcal{P}\}$.



The convex hull

... is a point set? A convex polygon? A subset?

- For a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$, $\text{conv}(\mathcal{P}) = \dots$?
- *Definition 2.3:* $\text{conv}(\mathcal{P}) = \{x \in \mathbb{R}^2 \mid x \text{ is a convex combination of } \mathcal{P}\}$.
- *This is an infinite set, but our algorithm(s) only give us points from \mathcal{P} !?*

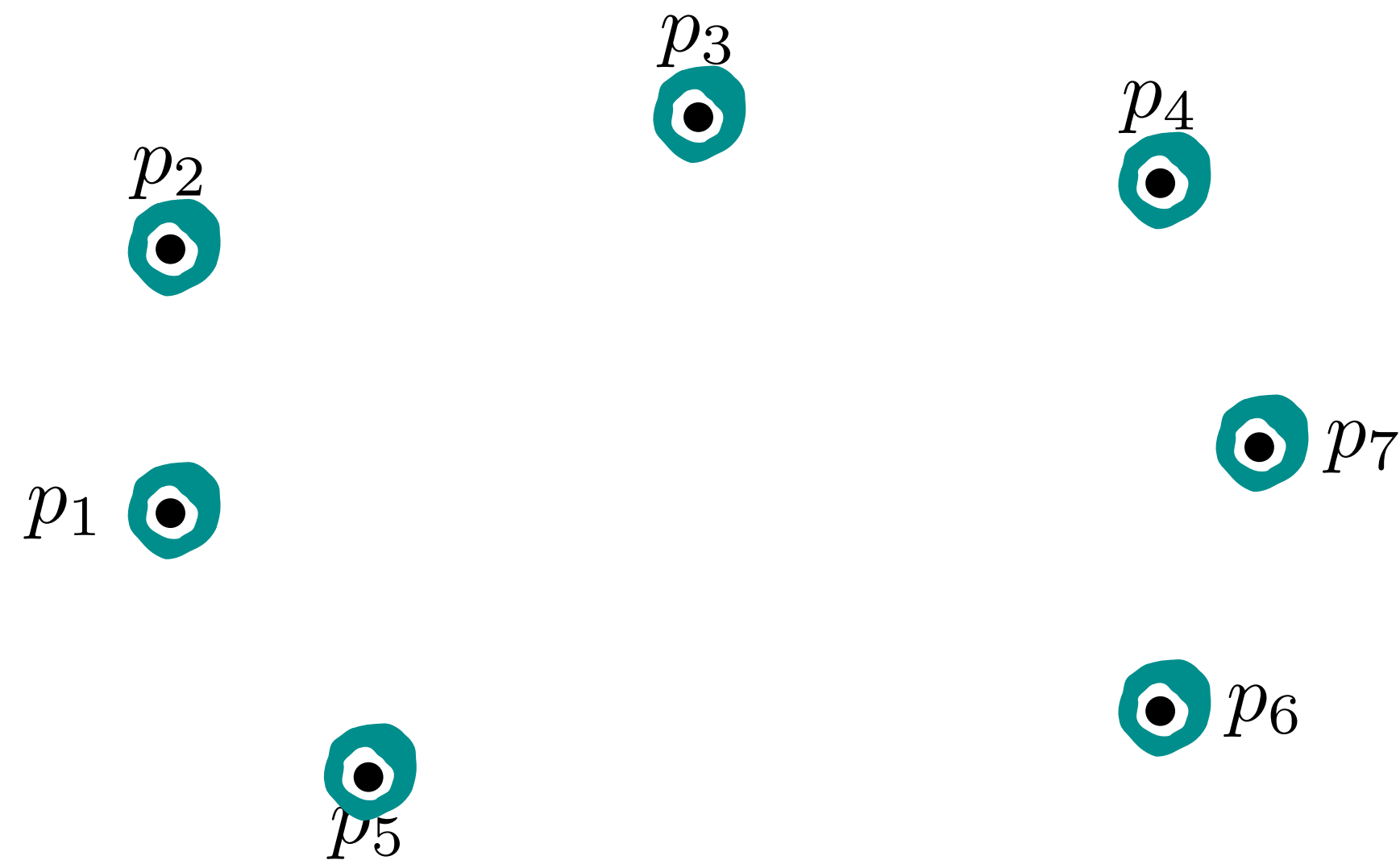


The convex hull

... is a point set? A convex polygon? A subset?

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- $\text{conv}(\mathcal{P}) = \mathcal{P}$???

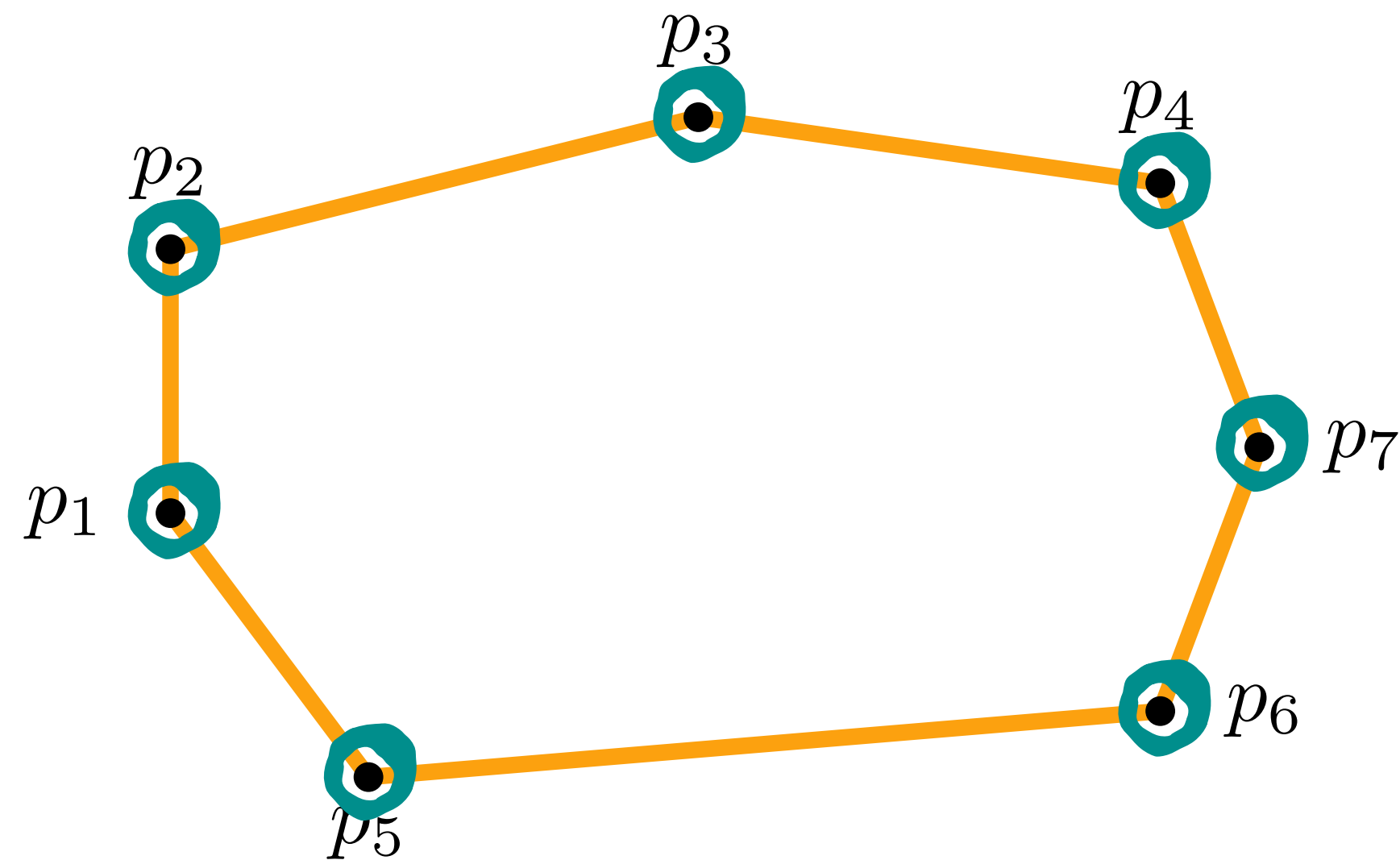
Point sets like this one
are referred to as being
in “convex position”.



The convex hull

... is a point set? A convex polygon? A subset?

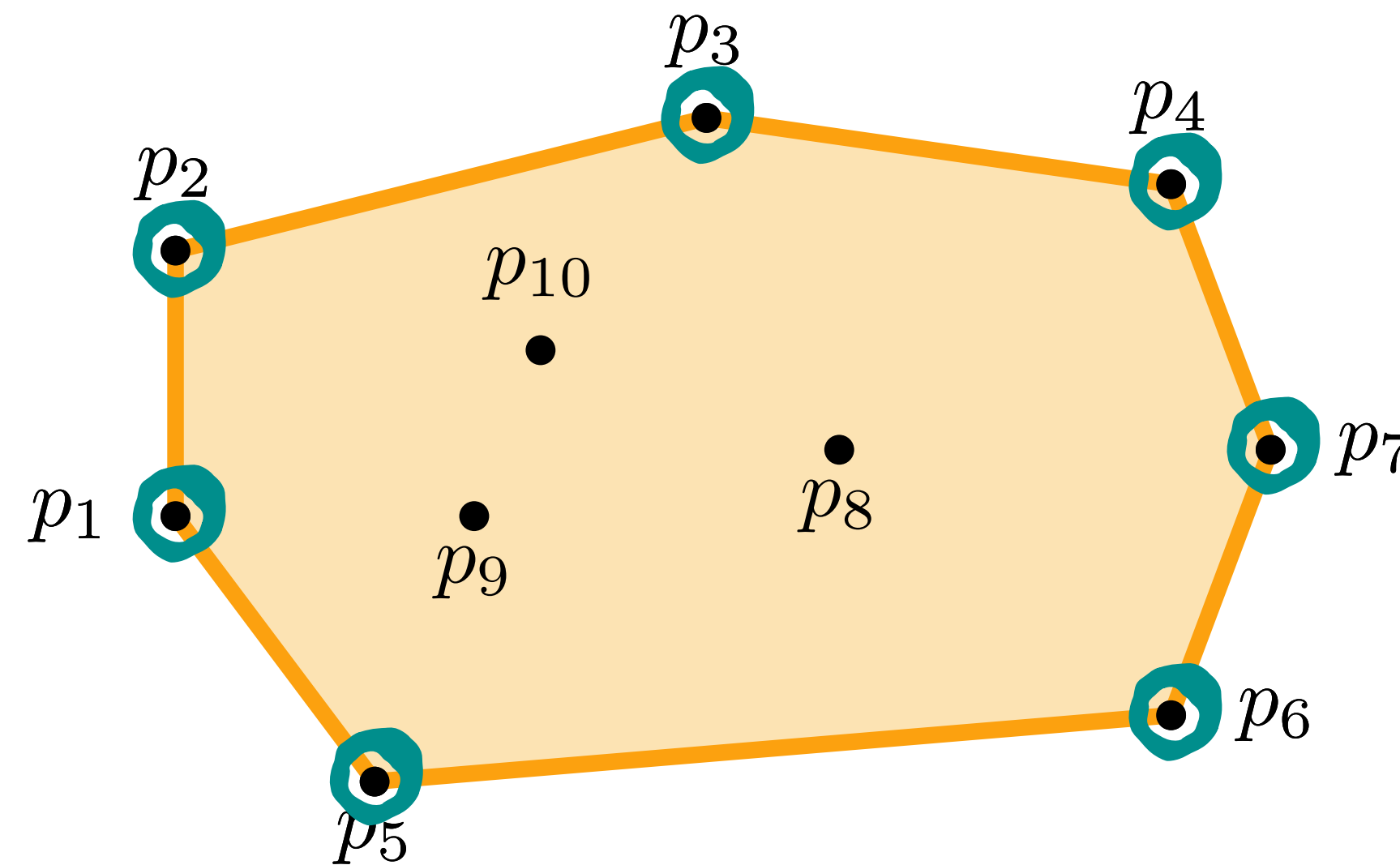
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- *This is an infinite set, but our algorithm(s) only give us points from \mathcal{P} !?*
- $\text{conv}(\mathcal{P}) \stackrel{\text{lightning bolt}}{\neq} \mathcal{P} \text{ ???}$



The convex hull

... is a (usually infinite) point set, described by a convex polygon.

- Given a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
- *Definition 2.3:* $\text{conv}(\mathcal{P}) = \{x \in \mathbb{R}^2 \mid x \text{ is a convex combination of } \mathcal{P}\}$
- *Our algorithms compute the boundary set $\delta \text{conv}(\mathcal{P})$ as a convex polygon!*

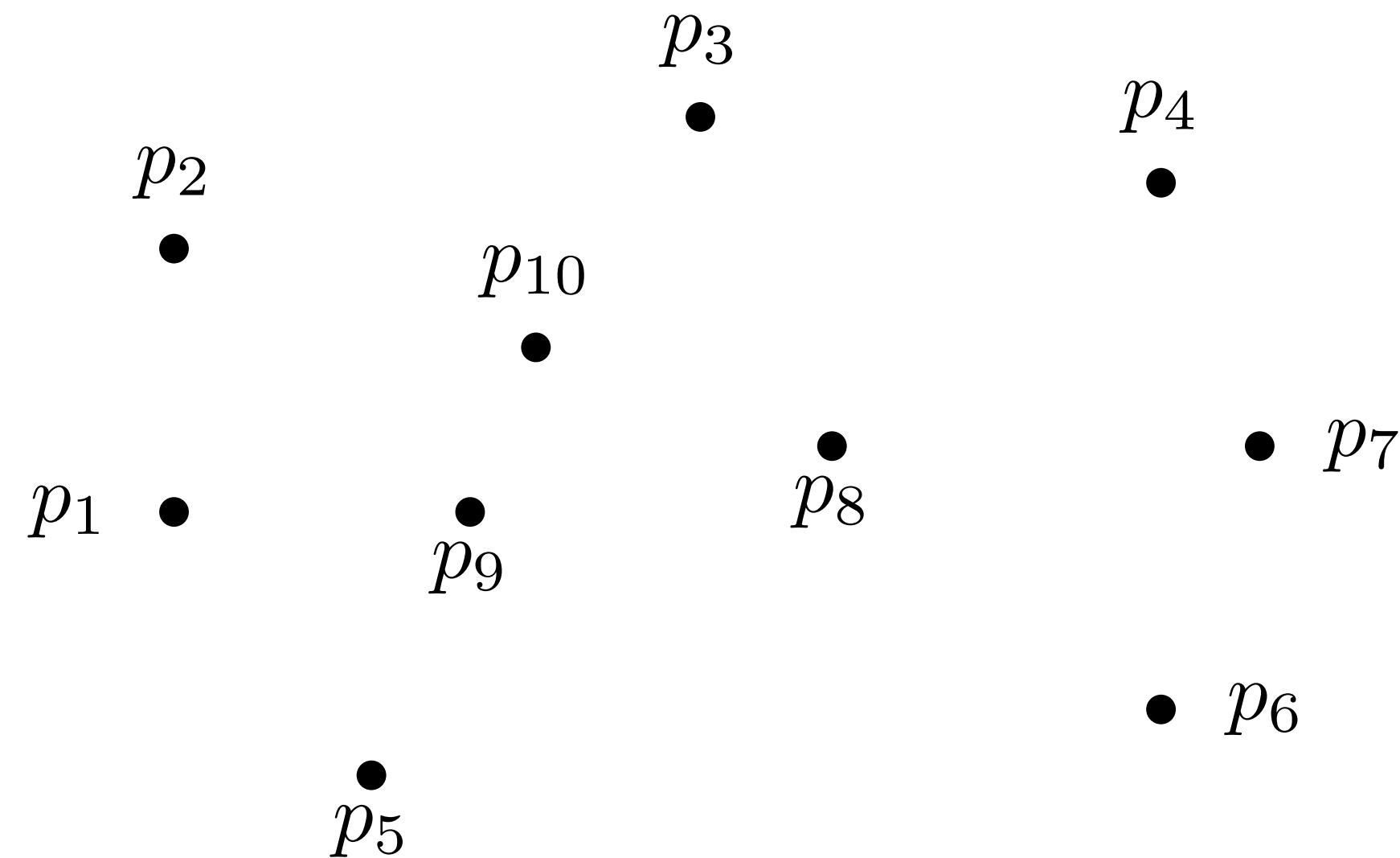


Detour: Point sets & Polygons

Point sets & Polygons

What's the difference?

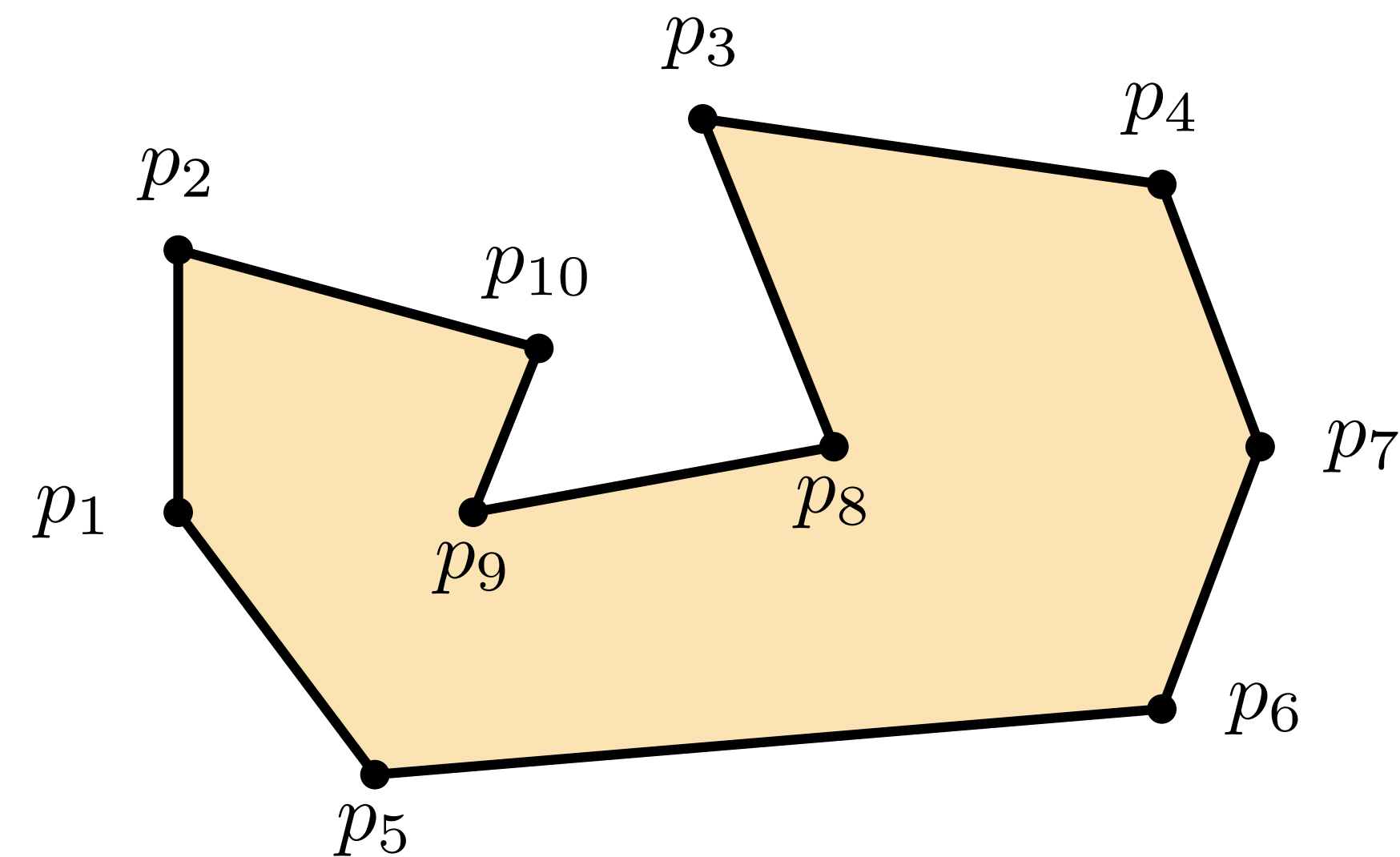
- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
- What's a polygon P using these points?



Point sets & Polygons

What's the difference?

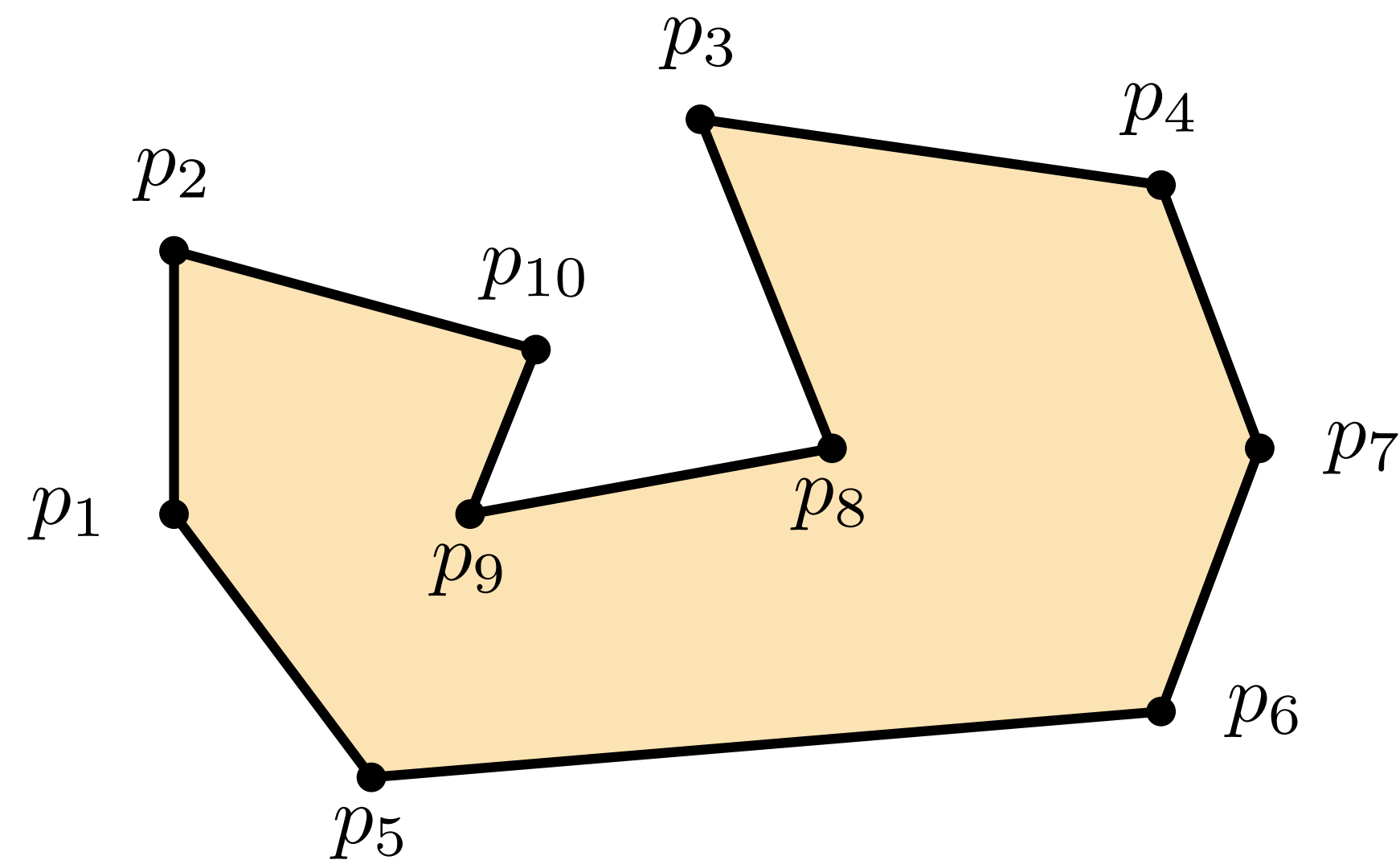
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Point sets & Polygons

What's the difference?

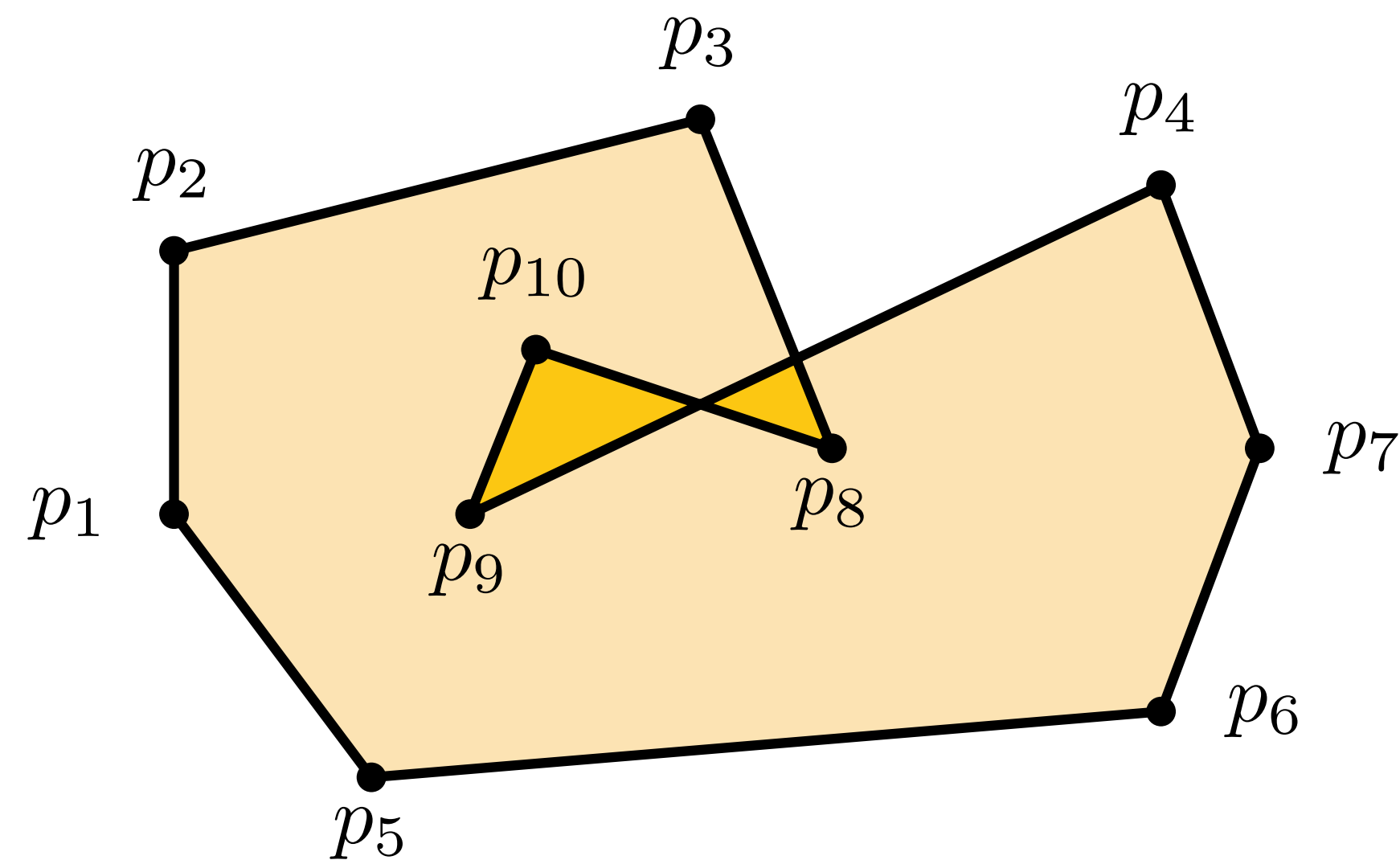
- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
- An ordered sequence $P := p_{\pi(1)}, p_{\pi(2)}, \dots, p_{\pi(n)} \in (\mathbb{R} \times \mathbb{R})^n$ is a polygon.



Point sets & Polygons

What's the difference?

- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
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Point sets & Polygons

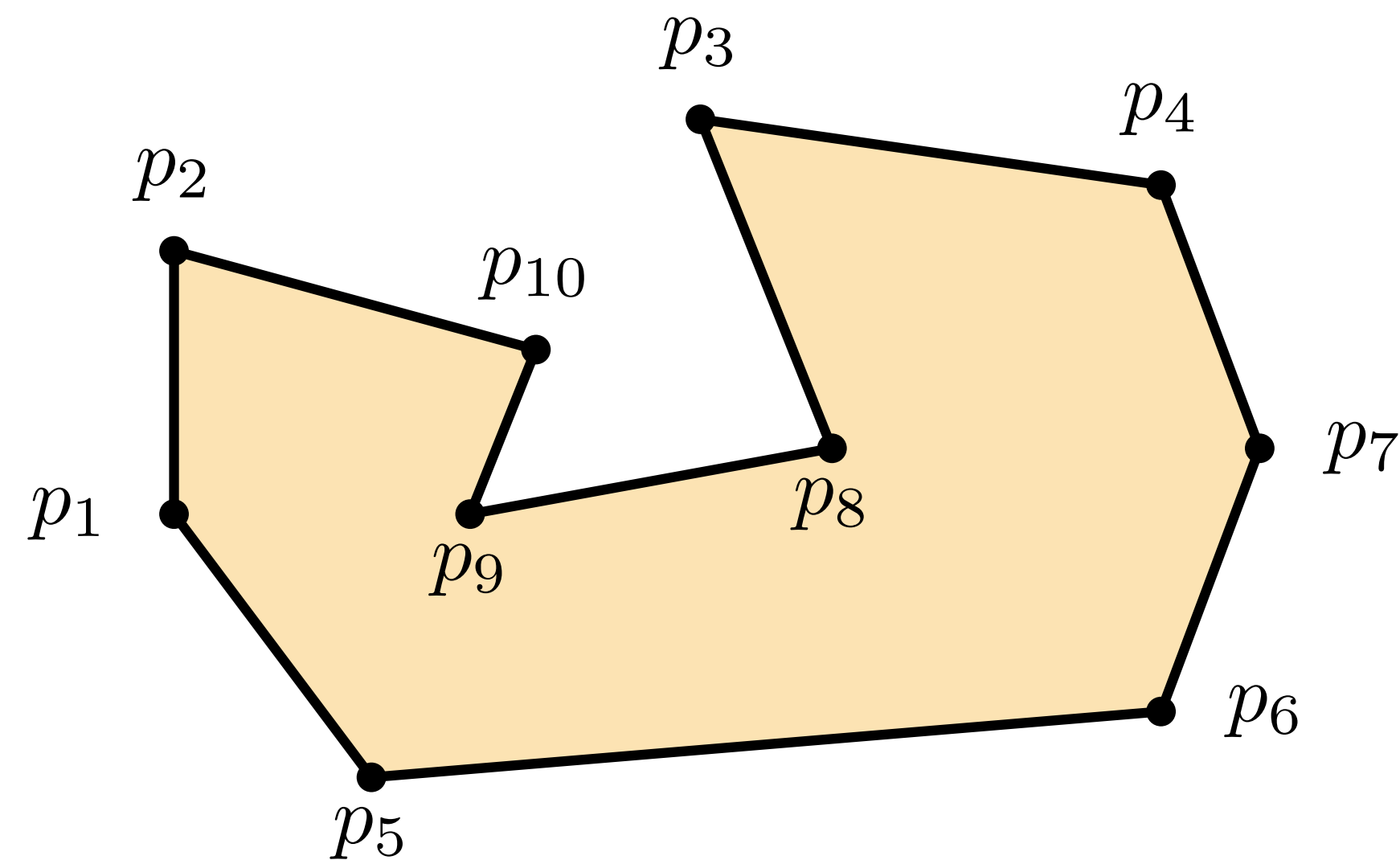
What's the difference?

- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
- A *simple* polygon P does not self-intersect.

Note: We will encounter further types of polygons later on :)

Typically, we describe polygons in CCW order:

$(p_1, p_5, p_6, p_7, p_4, p_3, p_8, p_9, p_1, p_2)$



Homework Sheet #1

Please submit your handwritten answers in pairs, using the box in front of IZ338 before the exercise timeslot on the due date above. Make sure to include your full names, matriculation numbers, and the programmes that you are enrolled in.

In accordance with the *guidelines* of the TU Braunschweig, using AI tools to solve any part of the exercises is **not permitted**.

Exercise 1. (5 points)

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$, i.e.,

$$\text{conv}(\text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q})) = \text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edge cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). (15 points)

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \dots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$ and a point $q \in \mathbb{R}^2$, does P contain q ? You may assume that the point set $\{p_1, p_2, \dots, p_n, q\}$ obeys general position.

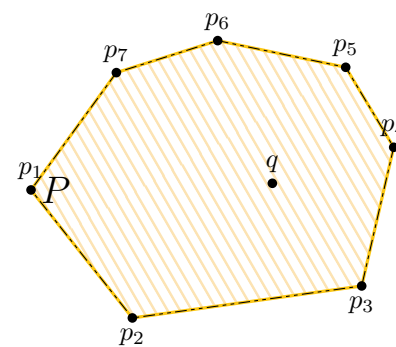


Figure 1: Example: the point q is located inside the convex polygon P .

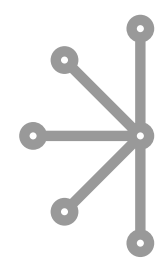
Exercise 3 (Farthest Point Pairs). (5+10 points)

Given a set \mathcal{P} of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in \mathcal{P} if

$$\forall u, v \in \mathcal{P}: |p - q| \geq |u - v|.$$

The Euclidean distance between p and q is then also called the *diameter* of \mathcal{P} .

- a) Prove that all farthest pairs in \mathcal{P} are vertices of the convex hull $\text{conv}(\mathcal{P})$.
- b) Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of \mathcal{P} up to a constant factor. Briefly explain approximation factor and runtime!

 **Computational Geometry – Sheet 1**
Prof. Dr. Sándor P. Fekete
Peter Kramer

Three weeks!

Winter 2024/2025

Due 14.11.2024
Discussion 21.11.2024

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These sheets are about logical deduction, so LLMs will most likely just generate plausible-sounding garbage.

Please save us the headache :)

Translation services such as DeepL are allowed, of course. If you so prefer, you may write your solutions in german.

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Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \dots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$ and a point $q \in \mathbb{R}^2$, does P contain q ? You may assume that the point set $\{p_1, p_2, \dots, p_n, q\}$ obeys general position.

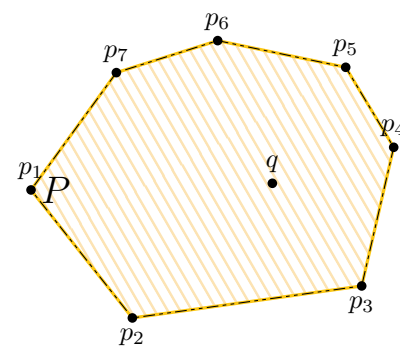


Figure 1: Example: the point q is located inside the convex polygon P .

Exercise 3 (Farthest Point Pairs). (5+10 points)

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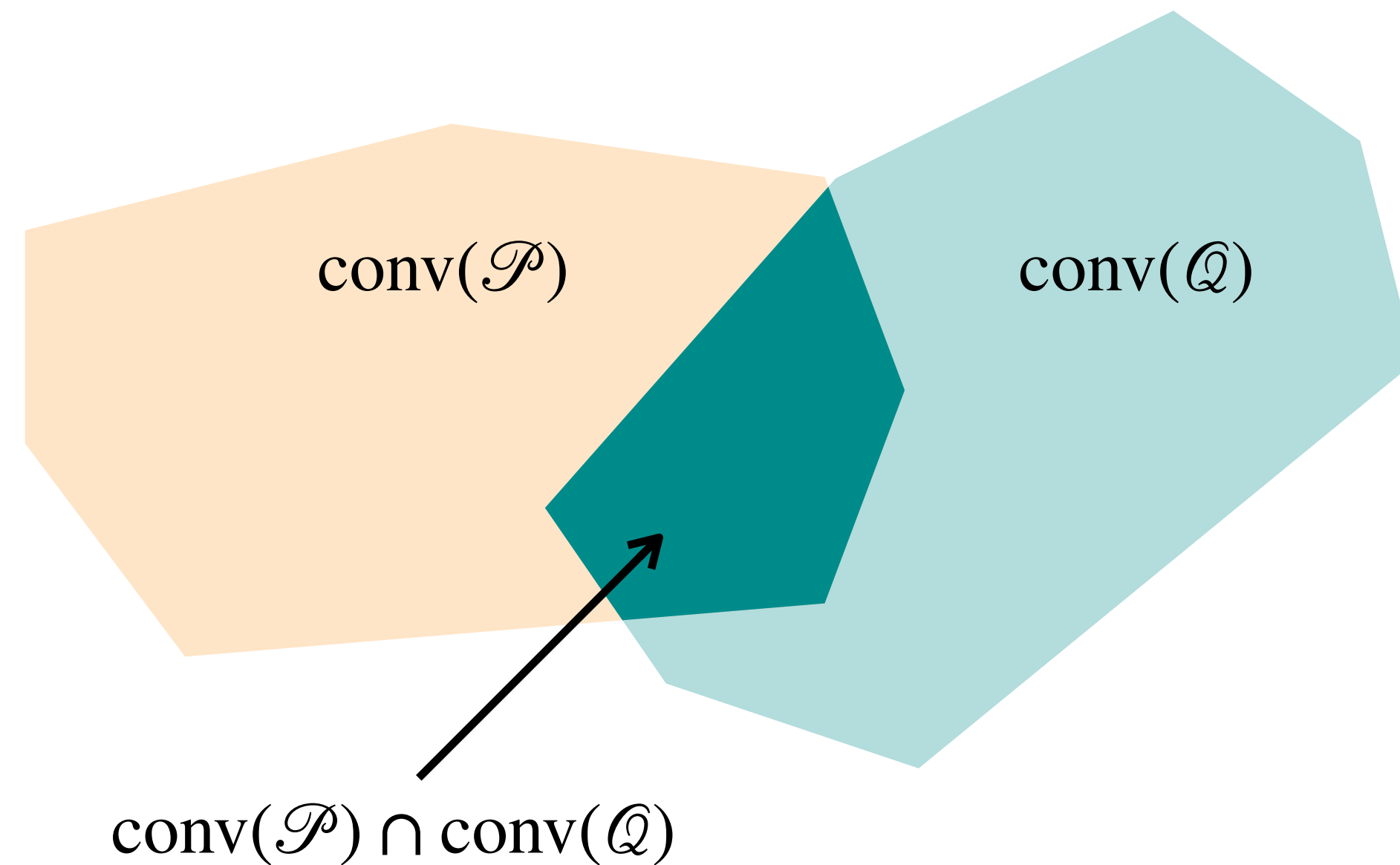
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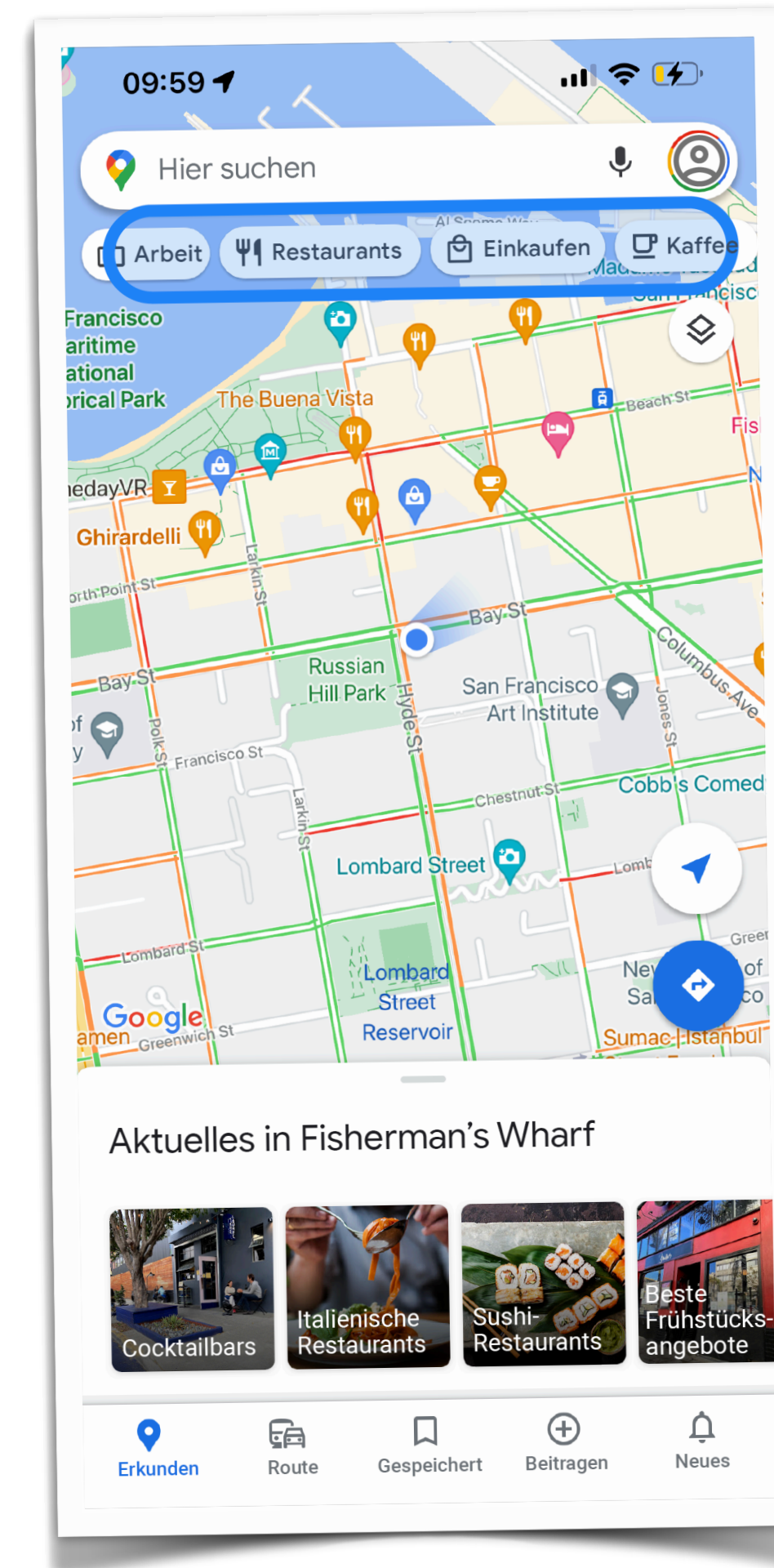
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Point Location Problems

“Where am I?”

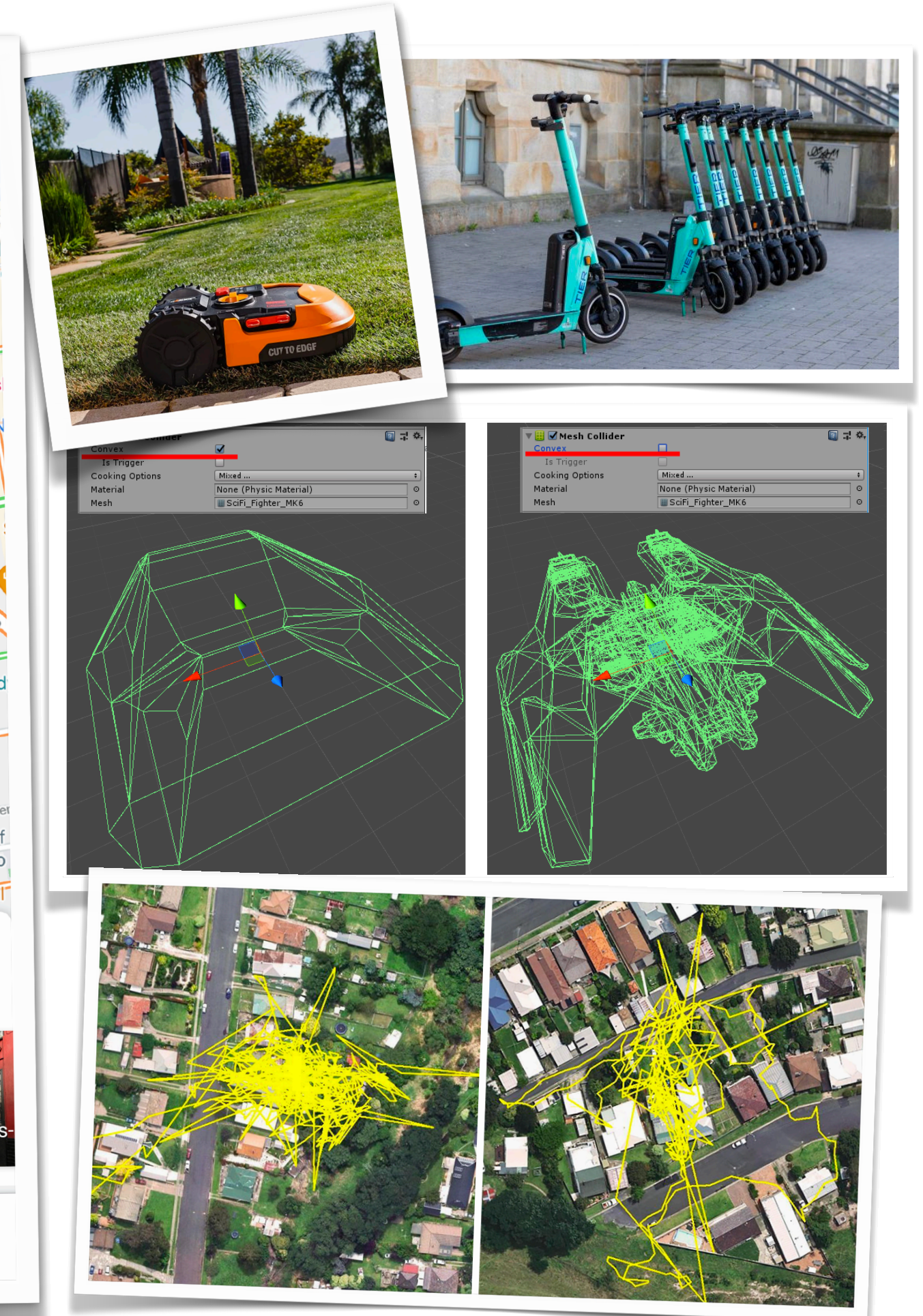
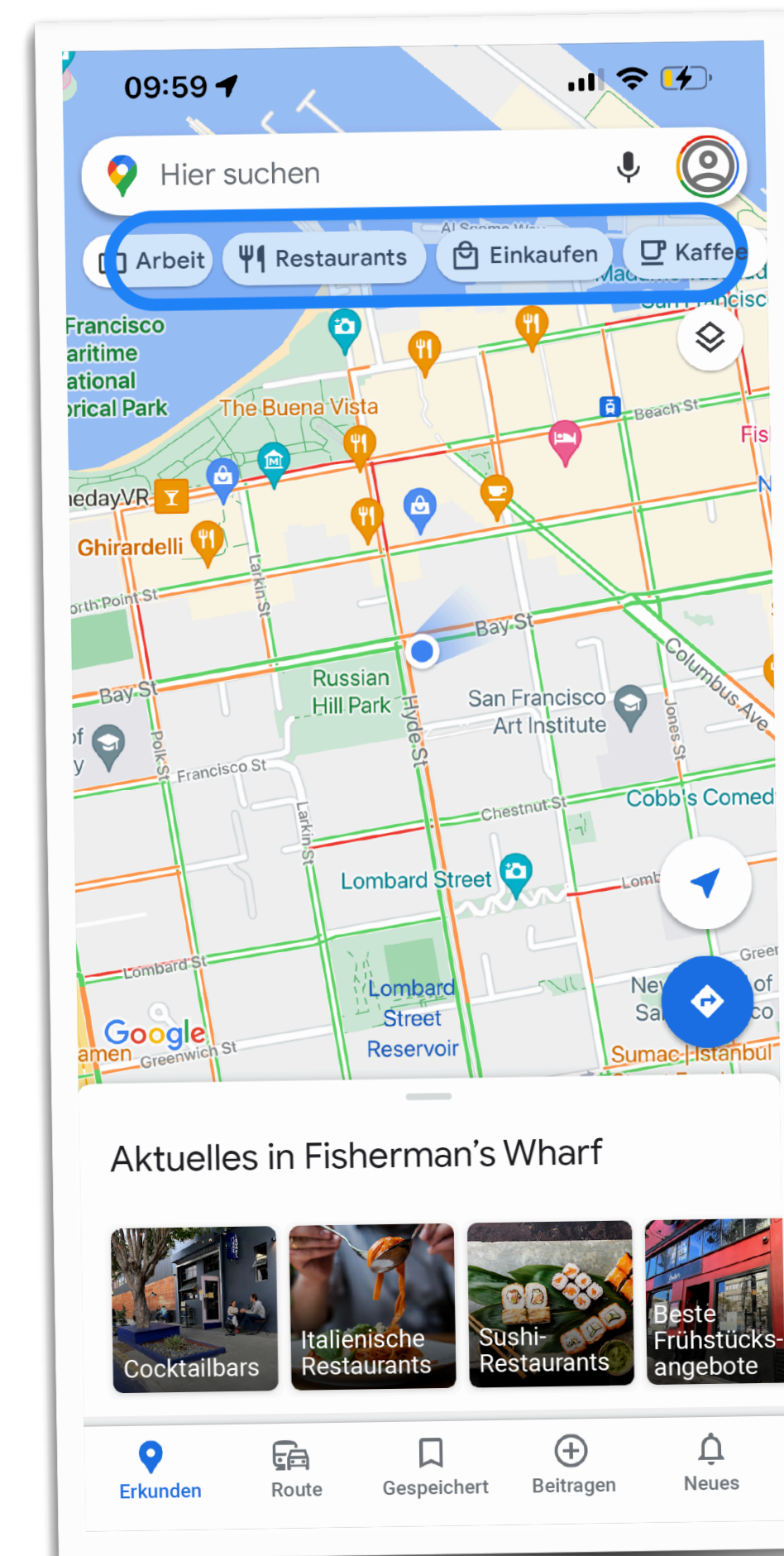
- Given geometric information such as a map in the plane, how can we decide **where** we are?



Point Location Problems

“Where am I?”

- Given geometric information such as a map in the plane, how can we decide **where** we are?
- “*In which country am I right now?*”
“*Can I leave this scooter here?*”
“*Do these virtual objects collide?*”
- “*Is point p inside of region P ?*”



Applications: Geofencing, Navigation, Simulation Software, Outlier Detection, ...



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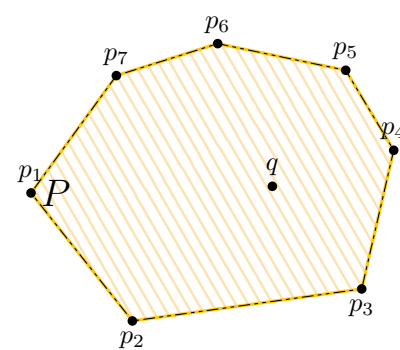


Figure 1: Example: the point q is located inside the convex polygon P .

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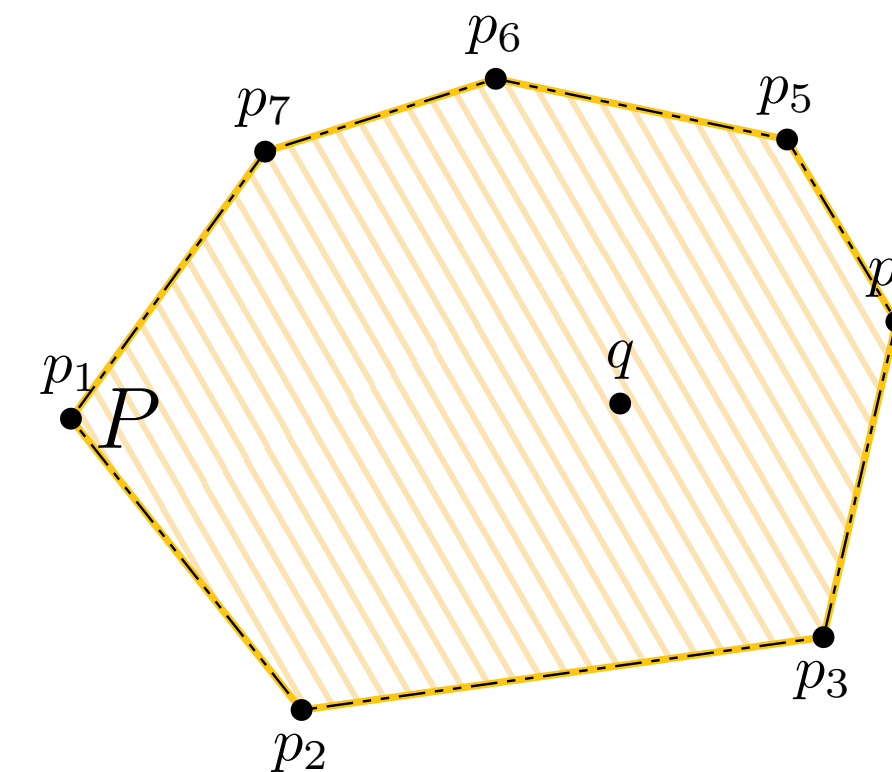


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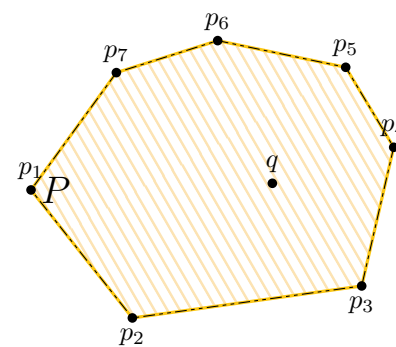


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The Euclidean distance between p and q is then also called the *diameter* of \mathcal{P} .

- Prove that all farthest pairs in \mathcal{P} are vertices of the convex hull $\text{conv}(\mathcal{P})$.
- Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of \mathcal{P} up to a constant factor. Briefly explain approximation factor and runtime!

Exercise 3 (Farthest Point Pairs).

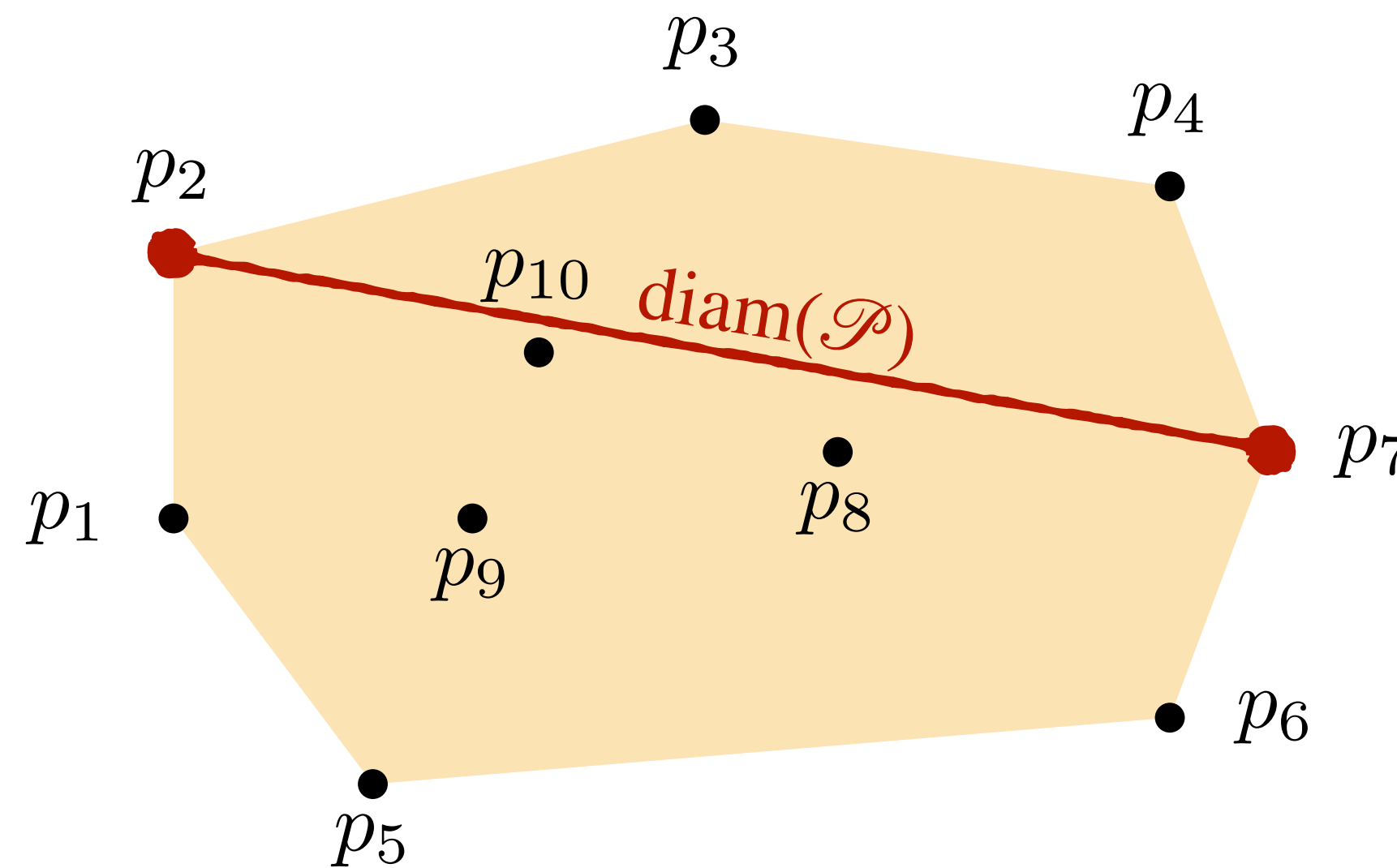
(5+10 points)

Given a set \mathcal{P} of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in \mathcal{P} if

$$\forall u, v \in \mathcal{P} : |p - q| \geq |u - v|.$$

The Euclidean distance between p and q is then also called the *diameter* of \mathcal{P} .

- Prove that all farthest pairs in \mathcal{P} are vertices of the convex hull $\text{conv}(\mathcal{P})$.
- Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of \mathcal{P} up to a constant factor. Briefly explain approximation factor and runtime!



For a c -approximation, argue that your algorithm's output is

- at most $c \cdot \text{diam}(\mathcal{P})$, and
- at least $\frac{1}{c} \cdot \text{diam}(\mathcal{P})$.

TL;DL

The important parts

- **Sign up for the mailing list!**
- One tutorial a week (except holidays).
- Biweekly homework: **Find partners!**
- First **sheet is out now**, due in 3 weeks.
- Material can be found on the website.
 - Slides, videos, and homework sheets.
- Any questions? Ask Peter or Kai:
 - kramer@ibr.cs.tu-bs.de
 - kobbe@ibr.cs.tu-bs.de

October						
28	29	30	31			

November						
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

Extra slides

Big O Notation

Catching up, if necessary.

- **If you speak German:** There is plenty of material on the course website of “Algorithmen und Datenstrukturen”
- **If you don't:** There is plenty of other courseware (e.g., from MIT) available!
- **There also exist a variety of short-form explanation videos, such as:**

